Does Voluntary Certification Promote Sustainability Investment?

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June 20, 2026

Abstract

Despite the prevalence of third-party sustainability certification, empirical evidence on whether such certification improves actual sustainability performance remains mixed. Motivated by this puzzle, we develop a game-theoretic model in which a firm chooses whether to make a costly sustainability investment and whether to pursue certification before selling in a regulated market. Consumers observe only price and certification status, but neither the firm's investment level nor its certification attempt. Certification plays two strategic roles: signaling sustainability to consumers and helping the firm mitigate regulatory risk before market entry. We show that lowering certification cost can either encourage or discourage investment. The latter occurs when regulation is sufficiently stringent: the firm may use certification as a substitute for investment, relying on its risk-mitigation effect rather than proactively improving sustainability. Our results suggest that policymakers carefully assess the joint impact of third-party certification and the regulatory environment. When regulation is already stringent, policymakers may achieve better outcomes by improving certification accuracy and consumer awareness than by reducing certification costs.

Key words: sustainability investment, certification, signaling game.

1. Introduction

Voluntary sustainability certification has become increasingly prevalent as firms face growing pressure from consumers, regulators, and other stakeholders to demonstrate responsible environmental and social practices. For example, ISO 14001 certifies environmental management systems, ENERGY STAR certifies energy-efficient products and buildings, and Forest Stewardship Council (FSC) certifies responsible forest management and sourcing. Such certifications are widely used across industries to verify and communicate the sustainability attributes of firms' processes and products, with the global certification market projected to reach $4.25 billion by 2030 (PR Newswire 2024). However, empirical evidence suggests that adoption of third-party certification is not always associated with improved sustainability performance (Yin and Schmeidler 2009, Ferrón-Vílchez 2016) and there is a decoupling between certification adoption and substantive improvement (Aravind and Christmann 2011).[1]

[1] Aravind and Christmann (2011) empirically show that ISO 14001-certified facilities do not significantly outperform uncertified facilities in environmental performance unless certified firms ensure high-quality implementation.

To understand the equivocal impact of certification, we distinguish between two strategic effects of certification.

(i) Certification has a *signaling* effect for consumers. Sustainability investment is often difficult for consumers to observe directly. Third-party certification can reduce this information asymmetry by providing a signal of the firm's sustainability level. This allows firms to strengthen consumer trust and capture a price premium from a sustainability-conscious market. For instance, ISO 14001 enables firms to demonstrate a commitment to environmental responsibility and enhances reputation with customers (International Organization for Standardization 2026). Manufacturing firms can use the ENERGY STAR label to signal energy efficiency and environmental benefits of their products. See the second column of Table 1 for additional examples and the sustainability attributes signaled by each certification.

(ii) Sustainability-related certification also has a *risk-mitigation* effect. Although such certifications are often voluntary, going through the certification process helps a firm assess its violation risk when selling in a regulated market. Because environmental and social compliance requirements are often complex and vary by region, a firm may remain uncertain whether its product or production process fully satisfies the relevant standards, even after undertaking internal process improvements. For instance, ISO 14001 "provides tools to identify, assess, and manage environmental risks" and helps "organizations to avoid potential fines, penalties, and legal actions" (International Organization for Standardization 2026, "Regulatory compliance" bullet in FAQ). See the third column of Table 1 for specific examples and the regulatory risks that each certification helps mitigate.

In this paper, we develop an analytical model to study how these two distinct effects jointly shape a firm's incentive to invest in sustainability improvement. Specifically, we consider a firm that chooses whether to make a costly sustainability improvement and whether to pursue third-party certification. Selling a noncompliant product in the market subjects the firm, if caught, to an external regulatory penalty. Depending on whether it is certified, the firm then sets its selling price and may choose not to enter the focal market upon certification failure.[2] Consumers have a higher willingness to pay for a compliant product, but when making purchases, they observe only the product's selling price and certification status, *without* observing the firm's sustainability investment or whether it ever attempted to obtain certification.

The unobservability of both the firm's investment and certification choices leads to a challenging signaling game with an endogenous type. Using a novel solution method, we fully characterize the

[2] Because we focus on voluntary certifications, a firm that fails certification may still continue selling in the market, albeit with greater regulatory risk. For example, ENERGY STAR certification imposes energy-efficiency requirements that are typically 10–30% stricter than the U.S. Department of Energy's mandatory standards (Agency 2021). Thus, products that are not ENERGY STAR certified can still be sold in the US market (WebstaurantStore 2024); however, failing to meet ENERGY STAR standards may indicate a higher risk of falling short of mandatory energy-efficiency requirements and therefore facing noncompliance penalties.

Table 1 Examples of Sustainability Certifications and Their Signaling and Risk-Mitigation Effects

Certification	Attribute Signaled	Compliance Risk Mitigated
FSC	Responsible forest management; sustainable sourcing	Deforestation and timber sourcing regulations, e.g., EUDR
ENERGY STAR	Energy efficiency; reduced energy consumption	Energy-efficiency standards for appliances, buildings, and equipment
Fair Trade	Fair labor practices; ethical sourcing; producer welfare	Human rights and supply chain due diligence requirements, e.g., CSDDD
ISO 14001	Environmental management systems; environmental responsibility	Industrial emissions, wastewater discharge, and hazardous waste management requirements, e.g., the EU Industrial Emissions Directive
ISO 45001	Occupational health and safety management	Workplace health and safety regulations, e.g., EU OSH Framework Directive
OEKO-TEX Standard 100	Textile chemical safety; absence of harmful substances	Chemical safety restrictions, e.g., REACH

firm's investment and certification decisions in equilibrium. Our main result is that a lower certification cost may either encourage or discourage firm investment. The latter unintended outcome occurs when the external regulation is sufficiently stringent. Through its signaling effect, certification helps the firm capture investment returns as a sustainability premium, thereby encouraging investment. By contrast, through its risk-mitigation effect, certification may dampen investment incentives by allowing the firm to assess violation risk without improving and then condition market entry on the certification outcome. This effect dominates when regulatory stringency is sufficiently high because the market-entry flexibility provided by certification becomes increasingly beneficial. We also show that this unintended consequence can be alleviated by improving certification accuracy and consumer awareness, and that greater regulatory penalties do not always encourage investment.

From managerial and policy perspectives, our research offers important implications. First, we show that sustainability investment and certification may act as substitutes rather than complements when firms use certification to mitigate market-entry risk, instead of proactively improving their sustainability performance. This finding offers an explanation for empirical observations that certification has mixed effects on actual sustainability performance and that certification adoption is often decoupled from substantive improvements (Aravind and Christmann 2011). By identifying when certification discourages investment, our analysis suggests that policymakers should carefully assess the joint impact of third-party certification and the local regulatory environment. In already stringent regulatory environments, resources may be better directed toward improving certification accuracy and consumer awareness rather than reducing certification costs. Moreover, stronger punitive regulation may not always induce sustainability investment when firms view certification and investment as substitutes.

Methodologically, we develop an approach tailored to signaling games with endogenous type choice and noisy signals. Although developed for certification status in our model, the approach may apply to other settings with similar informational frictions.

2. Literature Review

There is an extensive literature on promoting sustainable and socially responsible operations (Lee and Tang 2018, Sunar and Swaminathan 2022). Existing studies have examined various instruments, such as standards-setting policies (Wang et al. 2021), strategic sourcing (Agrawal and Lee 2019, Tang and Song 2023), supply chain financing (Chen et al. 2023), carbon offsetting (Gao and Souza 2022), as well as monitoring-based interventions, including third-party assessment (Nguyen et al. 2019), environmental inspections (Kim 2015), and social audits of labor and workplace compliance (Plambeck and Taylor 2016, Caro et al. 2018, Chen et al. 2020, Fang and Cho 2020).

Among this broad literature, the strand on sustainability certification is most closely related to our work. Chen and Lee (2017) compare certification, auditing, and deferred payment mechanisms for inducing suppliers to adopt sustainable practices from a buyer's perspective. Different from their supply chain setting, we study a single firm's endogenous choices of sustainability improvement and voluntary certification, where certification influences the belief of end consumers. Murali et al. (2019) examine how voluntary certification and mandatory environmental regulation shape green product development by competing firms when consumers discount self-declared environmental claims. Guo et al. (2017) examine how a firm uses price and information-sharing mechanisms, including third-party certification, to signal its exogenously given and privately known social responsibility type to consumers. Our work differs from Murali et al. (2019) in that we endogenize the credibility of voluntary certification through consumers' Bayesian beliefs about firm investment and certification attempts, while abstracting from competition. We depart from Guo et al. (2017) by allowing the firm to endogenously choose its private sustainability level and then signal this choice through price and certification. To our knowledge, this is the first paper to develop a signaling-game model in which a firm privately chooses both its sustainability level and certification strategy, whereas traditional signaling games typically treat private information as exogenous.

Certification has also been studied in broader quality contexts. For example, Hwang et al. (2006) compare voluntary quality inspection and mandatory certification in procurement. Chen and Deng (2013) show that noisy certification can mitigate underinvestment by efficient suppliers. Li et al.

(2023) study platform-provided inspection to certify seller authenticity and show that inspection affects sellers' price-signaling incentives beyond merely detecting counterfeits. In the context of generic drug manufacturing, Swinney et al. (2024) study reliability certification that provides information about manufacturers' supply disruption risk and examine its implications for reliability investment. Guan et al. (2025) examine two distinct quality disclosure mechanisms on online selling platforms, platform certification and seller disclosure. Unlike certification of product or supply quality, our paper is motivated by sustainability-related certification with a distinct role: it helps firms assess and mitigate regulatory penalty risk in regulated markets. This risk-mitigation effect is central to how certification shapes a firm's sustainability investment incentives.

From a modeling and methodological perspective, our work relates to the literature on signaling a firm's private type through instruments beyond pricing, including advertising (Milgrom and Roberts 1986), quality provision (Guo and Jiang 2016), price and quantity guarantee (Gümüş et al. 2012), after-sales service contracting (Bakshi et al. 2015), crowdfunding campaign (Chakraborty and Swinney 2021) and, in socially responsible operations, supplier self-reporting (Lu and Tomlin 2022) and greenwashing (Wei et al. 2026). Our model differs from this literature in that the firm's private type is endogenously chosen, as in In and Wright (2018), but is also intertwined with a noisy certification signal. This nonstandard signaling environment introduces additional complexity and requires a tailored approach to the equilibrium analysis.

3. The Model

3.1. Sustainability Investment

We consider the firm's sustainability level, denoted by θ_x, as the probability that no sustainability violation occurs, where $x \in \{H, L\}$ represents the firm's investment level in sustainability and $0 \leq \theta_L < \theta_H \leq 1$. As such, $x = H$ and $x = L$ represent high and low sustainability choices, respectively, resulting in higher and lower probabilities of compliance with environmental and sustainability regulations. Modeling sustainability as a binary decision is standard in the literature and reflects the practical choice of whether to adopt a sustainable technology or production process (Agrawal and Lee 2019, Wang et al. 2023). A higher sustainability level entails a higher investment cost. Let $c_H > c_L$ denote the costs associated with the high and low sustainability choices, respectively. For ease of exposition, we normalize $c_L = 0$ and refer to the two choices as "investment" and "no investment." Thus, the firm invests if it chooses $x = H$ and incurs cost c_H, and does not invest if it chooses $x = L$ at zero cost. Equivalently, $x = L$ can be interpreted as the firm's current sustainability level, and the firm decides whether to improve from this level, for example by adopting a cleaner technology, at cost c_H. A firm with sustainability level x knows θ_x, but cannot ascertain whether it is entirely free from violation risk unless $\theta_x = 1$. This setting is common in the literature and reflects the practice that firms may invest to reduce violation risk, while environmental and social responsibility standards may still be inadvertently violated (e.g., due to imperfect technology).

3.2. Sustainability Certification

The firm can pursue a third-party certification. Let $y \in \{A, N\}$ represent the firm's certification decision, where $y = A$ if the firm chooses to apply for the certification and $y = N$ otherwise. If the firm applies for certification, regardless of whether it passes or fails, it incurs a cost c_A, which includes administrative expenses and fees charged by the certifier.[3]

Certifications are often imperfect. Following the literature (Chen and Lee 2017), we assume that if a violation exists, the certifier detects it with probability $t > 0$, in which case the firm fails certification; with probability $1-t$, the certifier overlooks the violation and the firm mistakenly passes certification. If no violation exists, the firm always passes certification. We refer to t as certification accuracy. As such, the probability of a firm with investment level x passing the certification is given by $\gamma_x = \theta_x + (1-\theta_x)(1-t)$. Consequently, a firm with a higher investment level is more likely to obtain certification, that is, $\gamma_L < \gamma_H$.

In practice, consumers are typically unaware of whether a firm has attempted to obtain certification. Instead, they observe only the firm's certification status. We define S as the firm's certification status, where $S = 1$ indicates that the firm is certified and $S = 0$ indicates that it is uncertified. Let f_S denote the probability distribution of S, which is determined by the firm's joint decisions on $(x, y) \in Z$ where we define $Z = \{H, L\} \times \{A, N\}$ as the joint strategy space. We can write $f_S(x, y)$ as

$$S = 1 \text{ with probability } f_1(x,y) = \begin{cases} 0 & \text{if } y = N, \\ \gamma_x & \text{if } y = A, \end{cases} \tag{1}$$

$$\text{and } S = 0 \text{ with probability } f_0(x,y) = 1 - f_1(x,y). \tag{2}$$

Thus, $S = 1$ only when the firm applies for certification ($y = A$) and passes (which occurs with probability γ_x).

Based on the certification status S, its investment and certification decisions (x, y), the firm updates its belief regarding its violation risk. By Bayes' rule, the violation probability is given by

$$\psi(x,y,S) = \begin{cases} \frac{(1-t)(1-\theta_x)}{\theta_x + (1-t)(1-\theta_x)}, & \text{if } S = 1, \\ 1, & \text{if } y = A \text{ and } S = 0, \\ 1 - \theta_x, & \text{if } y = N. \end{cases} \tag{3}$$

[3] For example, a small to medium-sized enterprise applying for ISO 14001 Certification typically incurs an initial cost ranging from \$8,000 to \$25,000. This non-refundable amount covers the external auditor's fees, as well as significant internal or consulting costs for preparation (Group 2025).

Since $\frac{(1-t)(1-\theta_x)}{\theta_x+(1-t)(1-\theta_x)} < 1-\theta_x$, passing the certification implies lower violation risk. In contrast, applying for certification and failing, i.e., $y=A$ and $S=0$, implies noncompliance with probability one. As formalized below, based on $\psi(x,y,S)$, the firm determines its pricing decision, with the option of exiting the market. Therefore, our model captures an important effect of certification discussed in the introduction: certification helps firms identify potential violations and thereby mitigate regulatory penalty risk (International Organization for Standardization 2026).

3.3. Consumer Utility

The market consists of a unit mass of infinitesimal consumers. Consumers have a baseline valuation v for the product. In addition, they are willing to pay a premium r if the product (or its production process) complies with sustainability standards.[4] To elucidate our primary results, we focus on a base model in which all consumers are sustainability-conscious, willing to pay the premium r for a compliant product. In §5.2, we examine a more general case in which only a proportion of consumers in the market are sustainability-conscious.

A major challenge in modeling is the fact that consumers do not directly observe the firm's investment level or whether the firm has attempted to apply for certification, that is, both the firm's decisions (x,y) are *unobservable*. We therefore adopt the Perfect Bayesian Equilibrium (PBE) as our solution concept. By the definition of PBE, consumers hold a given belief structure, which is consistent with the firm's optimal choices of (x,y) on equilibrium path.

Let μ_{xy} denote consumers' on-path belief that the firm chooses some $(x,y)\in Z$, and we write $\boldsymbol{\mu}=(\mu_{HA},\mu_{HN},\mu_{LA},\mu_{LN})$ as the belief vector. Given any $\boldsymbol{\mu}$ and according to Bayes' rule, consumers update their belief about the violation probability based on the product's certification status S on the equilibrium path. Upon observing a certified product $(S=1)$, the no-violation probability is calculated as

$$\vartheta_1(\boldsymbol{\mu})=\frac{\mu_{HA}\gamma_H}{\mu_{HA}\gamma_H+\mu_{LA}\gamma_L}\frac{\theta_H}{1-(1-\theta_H)t}+\frac{\mu_{LA}\gamma_L}{\mu_{HA}\gamma_H+\mu_{LA}\gamma_L}\frac{\theta_L}{1-(1-\theta_L)t}.$$

The first and second terms correspond, respectively, to the cases in which the certified product has a high sustainability level and a low sustainability level.[5] Similarly, upon observing an uncertified status $(S=0)$, the no-violation probability is given by

$$\vartheta_0(\boldsymbol{\mu})=\frac{\mu_{HN}\theta_H}{\mu_{HA}(1-\gamma_H)+\mu_{LA}(1-\gamma_L)+\mu_{HN}+\mu_{LN}}+\frac{\mu_{LN}\theta_L}{\mu_{HA}(1-\gamma_H)+\mu_{LA}(1-\gamma_L)+\mu_{HN}+\mu_{LN}},$$

[4] For instance, according to a 2024 survey by PwC, over 80% of consumers indicated they would pay more for sustainably produced goods, with an average premium of 9.7% (PricewaterhouseCoopers 2024).

[5] Note that $\vartheta_1(\boldsymbol{\mu})$ is defined *only* on the equilibrium path. For an equilibrium in which the firm does not apply for certification, belief consistency requires $\mu_{HA}=\mu_{LA}=0$. Consequently, $\vartheta_1(\boldsymbol{\mu})$ is not defined, because consumers are not expecting a certified status on the equilibrium path. As discussed later, following the spirit of the Intuitive Criterion (Cho and Kreps 1987), we assume that upon observing an off-equilibrium certified status $S=1$, consumers believe that the deviation comes from an investing firm, i.e., $\mu_{HA}=1$.

where the first and second terms represent, respectively, the cases in which the uncertified product is supplied by a high– and a low–sustainability firm that did not apply for certification. (Recall that when the uncertified status is due to certification failure, the no-violation probability is zero.) As a result, for any certification status $S \in \{0,1\}$, consumers buy the product if and only if (iff) their expected utility $U = v + r\vartheta_S(\boldsymbol{\mu}) - p \geq 0$ where p represents the product's selling price. This leads to a demand function as follows.

$$d(p, S; \boldsymbol{\mu}) = \begin{cases} 1 \text{ if } p \leq v + r\vartheta_S(\boldsymbol{\mu}), \\ 0 \text{ if } p > v + r\vartheta_S(\boldsymbol{\mu}). \end{cases} \tag{4}$$

By associating d with $\boldsymbol{\mu}$, we emphasize that the demand is based on given consumers' belief. In equilibrium, the belief must be consistent with the firm's optimal decisions on (x, y).

3.4. Firm Profit

The firm solves a two-stage problem, first making the investment and certification decisions (x, y), and then choosing a selling price p based on the realized certification status S. Because the selling price is contingent on certification status, we let p_S denote the price charged when the firm has status $S \in \{0,1\}$. Thus, the firm charges p_1 if certified and p_0 if uncertified.

In addition to sales revenue, the firm incurs a penalty cost g (e.g., legal fines and reputational loss) if a violation is discovered by regulators or nongovernmental organizations (NGOs), which often monitor firms for sustainability compliance.[6] We assume that a violation, if any, is detected with exogenous probability λ, and define $G = \lambda g$ as the expected penalty conditional on a violation occurring. For brevity, we refer to the expected penalty G simply as the penalty, with the understanding that it accounts for the probability that a violation is exposed.

Recall that certification status allows the firm to update its violation probability to $\psi(x, y, S)$. Thus, given investment and certification decisions (x, y) and certification status S, the firm expects to incur penalty cost $\psi(x, y, S)G$ if it sells the product. By contrast, it can avoid the penalty by choosing not to enter the focal market, which is equivalent in our model to setting a prohibitively high price p_S.[7]

We now formulate the firm's problem with decision variables $(x, y) \in Z$ and $\mathbf{p} = (p_1, p_0) \in [0, \infty)^2$. For ease of notation, we normalize the unit production cost to zero, since it does not bring much

[6] For example, the U.S. Department of Energy enforces mandatory energy-efficiency standards, and firms distributing noncompliant products face civil penalties of \$575 per unit, with additional daily fines (Office 2025).

[7] Note that we focus on voluntary certifications rather than certifications required for market entry. For example, a product that fails to meet ENERGY STAR certification standards can still be sold in the U.S. market (WebstaurantStore 2025), because ENERGY STAR imposes stricter requirements than the mandatory energy-efficiency standards set by the DOE (Agency 2021). Yet, certification failure may indicate greater noncompliance risk.

additional insight. Conditional on each status S, the firm chooses a price $p_S \geq 0$ to maximize its second-stage profit written as

$$\pi(p_S, x, y, S; \boldsymbol{\mu}) = p_S d(p_S, S; \boldsymbol{\mu}) - \mathbb{I}(d(p_S, S; \boldsymbol{\mu}) > 0)\psi(x, y, S)G, \tag{5}$$

where $\mathbb{I}(\cdot)$ is an indicator function such that we incorporate the option of not entering the market to avoid regulatory penalties. Specifically, the firm can set a sufficiently high p_S such that $d(p_S, S; \boldsymbol{\mu}) = 0$.

Taking the expectation over S and incorporating the investment and certification costs, we can write the firm's expected profit as

$$\Pi(\mathbf{p}, x, y; \boldsymbol{\mu}) = \sum_{S\in\{0,1\}} f_S(x, y)\pi(p_S, x, y, S; \boldsymbol{\mu}) - c_H\mathbb{I}(x = H) - c_A\mathbb{I}(y = A), \tag{6}$$

where $f_S(x, y)$ is the probability mass function of S as defined in (1) and (2). Given a belief vector μ, the firm solves the following problem:

$$\max_{(x,y)\in Z,\, \mathbf{p}\in[0,\infty)^2} \Pi(\mathbf{p}, x, y; \boldsymbol{\mu}). \tag{7}$$

We assume that when indifferent between two options, the firm follows a tie-breaking order $(H, A) \succ (H, N) \succ (L, A) \succ (L, N)$, reflecting a preference for higher sustainability investment and, conditional on investment, for certification. We summarize the sequence of events in Figure 1, emphasizing that consumers observe only one of two certification statuses, certified and uncertified.

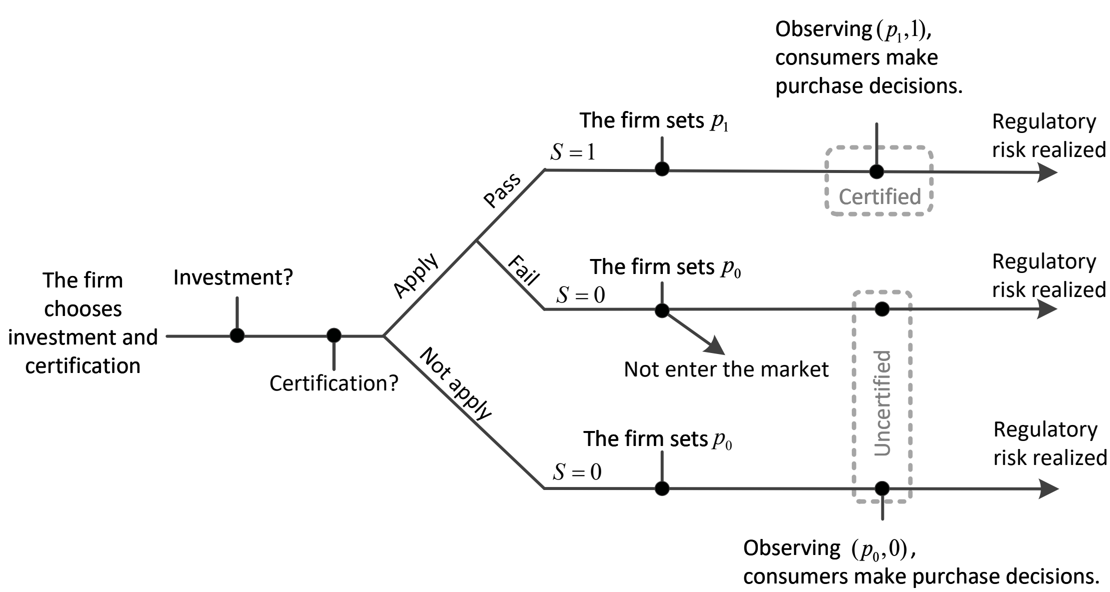


Figure 1 Sequence of Events

To focus on realistic situations, we impose a mild assumption that $G \leq \frac{v+\theta_L r}{1-\theta_L}$. Under this assumption, the firm always finds it profitable to stay in the market if it operates without pursuing certification ($y=N$) or if it applies and gets certified ($y=A$ and $S=1$). In contrast, if the firm applies but fails certification ($y=A$ and $S=0$), it may or may not be profitable to sell in the market, depending on the regulatory stringency. This modeling choice enables us to capture the fact that firms often seek certifications to mitigate violation risk, as discussed in the introduction.

In the following remarks, we reflect on some important model features and assumptions.

Remark 1. Motivated by firm investment and third-party certification for sustainability compliance, our model best fits settings in which a low sustainability level increases the risk of regulatory noncompliance and exposes the firm to potential penalties.[8] This differs from settings in which certifications primarily attest to premium product quality rather than regulatory compliance. In such settings, a firm selling a lower-quality product without certification may lose market appeal, but it is not necessarily subject to regulatory penalties.

Remark 2. Sustainability-related process improvements often take substantial time. We therefore focus on a single selling season and do not model the option of rectifying issues and reapplying for certification within the same period. Our main question is whether lower certification costs encourage firms to invest proactively before entering the market. This question remains relevant even if a firm could improve after failing certification, because such ex post rectification would still represent delayed sustainability investment. Moreover, the question remains relevant when a firm chooses not to enter the focal market upon certification failure. In practice, firms may continue selling in less regulated regions after forgoing sales in a regulated market due to compliance concerns.[9] Thus, a noninvesting firm that exits the focal market may still generate negative environmental or social impacts if it redirects sales to less regulated markets. In §5.1, we consider an alternative metric that accounts for sustainability damage only when a noncompliant firm enters the focal market, and show that our main results continue to hold.

3.5. Solution Approach

A major challenge in analyzing our model is that consumers observe neither the firm's sustainability investment x nor its certification decision y. However, consumers can potentially infer (x, y) through the observed certification status S and the price p_S. The model can therefore be viewed as a signaling game with endogenous private choices. Such games often admit many PBEs because consumers'

[8] In this regard, our model may also apply to certification settings involving health- or safety-related product defects that can generate social harm and trigger regulatory penalties.

[9] Based on our interview with an executive at a major international home-appliance manufacturer, the firm redirected sales to less regulated markets after failing to obtain certification due to concerns about compliance with EU regulations.

beliefs are not fully pinned down off the equilibrium path. In and Wright (2018) propose Reordering Invariance (RI) as an equilibrium refinement for such games. The idea is to consider a reordered game in which the sender first commits to observable actions and then privately makes unobservable choices. This refinement is reasonable because, when the sender receives no new payoff-relevant information between these choices, the order in which it makes its own observable and unobservable decisions should not affect its optimal strategy.

The RI refinement, however, does not directly apply to our setting. First, the observable certification status S is not directly chosen by the firm; rather, it is a stochastic outcome generated by the firm's investment and certification decisions. Second, the firm's price is contingent on the realized certification status, so the realization of S provides payoff-relevant information before the firm sets price. Inspired by RI, we evaluate the firm's unobservable decisions (x,y) as if the firm had chosen the contingent pricing plan (p_1,p_0) induced by the candidate equilibrium. Because prices are chosen after the certification status is realized, we additionally require each price p_S to be ex post optimal conditional on the realized status S.

Specifically, we evaluate every candidate strategy $(x^*,y^*)\in Z$ and determine whether and when it can be supported as an equilibrium. The analysis proceeds in two steps. First, given fixed (x^*,y^*), we determine the equilibrium prices p_S^* for each certification status S, taking into account consumers' purchase decisions induced by that price. Second, given these prices, we verify that no alternative action $(x',y')\in Z$ yields a higher expected payoff than the candidate strategy (x^*,y^*). Throughout this procedure, consumers' on-path belief is required to be consistent with the candidate strategy. For instance, to identify any equilibrium where the firm chooses to invest and apply for certification, that is, $(x^*,y^*)=(H,A)$, then the on-path belief must satisfy $\mu_{HA}=1$ and $\mu_{HN}=\mu_{LA}=\mu_{LN}=0$.

The following proposition formalizes our solution approach. The proof of Proposition 1 and further technical details are provided in Appendix A.

Proposition 1 **(Solution Approach)**. *For a given $(x^*,y^*)\in Z$, let $\boldsymbol{\mu}^*$ be a unit vector with entry $\mu_{x^*y^*}=1$ and the other entries being zero. Let $\mathbf{p}^*=(p_0^*,p_1^*)$ satisfy*

$$p_S^*\in\arg\max_{p_S\in[0,\infty)}\pi(p_S,x^*,y^*,S;\boldsymbol{\mu}^*)\quad \forall S\in\{0,1\}. \tag{C1}$$

The strategy (x^*,y^*), together with $\mathbf{p}^*$ and belief vector $\boldsymbol{\mu}^*$, constitutes a PBE if one of the following two conditions holds: (i) if $y^*=A$,

$$\Pi(\mathbf{p}^*,x^*,y^*;\boldsymbol{\mu}^*)\geq\Pi(\mathbf{p}^*,x',y';\boldsymbol{\mu}^*)\quad\forall(x',y')\in Z, \tag{C2}$$

or (ii) if $y^*=N$,

$$\begin{aligned}\Pi(\mathbf{p}^*,x^*,y^*;\boldsymbol{\mu}^*)\geq f_1(x',y')\max_{\hat{p}_1\in[0,\infty)}\pi(\hat{p}_1,x',y',1;\hat{\boldsymbol{\mu}})+f_0(x',y')\pi(p_0^*,x',y',0;\boldsymbol{\mu}^*)\\ -c_H\mathbb{I}(x'=H)-c_A\mathbb{I}(y'=A)\quad\forall(x',y')\in Z,\end{aligned} \tag{C3}$$

where $\hat{\boldsymbol{\mu}}$ is an off-equilibrium belief vector specified as $\hat{\mu}_{HA}=1$ and the other entries being zero.

Condition (C1) ensures sequential rationality at the pricing stage: for each realized certification status S, the price p_S^* is optimal conditional on that status. Conditions (C2) and (C3) ensure that the candidate investment and certification choices (x^*, y^*) are optimal relative to all alternative strategies, and hence consistent with the belief $\boldsymbol{\mu}^*$. In evaluating deviations, the price $\mathbf{p}^*$ is held fixed whenever possible, in the spirit of RI. The distinction between (C2) and (C3) arises because, under a no-certification equilibrium, (H, N) or (L, N), the firm is expected to be uncertified on the equilibrium path, i.e., $S = 0$. Thus, a deviation to certification may generate an off-path certified status $S = 1$, for which consumers' beliefs and the corresponding price are not pinned down by the on-path equilibrium. As formalized in (C3), following the spirit of the Intuitive Criterion (Cho and Kreps 1987), we specify the off-path belief $\hat{\boldsymbol{\mu}}$ by setting $\hat{\mu}_{HA} = 1$, because a certified outcome is most plausibly generated by a firm that has both invested and applied for certification. The associated price $\hat{p}_1$ is then chosen optimally under this off-path belief.

By verifying the conditions in Proposition 1, we identify when each candidate strategy can emerge as a PBE. When multiple PBEs exist, we select the sender-preferred PBE, defined as the equilibrium that yields the highest profit to the firm, following the literature (Kamenica and Gentzkow 2011, Bimpikis and Mantegazza 2023).

4. Results

4.1. Equilibrium Selling Prices

It is instructive to first examine the firm's equilibrium selling prices, as summarized in Lemma 1. Note that the lemma is derived as part of the full equilibrium characterization in Appendix C, where the equilibrium strategies and prices are proved jointly.

Lemma 1. *On the equilibrium path, the firm's optimal selling prices, conditional on its investment and certification decisions (x^*, y^*) and certification status S, are given in Table 2.*

Table 2 Equilibrium selling prices conditional on (x, y) and S.

	certified?		Invest $(x^* = H)$	Not invest $(x^* = L)$
Apply for certification $(y^* = A)$	$S = 1$	p_1^*	$v + \frac{\theta_H}{1-(1-\theta_H)t} r$	$v + \frac{\theta_L}{1-(1-\theta_L)t} r$
	$S = 0$	p_0^*	$\begin{cases} v, & \text{if } G \leq v \\ (v, +\infty), & \text{if } G > v \end{cases}$	
Not apply for certification $(y^* = N)$	$S = 0$	p_0^*	$v + \theta_H r$	$v + \theta_L r$

In general, the equilibrium prices consist of the baseline product value v plus a premium determined by consumers' belief regarding the firm's sustainability level. Recall that, in equilibrium, consumers form beliefs consistent with the firm's optimal investment and certification decisions. When the firm chooses not to apply for certification ($y^* = N$) and hence is not certified ($S = 0$), the equilibrium price p_0^* is $v + \theta_H r$ if it invests in sustainability ($x^* = H$) and $v + \theta_L r$ otherwise ($x^* = L$). Because consumers correctly anticipate the firm's investment choice in equilibrium, a higher sustainability level allows the firm to charge a higher price since $\theta_L < \theta_H$. When the firm applies for certification and passes it ($y^* = A$ and $S = 1$), the optimal price $p_1^* = v + \frac{\theta_x r}{1-(1-\theta_x)t}$ for $x \in \{H, L\}$. Note that p_1^* increases with the certification accuracy t, and as long as $t > 0$, $p_1^* = v + \frac{\theta_x r}{1-(1-\theta_x)t}$ is greater than p_0^* under $y^* = N$ and $S = 0$. That is, for any investment choice x, applying for *and* passing certification enhances consumers' belief in the firm's sustainability level, enabling the firm to charge a higher price, and the enhancement is stronger as the certification becomes more accurate.

By contrast, if the firm applies but fails certification ($y^* = A$ and $S = 0$), the firm may either sell or not sell in the market, depending on whether the expected penalty G is greater than the baseline value v. If $G \leq v$, the risk of penalty is low relative to the product value, and the firm will continue to sell in the market but set the price equal to v, because failure implies noncompliance and thus eliminates any price premium that can be commanded in the market. If $G > v$, the penalty risk dominates the sales revenue, and thus the firm will shut down sales by setting any prohibitively high price $p_0^* \in (v, +\infty)$.

Although the selling prices discussed above are intuitive for each possible equilibrium choice (x^*, y^*) since equilibrium requires consistency in consumer beliefs, it is somewhat involved to identify under what conditions each equilibrium emerges, as discussed in the next subsection.

4.2. Equilibrium Investment and Certification Choice

4.2.1. Value of Investment with Fixed Certification Choice. Next, we examine the values of investment when the firm's certification choice is fixed. Let Δ_A denote the value of investment if the firm pursues certification, and Δ_N denote the value of investment without certification.

To fix ideas, consider the case of perfect certification, i.e., $t = 1$. Absent certification, consumers are unable to distinguish whether the firm has made an investment or is compliant. As a result, the firm cannot credibly command the premium r. Pricing alone is not sufficient to signal the firm's investment choice: consumers recognize that a non-investing firm may charge $v + \theta_H r$ instead of $v + \theta_L r$, and consequently an investing firm may fail to capture the premium $(\theta_H - \theta_L)r$. Moreover, the firm cannot condition its market entry decision on certification outcomes. Hence, the value of investment arises solely from reducing the expected penalty, that is, $\Delta_N = (\theta_H - \theta_L)G$.[10]

[10] Recall that we focus on the parameter range in which it is profitable to remain in the market absent certification.

With perfect certification, the firm's actual compliance will be fully revealed. For any investment level $x \in \{H, L\}$, the firm passes certification with probability θ_x. If certified, the firm can charge the premium price $v+r$. If certification fails, the firm will choose between selling at the baseline price v while incurring the (expected) penalty G and exiting the market with zero profit. Taken together, the firm's expected profit is

$$\theta_x(v+r)+(1-\theta_x)\max(v-G,0)$$

where the maximum operator captures the option to exit the market following certification failure. Therefore, if the firm pursues certification, the value of investment, which increases sustainability level from θ_L to θ_H, is

$$\Delta_A=(\theta_H-\theta_L)\big(r+\min(G,v)\big).$$

Our model captures the dual effects of certification as discussed in the introduction. When $G \le v$, the firm always sells in the market and the difference $\Delta_A-\Delta_N=(\theta_H-\theta_L)r$ reflects the *signaling effect* of certification. When $G>v$, certification enables the firm to avoid the penalty G by shutting down sales upon certification failure, resulting in a *risk mitigation* effect. Because of this effect, the investment value under certification $\Delta_A=(\theta_H-\theta_L)(r+\min(G,v))$ becomes independent of G once $G>v$, whereas the investment value without certification $\Delta_N=(\theta_H-\theta_L)G$ always increases with G. As such, when G is sufficiently large, certification may reduce the value of investment, i.e., $\Delta_A<\Delta_N$. Intuitively, in this case, certification is pursued primarily to screen violation risk for market entry decisions, and this risk-mitigation effect dampens the firm's incentive to invest.

Lemma 2. *(i) Suppose that the firm never pursues certification. In equilibrium, the firm chooses to invest (H,N) if $c_H \le \Delta_N$, and not to invest (L,N) if $c_H > \Delta_N$, where $\Delta_N=(\theta_H-\theta_L)G$.*

(ii) Suppose that the firm always pursues certification. In equilibrium, the firm chooses to invest (H,A) if $c_H \le \Delta_A$, and not to invest (L,A) if $c_H>\Delta_A$, where

$$\Delta_A=(\theta_H-\theta_L)\left[t\left(\frac{\theta_H r}{1-(1-\theta_H)t}+\min(G,v)\right)+(1-t)G\right]. \tag{8}$$

(iii) $\Delta_A \ge \Delta_N$ iff $G \le \eta$ where $\eta = v+\frac{\theta_H r}{1-(1-\theta_H)t}$.

More generally, Lemma 2 provides expressions for Δ_A and Δ_N by allowing any certification accuracy $t \in (0,1]$. In particular, the expression for Δ_A is more involved because certification may fail to detect a violation with probability $1-t$, which affects consumers' beliefs about the actual sustainability level of a certified firm. Yet, the main observation remains: There exists a threshold $\eta := v+\frac{\theta_H r}{1-(1-\theta_H)t}$ such that $\Delta_A<\Delta_N$ iff $G>\eta$.

Figure 2 illustrates how Δ_A and Δ_N vary with G for $t=1$ and $t=0.5$, respectively. When the risk-mitigation effect is active, that is, when certification failure leads to no market entry, the slope of Δ_A becomes flatter: Δ_A is constant when $t = 1$ and less sensitive to G when $t < 1$. This reduces the value of investment under certification relative to that without certification. In addition, the tipping point η increases with t, implying that less accurate certification is more likely to reduce the firm's investment incentive.

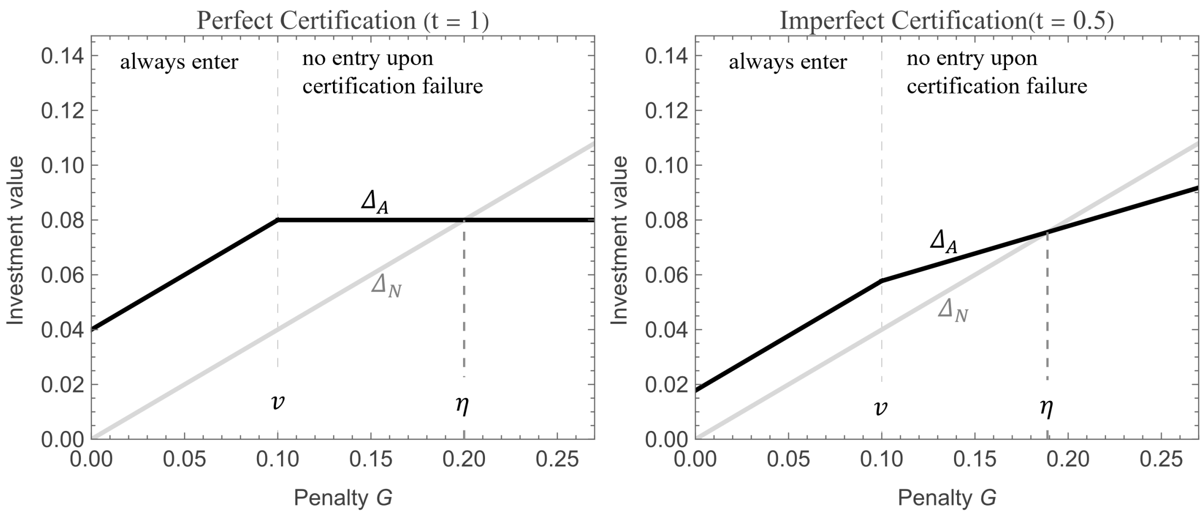


Figure 2 Values of Investment with and without certification as functions of penalty G

To formally establish the effect of certification on investment, however, we need to characterize the firm's equilibrium investment and certification decisions, which are jointly determined in the game. The equilibrium structure differs across two parameter regimes, depending on whether $G \leq \eta$ or $G > \eta$. We therefore discuss these two regimes separately below.

4.2.2. Low Penalty Regime ($G \leq \eta$). We first discuss the equilibrium when $G \leq \eta$, in which case $\Delta_A \geq \Delta_N$, so certification amplifies the investment value. We define $\hat{c}$ as the *cutoff investment cost* such that the firm chooses to invest iff $c_H \leq \hat{c}$. A higher $\hat{c}$ thus indicates a greater firm incentive to invest. Proposition 2 fully characterizes the firm's investment and certification decisions in equilibrium. Throughout the paper, we use "increase" and "decrease" in the weak sense. Figure 3 illustrates the proposition for different pairs of c_A and c_H. The left panel depicts the case of perfect certification ($t = 1$), while the right panel depicts the case of imperfect certification ($t = 0.5$). The thick piecewise-linear curve represents the cutoff investment cost $\hat{c}$, below which the firm prefers to invest.

Proposition 2. *Under low penalty ($G \leq \eta$), the firm's equilibrium investment and certification decisions are characterized as follows:*

i. When $c_H \leq \Delta_N$, $(x^*, y^*) = (H, A)$ if $c_A \leq \rho_H$, and $(x^*, y^*) = (H, N)$ otherwise.

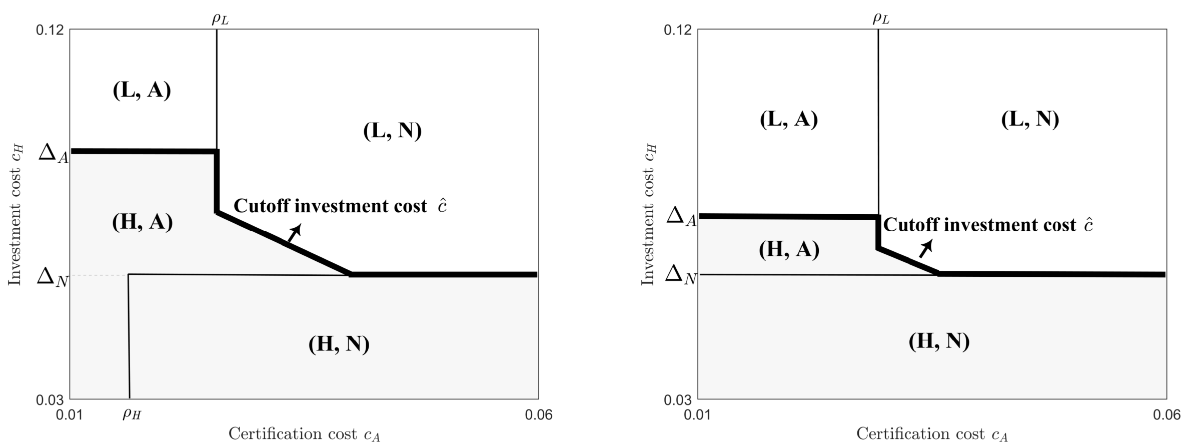
ii. When $c_H > \Delta_A$, $(x^*, y^*) = (L, A)$ if $c_A \leq \rho_L$, and $(x^*, y^*) = (L, N)$ otherwise.

iii. When $\Delta_N < c_H \le \Delta_A$, $(x^*, y^*) = (H, A)$ *if* $c_A \le \min\{\rho_L, \rho_1\}$, *and* $(x^*, y^*) = (L, N)$ *otherwise.*

For the above cases, the cutoff investment cost $\hat{c}$ *is given by*

$$\hat{c} = \begin{cases} \Delta_A, & c_A \le \rho_L, \\ \Phi_l - c_A, & \rho_L < c_A \le \rho_1, \\ \Delta_N, & c_A > \rho_1, \end{cases} \tag{9}$$

where Φ_l *is independent of* c_A, *and thus* $\hat{c}$ *is decreasing in* c_A.[11] *Importantly, the firm's equilibrium sustainability level* θ_{x^*} *is decreasing in* c_A, *that is, a lower certification cost encourages investment.*



(a) $t = 1$ (b) $t = 0.5$

Notes. H : Invest; L: Do not invest; A : Apply for certification; N: Do not apply for certification.

Figure 3 Illustration of the firm's optimal solutions under low penalty ($G \le \eta$)

We explain the equilibrium structure based on the investment cost c_H relative to Δ_A and Δ_N.

Low Investment Cost ($c_H \le \Delta_N$). In this case, investment is worthwhile even purely for reducing expected penalty. Consequently, the firm always invests, and consumers anticipate this; thus, certification does not create value through signaling the firm's investment choice. However, in equilibrium the firm pursues certification if the certification cost c_A is lower than a threshold ρ_H. When $G > v$, this is intuitive because certification provides value for the firm to mitigate penalty risk before market entry. More intriguingly, when $G \le v$ such that the firm always enters the market, certification brings no signaling or risk-mitigation value. Despite that, the firm still pursues certification in equilibrium. This is because the invest-without-certification strategy (H, N) may not be sustained in equilibrium. For a sufficiently low c_A, the firm can benefit from deviating from (H, N) to the

[11] The closed-form expressions of Φ_l, ρ_H, ρ_L and ρ_1 are provided in Appendix C.

invest-with-certification strategy (H, A). Even if certification fails, the associated loss is mitigated because consumers cannot distinguish whether an uncertified status arises from certification failure or from not participating in certification.[12]

High Investment Cost ($c_H > \Delta_A$). In this case, the investment cost exceeds the value of investment, regardless of whether the firm pursues certification. Thus, the firm never invests and consumers anticipate this. Similar to the previous case, although certification does not signal the firm's investment decision, in equilibrium the firm will apply for certification if its cost is sufficiently low, i.e., $c_A \leq \rho_L$. The rationale is similar. The firm can benefit from the risk-mitigation effect (when $G > v$) and, even without the risk-mitigation effect (when $G \leq v$), the unobservability of the certification decision creates an incentive for the firm to deviate from (L, N) to (L, A). Note that $\rho_L > \rho_H$; that is, a non-investing firm has a stronger incentive to obtain certification. This is because certification provides greater benefits to such a firm through the risk-mitigation effect and consumers' inability to distinguish the reasons for an uncertified status.

Moderate Investment Cost ($\Delta_N < c_H \leq \Delta_A$). In this case, the firm would invest only when certification credibly signals its investment choice. Thus, the equilibrium is effectively chosen between (H, A) and (L, N). In other words, certification and sustainability investment are *complementary*. The invest-with-certification strategy (H, A) is played in equilibrium when both the investment cost c_H and certification cost c_A are low, or $c_H \leq \hat{c}$ as shown in Figure 3.

Importantly, the cutoff threshold $\hat{c}$ is decreasing in c_A. Put differently, within the low-penalty regime $G \leq \eta$, a lower certification cost strengthens the firm's incentive to invest and improves the equilibrium sustainability level θ_{x^*}, because it makes it easier for the firm to signal its investment and thereby capture the return on investment through certification.

Before turning to the high-penalty regime, we present several corollaries that highlight additional implications of the equilibrium results in the low-penalty regime.

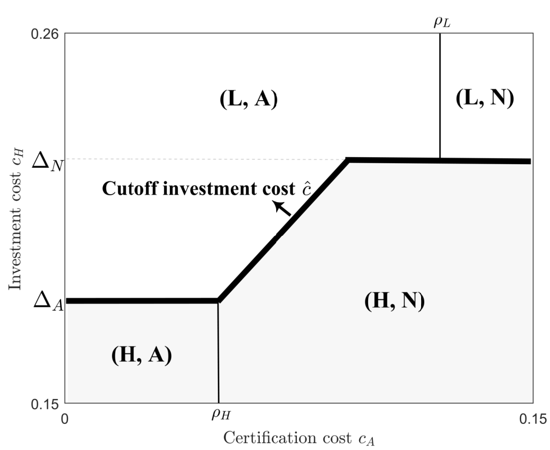
Corollary 1. *Under a low penalty ($G \leq \eta$) and moderate certification cost ($\rho_L < c_A \leq \rho_1$), the firm's equilibrium certification decision is non-monotone in the investment cost: as c_H increases, the firm switches from not applying for certification to applying, and then back to not applying.*

[12] Within the case discussed here, if the firm's certification decision y were observable, the equilibrium strategy would always be (H, N), because certification does not improve the firm's expected profit. To see this, consider the case of perfect certification ($t = 1$) with a low penalty $G \leq v$. If the firm does not pursue certification, it sells at a price $v + \theta_H r$ and incurs an expected penalty $(1 - \theta_H)G$, yielding an expected profit of $v + \theta_H r - (1 - \theta_H)G$, net of any certification cost. If the firm instead pursues certification, its expected profit is $\theta_H(v + r) + (1 - \theta_H)(v - G)$, where $v + r$ and v are the equilibrium prices following certification success and failure, respectively. Hence, certification does not increase expected profit. However, when consumers do not observe the firm's certification decision y, certification can become a profitable deviation from (H, N). In particular, upon certification failure, the firm may still command a price higher than v, as consumers cannot distinguish whether the uncertified outcome arises from certification failure or from not participating in certification. This creates an incentive to deviate.

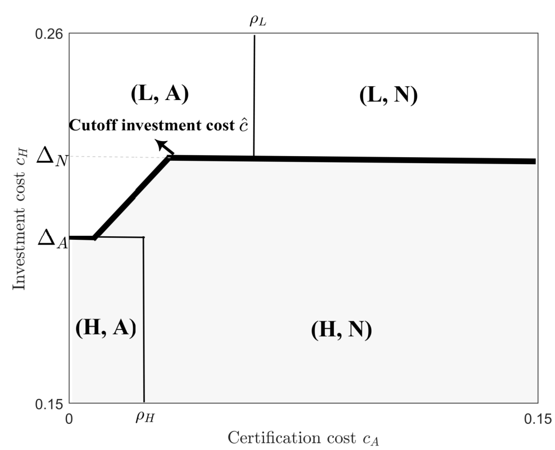
As shown in Figure 3, there exists an intermediate range of c_A within which the equilibrium switches from (H,N) to (H,A) and then to (L,N) as the investment cost c_H increases; see the wedge-shaped (H,A) region. In other words, the firm applies for certification only when the investment cost is moderate. Intuitively, when the investment cost is either sufficiently low or sufficiently high, consumers can readily infer the firm's investment choice. Certification is therefore most valuable as a signal of the firm's investment level when the investment cost is moderate.

COROLLARY 2. *Suppose that the investment cost is sufficiently high or low ($c_H > \Delta_A$ or $c_H \leq \Delta_N$) and $G \leq v$. If the certification decision y were publicly observable, then the firm never pursues certification. Therefore, certification is pursued more often in equilibrium because of the unobservability of certification attempts.*

As briefly discussed earlier, when consumers can readily anticipate the firm's investment level (when c_H lies at one of the extreme ends) and the firm always enters the market (when $G \leq v$), certification creates no real value. Nevertheless, the firm may still pursue certification in equilibrium because the profit loss from certification failure is mitigated by consumers' inability to ascertain the cause of an uncertified status. By contrast, under a hypothetical benchmark in which the firm's certification application is publicly observable, the firm would never pursue certification. In this regard, our results imply that the unobservability of certification attempts leads certification to be used more often in equilibrium.



(a) $t = 1$



(b) $t = 0.5$

Notes. H : Invest; L: Do not invest; A : Apply for certification; N: Do not apply for certification.

Figure 4 Illustration of the firm's optimal solutions under high penalty ($G > \eta$).

4.2.3. High Penalty Regime ($G > \eta$). We now consider the case of $G > \eta$. Since $\eta > v$, in this case the firm always chooses not to enter the market following a certification failure; hence, certification, if pursued, serves as an instrument for mitigating market-entry risk (in addition to its signaling effect). Moreover, in this case $\Delta_A < \Delta_N$ so that investment is less valuable when the firm pursues certification, because certification is used primarily to assess violation risk before the market-entry decision, which weakens the firm's incentive to invest

The firm's investment and certification choices under the high-penalty regime are characterized in Proposition 3 and illustrated by Figure 4 for different pairs of c_A and c_H, where the thick piecewise-linear curve represents the cutoff investment cost $\hat{c}$. Specifically, Figure 4(a) demonstrates the case of perfect certification ($t = 1$) and Figure 4(b) considers imperfect certification ($t = 0.5$).

Proposition 3. *Under high penalty ($G > \eta$), the firm's equilibrium investment and certification decisions are characterized as follows:*

i. When $c_H \leq \Delta_A$, $(x^*, y^*) = (H, A)$ if $c_A \leq \rho_H$, and $(x^*, y^*) = (H, N)$ otherwise.

ii. When $c_H > \Delta_N$, $(x^*, y^*) = (L, A)$ if $c_A \leq \rho_L$, and $(x^*, y^*) = (L, N)$ otherwise.

iii. When $\Delta_A < c_H \leq \Delta_N$, $(x^*, y^*) = (L, A)$ if $c_A \leq \min\{\rho_H, \rho_2\}$, and $(x^*, y^*) = (H, N)$ otherwise.

For the above cases, the cutoff investment cost $\hat{c}$ is given by

$$\hat{c} = \begin{cases} \Delta_A, & \text{if } c_A \leq \min\{\rho_H, \rho_2\}, \\ \Phi_h + c_A, & \text{if } \min\{\rho_H, \rho_2\} < c_A \leq \rho_3, \\ \Delta_N, & \text{if } c_A > \rho_3, \end{cases} \tag{10}$$

where Φ_h is independent of c_A, and thus $\hat{c}$ is increasing in c_A.[13] *Importantly, the firm's equilibrium sustainability level θ_{x^*} is increasing in c_A, that is, a lower certification cost discourages investment.*

Low Investment Cost ($c_H \leq \Delta_A$). When the investment cost is lower than the investment value, regardless of the firm's certification choice, the firm always invests and consumers anticipate this in equilibrium. Under the high penalty regime, certification always serves as a risk-mitigation instrument. Thus, when the certification cost c_A is lower than a threshold ρ_H, the invest-without-certification strategy (H, N) is optimal to pay for the certification to assess violation risk before market entry.

High Investment Cost ($c_H > \Delta_N$) When the investment cost exceeds its value, regardless of the certification choice, the firm chooses not to invest, as anticipated by consumers. As in the previous case, the firm pursues certification if the certification cost is sufficiently low, i.e., $c_A \leq \rho_L$. Like before, $\rho_L > \rho_H$ because a non-investing firm benefits more from the risk-mitigation effect than an investing firm.

[13] The closed-form expressions of Φ_h, ρ_H, ρ_L, ρ_2, and ρ_3 are provided in Appendix C.

Moderate Investment Cost ($\Delta_A < c_H \le \Delta_N$) In the high-penalty regime, investment generates greater value without pursuing certification than when certification is pursued, i.e., $\Delta_A < \Delta_N$. Consequently, when c_H lies between Δ_A and Δ_N, the equilibrium is effectively determined between the not-invest-with-certification strategy (L, A) and the invest-without-certification strategy (H, N). In other words, certification and sustainability investment are now *substitutes*, because certification primarily serves to avoid violation penalty which makes investment less valuable to the firm. Therefore, strategy (H, N) is played in equilibrium rather than (L, A) when the certification cost is high and the investment cost is low. As such, the investment cutoff cost $\hat{c}$ becomes increasing in c_A, implying that more expensive certification actually encourages the firm to invest.

It is worth noting that our result that certification and investment can act as substitutes echoes the empirical findings mentioned earlier that certification adoption is often decoupled from actual sustainability improvement (Aravind and Christmann 2011). Our analysis provides a plausible explanation for this empirical observation: decoupling may arise when firms leverage certification primarily for risk mitigation. This risk-mitigation effect dampens their incentive to invest proactively and may dominate the signaling effect that otherwise motivates investment.

The following corollary highlights an additional interesting observation from the equilibrium results.

Corollary 3. *Under the high penalty regime with imperfect certification ($G > \eta$ and $t < 1$), there exists an intermediate range of c_A within which the firm's certification choice is non-monotone in the investment cost: as c_H increases, the firm's certification decision switches from applying, to not applying, and then back to applying.*

When certification is imperfect ($t < 1$), the equilibrium strategy can switch from (H, A) to (H, N) and then to (L, A); see the wedge-shaped intrusion of the (H, N) region between the (H, A) and (L, A) regions in Figure 4(b). Thus, certification is pursued when investment is either sufficiently inexpensive or sufficiently costly, but not when the investment cost is moderate. Such an indented (H, N) region does not arise when certification is perfect ($t = 1$). Under the high-penalty regime, certification and investment act as substitutes. Thus, less accurate certification makes investment relatively more attractive, expanding the overall investment region, i.e., the union of (H, A) and (H, N). At the same time, lower accuracy weakens certification's signaling value, because a non-investing firm can more easily mimic an investing firm. As a result, (H, A) becomes harder to sustain in equilibrium.[14] Consequently, for moderate investment costs, the firm may choose (H, N) rather than (H, A), as certification no longer provides a sufficiently credible signal of investment.

[14] The indented (H, N) region emerges because (H, A) does not exist as a PBE in this region.

4.3. Policy Implications

Our equilibrium results imply that a reduction in certification cost may either encourage or discourage investment decisions, which depends crucially on whether the penalty cost $G \leq \eta$ or $G > \eta$. The following proposition formalizes this result and further shows that the threshold η increases with certification accuracy.

Proposition 4. *As the certification cost c_A decreases, the firm is more likely to invest if $G \leq \eta$, but less likely to do so if $G > \eta$. That is, certification promotes sustainability investment if and only if $G \leq \eta$. Moreover, the threshold η is increasing in certification accuracy t.*

The proposition is illustrated by Figure 5, which shows that certification is more likely to discourage investment when it becomes less accurate. This result is driven by the dual effects of certification. Generally speaking, the signaling effect helps the firm command a sustainability premium in the market, thereby encouraging investment. By contrast, the risk-mitigation effect enables the firm to assess market-entry risk, which can substitute for investment and thereby discourage it. Higher certification accuracy strengthens the signaling effect more than the risk-mitigation effect. Thus, an increase in certification accuracy makes certification more likely to promote investment.

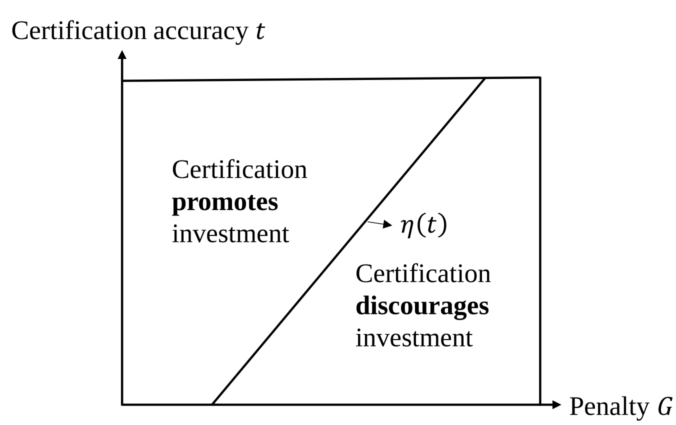


Figure 5 Threshold η: certification promotes vs. discourages investment

Overall, our results have important policy implications. Recall that G represents the expected penalty associated with selling a noncompliant product in the focal market, incorporating both the probability that the violation is detected by an external party and the resulting loss upon detection. Thus, a higher G reflects a more stringent regulatory environment, either because of more intense external scrutiny or a more severe penalty upon exposure. On the other hand, policymakers can often influence a firm's certification cost through subsidies and other supports.[15] Our result suggests that, given a sufficiently stringent regulatory environment, reductions in firms' certification costs may be counterproductive. Lower certification costs may induce a firm to opportunistically leverage certification to mitigate market-entry risk, rather than proactively invest to improve its inherent sustainability level. This unintended consequence can be mitigated, to some extent, by improving certification accuracy. In other words, when the regulatory penalty is already severe, policymakers may achieve better outcomes by allocating resources to improving the accuracy of certification processes rather than reducing firms' certification costs, which may trigger opportunistic behavior.

Next, we examine the impact of regulatory stringency on sustainability investment when the firm has option to pursue or not pursue certification. By combining the low- and high-penalty regimes and analyzing the cutoff investment cost as a function of penalty G, we obtain the following result.

PROPOSITION 5. *Holding other variables fixed, the cutoff investment cost $\hat{c}$ varies with the penalty G as follows: (i) If $c_A \leq \theta_H rt(1-\theta_H)$, $\hat{c} = \Delta_A$, which is increasing in G. (ii) If $\theta_H rt(1-\theta_H) < c_A \leq (1-\theta_L)(G-v)t$, $\hat{c}$ may be either increasing or decreasing in G. (iii) If $c_A > (1-\theta_L)(G-v)t$, $\hat{c} = \Delta_N$, which is increasing in G.*

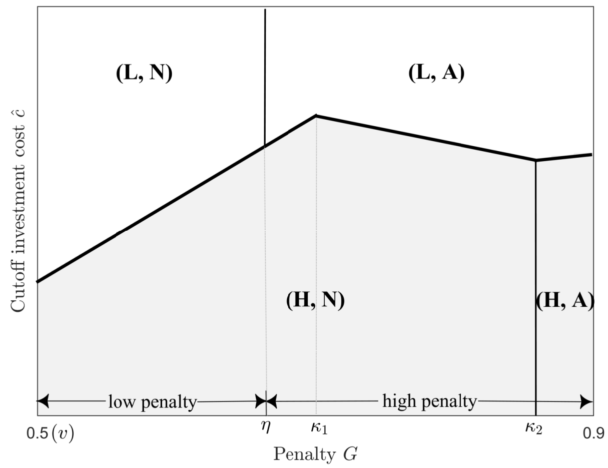
When the certification cost is sufficiently low or sufficiently high, the firm always pursues or does not pursue certification, respectively. As a result, in these two cases, the cutoff investment costs reduce to Δ_A and Δ_N, respectively, as characterized in Lemma 2. As shown earlier in Figure 2, both Δ_A and Δ_N are increasing in G. This suggests that more stringent regulatory monitoring or a higher penalty always encourages investment in industries where certification is either universally adopted or unavailable.

However, the impact of regulatory stringency is more nuanced when the certification cost lies in the intermediate range. As shown in Case (ii) of Proposition 5, a higher expected penalty G may possibly reduce the cutoff investment cost, thus reducing the firm's incentive to invest. To illustrate intuition, Figure 6 presents a representative example.[17] In this example, as G increases, the firm initially follows no-certification strategies, either (L, N) or (H, N). Consequently, the cutoff $\hat{c}$ coincides with Δ_N, thus increasing with G. However, after G increases into the high-penalty regime, the firm chooses

[15] For example, in Singapore and India, governments subsidize firms' adoption of certification standards, such as ISO 14001, by supporting consultancy and certification fees; see https://www.enterprisesg.gov.sg/financial-support/enterprise-development-grant and https://my.msme.gov.in/MyMsmeMob/MsmeScheme/Pages/0_2_2.html, accessed June 1, 2026.

[16] Given $(\theta_H - \theta_L)r + (1-\theta_H)\theta_L rt < c_A \leq (1-\theta_L)(G-v)t$, we can analytically show that there exist two tipping points such that $\hat{c}$ is increasing in G for $\eta < G \leq \kappa_1$ and $G > \kappa_2$, but decreasing in G for $\kappa_1 < G \leq \kappa_2$, where $\kappa_1 = v + \frac{c_A + \theta_H r - \theta_L r}{(1-\theta_L)t}$ and $\kappa_2 = v + \frac{c_A}{(1-\theta_H)t} + \frac{(\theta_H - \theta_L)(1-t)r}{(1-\theta_H)(1-(1-\theta_H)t)t}$.

[17] Depending on the range of c_A, the cutoff $\hat{c}$ can involve different cases and the resulting pattern can be somewhat different from Figure 6.



Notes. The figure is generated under the parameter region $(\theta_H - \theta_L)r + (1-\theta_H)\theta_L rt < c_A \leq (1-\theta_L)(G-v)t$, which falls within Case (ii) of Proposition 5.[16]

Figure 6 Non-monotone impact of penalty G on the cutoff investment cost for moderate certification costs

between pursuing certification without investment and investing without pursuing certification, that is, (L, A) and (H, N), since the two decisions become substitutable. In this case, increased regulatory stringency may nudge the firm more toward relying on certification to mitigate penalty risk, rather than investment, making the investment cutoff $\hat{c}$ decreasing in G. Eventually, when G is sufficiently large, the firm always uses certification to assess penalty risk, and the investment cutoff $\hat{c}$ reduces to Δ_A, becoming increasing in G again.

Our findings therefore suggest that regulators should carefully evaluate the joint effect of third-party certification and punitive regulatory measures. Stronger punitive measures may not always encourage sustainability investment, particularly when firms view certification and investment as substitutes for managing regulatory penalty risk.

5. Extensions

5.1. Alternative Sustainability Metric: Noncompliant Market Entry

In preceding discussions, we focus on whether the firm has greater or less incentive to make sustainability investment, whether or not it chooses to enter the market upon certification failure. As discussed in Remark 2, this is a relevant sustainability metric because firms often redirect sales to less regulated markets. We now examine an alternative metric that captures environmental or social damage only when the firm enters the focal market. Specifically, we consider the probability that a noncompliant firm enters the market, denoted by SD, as the expected sustainability damage. When the firm does not pursue certification, $SD = 1 - \theta_x$. When the firm pursues certification, SD depends on its market-entry decision after certification failure: $SD = 1 - \theta_x$ if the firm enters the market regardless of the certification outcome, and $SD = (1-\theta_x)(1-t)$ if the firm does not enter the market upon certification failure.

Let SD^* denote the expected sustainability damage in equilibrium. As a benchmark, we consider the case where certification is unavailable (or equivalently, $c_A = +\infty$), and let SD^N denote the expected sustainability damage in this no-certification benchmark.

PROPOSITION 6. *(i) Under the low penalty regime ($G \le \eta$), certification decreases the expected sustainability damage, i.e., $SD^* \le SD^N$. (ii) Under the high penalty regime ($G > \eta$), certification can strictly increase the expected sustainability damage, i.e., $SD^* > SD^N$, if $\Delta_A < c_H \le \Delta_N$ and $t \le \frac{\theta_H - \theta_L}{1-\theta_L}$.*

Proposition 6 establishes that certification has a similarly equivocal effect on sustainability damage as it does on the firm's investment choice. Under the low-penalty regime ($G \le \eta$), certification improves sustainability by inducing a weakly higher investment level. When $G \le v$, the firm always enters the market, so higher investment directly reduces sustainability damage. When $v < G \le \eta$, the firm exits the market upon certification failure. Thus, certification not only motivates investment but also filters out a noncompliant firm with probability t.

Under the high-penalty regime ($G > \eta$), however, certification can increase sustainability damage under the condition identified in Proposition 6(ii). Recall that $\Delta_A < c_H \le \Delta_N$ is the condition under which certification and investment act as substitutes, so that the firm chooses between (L, A) and (H, N). In this case, when certification is available, the firm may prefer to mitigate regulatory risk through certification rather than proactively improve its sustainability level. As a result, introducing certification may induce the firm to switch from (H, N) to (L, A), changing expected sustainability damage from $1-\theta_H$ to $(1-\theta_L)(1-t)$. Although certification filters out noncompliant firms with some probability, it also discourages investment. Therefore, if certification does not detect noncompliance with sufficient accuracy, i.e., $t \le \frac{\theta_H - \theta_L}{1-\theta_L}$, it may lead to an overall increase in sustainability damage.

The above finding reinforces the policy implications discussed earlier. In an already stringent regulatory environment, resources may be better directed toward improving the reliability of third-party certification processes, rather than reducing firms' certification costs.

5.2. Heterogeneous Consumers

In the base model, we assume that all consumers are sustainability conscious and willing to pay a premium r for a compliant product. We now extend the analysis to a more realistic setting in which only a fraction δ of consumers are sustainability conscious (SC), while the remaining fraction $1-\delta$ are non-sustainability conscious (NC). Only SC consumers value compliance and are willing to pay the premium r; NC consumers value the product only at the baseline level v, regardless of its sustainability level. In practice, δ can be interpreted as the degree of consumer awareness in the market.

The presence of two consumer segments introduces an additional layer of complexity for the firm's pricing decision. The firm effectively chooses between capturing the entire market by charging a price of v and targeting only the sustainability-conscious segment by charging a premium price which depends on those consumers' belief. It can be shown that, in equilibrium, a certified firm with investment level x will set a price as

$$p_1^* = \begin{cases} v, & \text{if } \delta \leq 1 - \frac{\theta_x r}{\theta_x r + \theta_x tv + v - tv}, \\ v + \frac{\theta_x r}{1-(1-\theta_x)t}, & \text{if } \delta > 1 - \frac{\theta_x r}{\theta_x r + \theta_x tv + v - tv} \end{cases} \tag{11}$$

and an uncertified firm, if entering the market, will set

$$p_0^* = \begin{cases} v, & \text{if } \delta \leq \frac{v}{\theta_x r + v}, \\ v + \theta_x r, & \text{if } \delta > \frac{v}{\theta_x r + v}. \end{cases} \tag{12}$$

As such, the firm opts to charge a premium price only when SC consumers account for a sufficiently large fraction of the market. Moreover, the size of the premium depends on these consumers' expectations, which are shaped by the signaling effect of certification. Indeed, we can show that when δ is small and $G \leq v$ – so that certification serves only as a signaling device rather than as a tool for mitigating market-entry risk – the firm either never uses certification, or no pure-strategy equilibrium exists. As proved in the proposition below, as long as δ is not too small, our main result continues to hold under a generalized regime threshold $\tilde{\eta}$.

Proposition 7. *Provided $\delta > \frac{v}{v+\theta_H r}$, there exists a threshold $\tilde{\eta} = \frac{\delta(\theta_H(r+tv)+v-tv)}{1-(1-\theta_H)t}$ such that a lower certification cost encourages investment if $G \leq \tilde{\eta}$ while discouraging investment if $G > \tilde{\eta}$.*

It follows that $\tilde{\eta}$ increases with the size of the SC segment, δ. In other words, certification is more likely to encourage investment when there are more SC consumers in the market. As illustrated in Figure 7, the region in which certification promotes investment expands as δ increases. Our results therefore indicate that the unintended investment-dampening effect of certification can be alleviated by improving consumer awareness in the market. In a stringently regulated market, raising consumer awareness may be more effective in inducing sustainability investment than reducing certification costs.

6. Conclusion

This paper studies how third-party certification shapes firms' sustainability improvement incentives when certification serves both as a market signal and as a tool for assessing regulatory risk. We develop a model in which a firm chooses whether to make sustainability investment and whether to

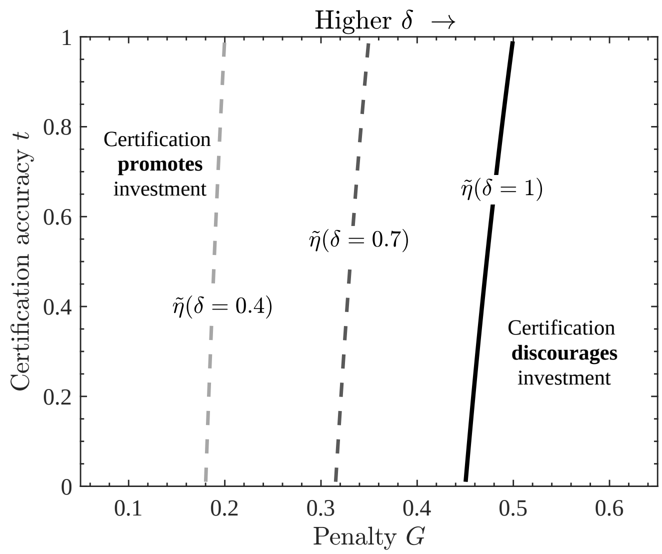


Figure 7 Effect of sustainability-conscious segment δ on the regime threshold $\tilde{\eta}$.

pursue certification before selling in a regulated market. Consumers observe price and certification status, but not the firm's investment or certification attempt. We find that lowering certification cost can either encourage or discourage firm investment; the latter occurs when regulation is sufficiently stringent such that the firm opportunistically leverages certification to evaluate market-entry risk, rather than proactively investing for improvement. We further show that improving certification accuracy and consumer awareness can mitigate this unintended consequence, whereas stronger regulatory penalties do not always promote sustainability investment.

Our analysis makes several simplifying assumptions that open avenues for future research. First, we focus on a monopolistic firm; an important direction is to examine how competition among multiple firms affects the role of certification. Second, we consider a firm selling directly to consumers. Future work could study supply chain settings in which a downstream firm sources from one or more suppliers that voluntarily adopt certification. Additionally, we develop a solution method for signaling games in which the sender makes private choices that generate noisy signals and signal-dependent actions. We hope this approach can be applied to other settings with similar informational frictions.

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Appendices for "Does Voluntary Certification Promote Sustainability Investment?"

A. PBE Definition and Solution Approach

This section introduces the PBE concept used in our analysis and outlines the procedure for solving the equilibrium. We first define the equilibrium components of the game and then describe the solution approach used to characterize the PBEs and identify the equilibrium outcomes reported in the main text.

A.1. PBE Definition

A Perfect Bayesian Equilibrium (PBE) in the game consists of the following elements:

- A strategy profile $((\mathbf{p}, x, y), \mathbf{d})$ consists of the firm's investment and certification decisions $(x, y) \in Z$, a pricing strategy $\mathbf{p} = (p_1, p_0))$ contingent on the realized certification status $S \in \{0, 1\}$, and consumers' purchase decisions $\mathbf{d} = (d(p_1, 1), d(p_0, 0))$ after observing price and certification status.
- A belief vector $\boldsymbol{\mu} = \{\mu_{HA}, \mu_{HN}, \mu_{LA}, \mu_{LN}\}$, where μ_{xy} represents consumers' posterior belief about the firm's investment and certification decisions after observing the price p_S and certification status S.
- The following consistency and optimality conditions: 1) strategies are sequentially rational given beliefs; 2) beliefs are updated using Bayes' rule on equilibrium path. Formally, a PBE takes the form of a strategy–belief profile

$$((\mathbf{p}^*, x^*, y^*), \mathbf{d}^*; \boldsymbol{\mu}^*).$$

We next characterize each component of the equilibrium profile and specify how it is derived from sequential rationality and belief consistency.

- Belief Consistency (Bayesian Updating):

  On the equilibrium path, for any observed (p_S, S) that occurs with positive probability,

  $$\mu_{xy}(p_S, S) = \Pr((x, y) \mid (p_S, S)). \tag{B}$$

- Consumers' Sequential Rationality:

  Given the belief vector $\boldsymbol{\mu}^*(p_S, S)$, consumers choose their purchase decision optimally:

  $$\forall p \in [0, \infty), S \in \{0, 1\}, \quad d^*(p_S, S; \boldsymbol{\mu}^*) \in \arg\max_{d \in \{0,1\}} \mathbb{E}_{(x,y)\sim \boldsymbol{\mu}^*(p_S, S)}[u(d; p_S, x, y, S)]. \tag{P$_C$}$$

- Firm's Sequential Rationality:

  Given $d^*(p_S, S; \boldsymbol{\mu}^*)$, the firm's equilibrium strategies (p_S^*, x^*, y^*) are optimal at each stage.

Stage 1 (Investment and Certification Decisions):

$$(x^*,y^*)\in\arg\max_{(x,y)\in Z}\mathbb{E}_{S\sim f}[\pi(p_S^*,x,y,S;\boldsymbol{\mu}^*)-c_H\mathbb{I}(x=H)-c_A\mathbb{I}(y=A)], \tag{P$_{M.1}$}$$

where $f_S(x,y)$ denotes the probability mass function of random variable $S\in\{0,1\}$ conditional on (x,y). Specifically, $f_1(x,y)=\begin{cases}0, & \text{if } y=N,\\ \gamma_x, & \text{if } y=A,\end{cases}$ and $f_0(x,y)=1-f_1(x,y)$.

Stage 2 (Price Decision):

$$\forall(x,y)\in Z \text{ and } \forall S\in\{0,1\},\quad p_S^*\in\arg\max_{p\in[0,\infty)}\pi(p_S,x^*,y^*,S;\boldsymbol{\mu}^*), \tag{P$_{M.2}$}$$

Solving for a PBE directly is challenging in dynamic Bayesian games with endogenous signaling, where the signal structure depends on the firm's own strategic choices and belief updates are nested across multiple decision margins—making it difficult to rule out implausible equilibria.

A.2. PBE Solution Approach

Because consumers observe neither the firm's investment decision x nor its certification decision y, equilibrium prices, consumer beliefs, and purchase decisions are jointly determined. Directly characterizing all PBEs is therefore analytically challenging.

To solve the model, we adopt a constructive approach and evaluate each candidate firm strategy $(x^*,y^*)\in Z$ separately. The equilibrium analysis proceeds in two steps.

First, for a given candidate strategy (x^*,y^*), we determine the equilibrium prices $p_S^*(x^*,y^*)$ for each realized certification status $S\in\{0,1\}$, taking into account the consumer purchase decisions induced by those prices. The corresponding on-path belief is specified to be consistent with the candidate strategy under consideration. For example, when evaluating the candidate strategy (H,A), the on-path belief satisfies $\mu_{HA}=1$, while all remaining belief components are equal to zero.

Second, given the induced prices and purchase decisions, we verify whether the candidate strategy (x^*,y^*) is optimal relative to all alternative strategies $(x',y')\in Z$. The following proposition formalizes this solution approach. For completeness, we restate Proposition 1.

Proposition 1. *For a given $(x^*,y^*)\in Z$, let $\boldsymbol{\mu}^*$ be a unit vector with entry $\mu_{x^*y^*}=1$ and the other entries being zero. Let $\mathbf{p}^*=(p_0^*,p_1^*)$ satisfy*

$$p_S^*\in\arg\max_{p_S\in[0,\infty)}\pi(p_S,x^*,y^*,S;\boldsymbol{\mu}^*),\quad \forall S\in\{0,1\}. \tag{C1}$$

The strategy (x^*,y^*), together with $\mathbf{p}^*$ and belief vector $\boldsymbol{\mu}^*$, constitutes a pure-strategy PBE if one of the following two conditions holds: (i) if $y^*=A$,

$$\Pi(\mathbf{p}^*,x^*,y^*;\boldsymbol{\mu}^*)\geq\Pi(\mathbf{p}^*,x',y';\boldsymbol{\mu}^*)\quad\forall(x',y')\in Z, \tag{C2}$$

or (ii) if $y^* = N$,

$$\begin{aligned}\Pi(\mathbf{p}^*, x^*, y^*; \boldsymbol{\mu}^*) \geq f_1(x', y') \max_{\hat{p}_1 \in [0,\infty)} \pi(\hat{p}_1, x', y', 1; \hat{\boldsymbol{\mu}}) + f_0(x', y')\pi(p_0^*, x', y', 0; \boldsymbol{\mu}^*) \\ - c_H \mathbb{I}(x' = H) - c_A \mathbb{I}(y' = A) \quad \forall (x', y') \in Z,\end{aligned} \tag{C3}$$

where $\hat{\boldsymbol{\mu}}$ is an off-equilibrium belief vector specified as $\hat{\mu}_{HA} = 1$ and the other entries being zero.

Verification.

The solution approach in Proposition 1 is consistent with the PBE definition introduced above.

- *Belief consistency* (B).

For each candidate strategy (x^*, y^*), the corresponding on-path belief vector $\boldsymbol{\mu}^*$ assigns probability one to the candidate strategy under consideration, i.e., $\mu_{x^*y^*} = 1$. Hence, the observed outcome (p_S^*, S) is consistent with Bayes' rule on the equilibrium path. For equilibria with $y^* = N$, off-path beliefs are specified by $\hat{\boldsymbol{\mu}}$ as in condition (C3). Therefore, the belief system satisfies (B).

- *Consumers' sequential rationality* (P_C).

Given the belief vector $\boldsymbol{\mu}^*$, consumers optimally choose whether to purchase after observing (p_S, S). The corresponding purchase decisions are explicitly incorporated when solving for the equilibrium prices p_S^*. Hence, consumers' purchase decisions satisfy (P_C).

- *Firm's sequential rationality* ($\mathrm{P}_{M.1}$)–($\mathrm{P}_{M.2}$).

Condition (C1) ensures that, for each realized certification status S, the equilibrium price p_S^* maximizes the firm's profit conditional on the candidate strategy (x^*, y^*), thereby satisfying ($\mathrm{P}_{M.2}$). Conditions (C2) and (C3) further guarantee that the candidate strategy (x^*, y^*) yields a weakly higher expected payoff than any alternative strategy in Z, thereby satisfying ($\mathrm{P}_{M.1}$). Therefore, the firm has no profitable deviation at either the pricing stage or the investment-certification stage.

Consequently, any strategy profile satisfying Proposition 1 constitutes a pure-strategy PBE.

B. Proof of Lemma 2

Proof. Fix the certification decision. The firm then chooses whether to invest.

For part (i), suppose the firm never applies for certification. The firm then chooses between (H, N) and (L, N). Consider the candidate equilibrium strategy (H, N). Following Proposition 1, let $\boldsymbol{\mu}^*$ be the belief vector satisfying $\mu_{HN}^* = 1$, with all other entries equal to zero. According to (4), under the belief $\boldsymbol{\mu}^*$, the demand function is given by $d(p_0, 0; \boldsymbol{\mu}^*) = \mathbb{I}(p_0 \leq v + \theta_H r)$. By condition (C1),

$$p_0^* \in \arg\max_{p_0 \in [0,\infty)} \{p_0 d(p_0, 0; \boldsymbol{\mu}^*) - c_H - \mathbb{I}(p_0 \leq v + \theta_H r)(1 - \theta_H)G\}.$$

Since profit is increasing in p_0 whenever demand is positive, the optimal price is $p_0^* = v + \theta_H r$, and the equilibrium profit is $\Pi(\mathbf{p}^*, H, N; \boldsymbol{\mu}^*) = v + \theta_H r - c_H - (1 - \theta_H)G$. To sustain (H, N) as the equilibrium strategy, condition (C3) requires that the equilibrium profit under (H,N) be no smaller than the profit from choosing (L,N). Substituting the corresponding profits into (C3) yields

$$v+\theta_H r-c_H-(1-\theta_H)G\geq v+\theta_H r-(1-\theta_L)G.$$

Rearranging the above inequality yields $c_H\leq(\theta_H-\theta_L)G$. Hence, (H,N) is sustained iff $c_H\leq\Delta_N\equiv(\theta_H-\theta_L)G$. Similarly, suppose (L,N) is the candidate equilibrium strategy. We obtain that (L,N) is sustained iff $c_H>\Delta_N\equiv(\theta_H-\theta_L)G$.

For part (ii), suppose the firm always applies for certification. The firm then chooses between (H,A) and (L,A). Consider the candidate equilibrium strategy (H,A). Following Proposition 1, let $\boldsymbol{\mu}^*$ be the belief vector satisfying $\mu^*_{HA}=1$, with all other entries equal to zero. Under the induced belief $\boldsymbol{\mu}^*$, according to (4), the demand functions are given by $d(p_1,1;\boldsymbol{\mu}^*)=\mathbb{I}(p_1\leq v+\frac{\theta_H r}{1-(1-\theta_H)t})$ and $d(p_0,0;\boldsymbol{\mu}^*)=\mathbb{I}(p_0\leq v)$. By condition (C1), $\mathbf{p}^*=(p_0^*,p_1^*)$ satisfies

$$p_S^*\in\arg\max_{p_S\in[0,\infty)}\pi(p_S,H,A,S;\boldsymbol{\mu}^*),\quad\forall S\in\{0,1\}.$$

Since profit is increasing in price whenever demand is positive, we obtain $p_1^*=v+\frac{\theta_H r}{1-(1-\theta_H)t}$ and $p_0^*=v$. Substituting the equilibrium prices into the profit function, we obtain the firm's profits from selling in the market conditional on the certification outcome

$$\pi(p_1^*,H,A,1;\boldsymbol{\mu}^*)=v+\frac{\theta_H r}{1-(1-\theta_H)t}-c_H-c_A-\frac{(1-\theta_H)(1-t)}{\theta_H+(1-\theta_H)(1-t)}G,$$

$$\pi(p_0^*,H,A,0;\boldsymbol{\mu}^*)=v-c_H-c_A-G.$$

If certification fails $(S=0)$, the firm may alternatively exit the market, in which case its profit is $-c_H-c_A$. Comparing the profits from remaining in and exiting the market following certification failure yields the threshold $G=v$. Therefore, the firm remains in the market following certification failure when $G\leq v$, and exits the market when $G>v$.

To sustain (H,A) as the equilibrium strategy, condition (C2) requires that the equilibrium profit under (H,A) be no smaller than the profit from choosing (L,A). We now distinguish two cases.

When $G\leq v$, substituting the corresponding profits into (C2) yields

$$\gamma_H(p_1^*-\psi(H,A,1)G)+(1-\gamma_H)(p_0^*-G)-c_H-c_A\geq\gamma_L(p_1^*-\psi(L,A,1)G)+(1-\gamma_L)(p_0^*-G)-c_A.$$

Rearranging the above inequality yields $c_H\leq\tau_1\equiv(\theta_H-\theta_L)\big(\frac{\theta_H rt}{1-(1-\theta_H)t}+G\big)$. Hence, (H,A) is sustained iff $c_H\leq\tau_1$. Similarly, suppose (L,A) is the candidate equilibrium strategy. We obtain that (L,A) is sustained iff $c_H>\tau_2\equiv(\theta_H-\theta_L)\big(\frac{\theta_L rt}{1-(1-\theta_L)t}+G\big)$. It is straightforward to show that $\tau_1>\tau_2$ since $\theta_H>\theta_L$. For the intermediate region $\tau_2<c_H\leq\tau_1$, both (H,A) and (L,A) are sustained in equilibrium. Following the sender-preferred equilibrium selection (Kamenica and Gentzkow 2011,

Bimpikis and Mantegazza 2023), we choose the equilibrium that yields the higher profit to the firm. Comparing the equilibrium profits under the two strategies, we find that (H, A) yields a higher profit if $c_H \leq (\theta_H - \theta_L)(G + r)$. Since $(\theta_H - \theta_L)(G + r) > \tau_1$, (H, A) yields a higher profit throughout the overlap region. Hence, the equilibrium cutoff is determined by the condition for sustaining (H, A), implying that $\Delta_A = \tau_1 = (\theta_H - \theta_L)\big(\frac{\theta_H r t}{1-(1-\theta_H)t} + G\big)$.

When $G > v$, the firm exits the market after certification failure. Substituting the corresponding profits into (C2) yields

$$\gamma_H(p_1^* - \psi(H, A, 1)G) - c_H - c_A \geq \gamma_L(p_1^* - \psi(L, A, 1)G) - c_A.$$

Rearranging the above inequality yields $c_H \leq \tau_3 \equiv (\theta_H - \theta_L)\big(t\frac{v+\theta_H r}{1-(1-\theta_H)t} + (1-t)G\big)$. Hence, (H, A) is sustained iff $c_H \leq \tau_3$. Similarly, suppose (L, A) is the candidate equilibrium strategy. We obtain that (L, A) is sustained iff $c_H > \tau_4 \equiv (\theta_H - \theta_L)\big(t\frac{v+\theta_L r}{1-(1-\theta_L)t} + (1-t)G\big)$. An overlap region $\tau_4 < c_H \leq \tau_3$ exists in which both (H, A) and (L, A) are sustained in equilibrium. Comparing the equilibrium profits under the two strategies, we find that (H, A) yields a higher profit if $c_H \leq (\theta_H - \theta_L)(r + tv + (1-t)G)$. Since $(\theta_H - \theta_L)(r + tv + (1-t)G) > \tau_3$, (H, A) yields a higher profit throughout the overlap region, implying that $\Delta_A = \tau_3 = (\theta_H - \theta_L)\big(t\frac{v+\theta_H r}{1-(1-\theta_H)t} + (1-t)G\big)$. $\square$

C. Proof of Lemma 1 and Propositions 2–5

This section characterizes the Perfect Bayesian Equilibria (PBE) of the game and provides the equilibria in the main text. In particular, the equilibrium prices shown in Lemma 1 and the equilibrium strategies characterized in Propositions 2–5 are all derived from the PBE analysis presented below.

Proof. The equilibrium analysis proceeds in two steps.

First, following the solution approach outlined above, we derive the conditions under which a candidate PBE exists. Specifically, for each candidate firm strategy $(x, y) \in Z$, we determine the corresponding equilibrium prices and consumer purchase decisions, and then verify whether the strategy under evaluation is optimal relative to all feasible deviations. This yields the parameter regions under which the candidate equilibrium can be sustained.

Second, multiple PBEs may coexist for some parameter values. In such cases, we adopt the sender-preferred refinement and select the equilibrium that yields the highest expected payoff to the firm (Kamenica and Gentzkow 2011, Bimpikis and Mantegazza 2023).

Step 1: PBE derivation. A PBE can be formally represented as

$$((\mathbf{p}^*, x^*, y^*), \mathbf{d}^*; \boldsymbol{\mu}^*),$$

where (x^*, y^*) denotes the firm's equilibrium investment and certification strategy; $\mathbf{p}^* = (p_1^*, p_0^*)$ denotes the firm's equilibrium pricing strategy contingent on the realized certification status $S \in$

$\{0,1\}$; $\mathbf{d}^* = (d^*(p_1,1), d^*(p_0,0))$ represents consumers' equilibrium purchase decisions; and $\boldsymbol{\mu}^*$ specifies the corresponding on-path beliefs.

To characterize the equilibrium, we proceed as follows.

First, for a given candidate strategy $(x^*, y^*) \in Z$, consumers' on-path beliefs assign probability one to the candidate strategy profile, i.e., $\mu_{x^*y^*} = 1$, while all remaining entries are equal to zero.

Second, given the candidate strategy, the corresponding on-path beliefs, and a specified consumer purchase decision, we determine the equilibrium prices $p_S^*(x,y)$ for each certification status $S \in \{0,1\}$ satisfying condition (C1).

Third, given the resulting prices, we verify whether the investment and certification strategy under evaluation is optimal relative to the other feasible strategies. The resulting conditions ensure that the candidate strategy is consistent with the specified beliefs and therefore can be sustained as an equilibrium.

It is worth noting that consumers' purchase decisions are not imposed independently of the firm's pricing strategy. Rather, consumers decide whether to purchase after observing (p_S, S), and the firm's equilibrium prices must be consistent with the specified purchase decisions. Accordingly, in the analysis below, we consider the following possible consumer purchase decisions: (i) $d(p_S,S) = 1, \forall S \in \{0,1\}$, (ii)$d(p_1,1) = 1, d(p_0,0) = 0$ and (iii) $d(p_S,S) = 0, \forall S \in \{0,1\}$.

We analyze these cases in turn.

Case 1: (H,A) with $\mu_{HA} = 1$.

In this case, the firm optimally chooses to invest in sustainability and apply for certification. Consumers' on-path beliefs satisfy $\mu_{HA} = 1$ and $\mu_{HN} = \mu_{LA} = \mu_{LN} = 0$. According to (4), consumers' optimal purchase decision is given by the indicator function $d(p_S,S) = \mathbb{I}(p_S \le \overline{p}_S(\boldsymbol{\mu})$, where $\overline{p}_S$ denotes the threshold price (i.e., the maximum willingness-to-pay). When consumers believe the firm has adopted the (H,A) strategy, i.e., $\mu_{HA} = 1$, the threshold prices for each certification status are given by

$$\overline{p}_1 = v + \frac{\theta_H r}{1-(1-\theta_H)t} \quad (\text{if } S=1), \quad \overline{p}_0 = v \quad (\text{if } S=0).$$

Given the candidate strategy (H,A) and the belief $\mu_{HA} = 1$, the firm's pricing strategy may induce different consumer purchase decisions across certification states. We therefore analyze three candidate PBEs, corresponding respectively to purchase in both states, purchase only following certification success, and no purchase in either state.

(i) PBE $((p_1^*, p_0^*, H, A), (1,1); \mu_{HA} = 1)$**.**

We begin by considering the candidate PBE under which the firm invests in sustainability and applies for certification, sells in the market under both certification status $S \in \{0,1\}$, and sets prices that induce consumer purchase in both states, i.e., $d(p_S,S) = 1$ for all $S \in \{0,1\}$.

Conditional on the candidate strategy (H, A), the belief $\mu_{HA} = 1$, and the purchase decision $d(p_S, S) = 1$ for all $S \in \{0, 1\}$, the equilibrium prices satisfy

$$p_S^* \in \arg\max_{p_S \in [0, \overline{p}_S)} \pi(p_S, H, A, S; \boldsymbol{\mu}^*), \quad \forall S \in \{0, 1\}, \tag{C.1}$$

$$s.t. \quad \Pi(p_1^*, p_0^*, H, A) \geq \Pi(p_1^*, p_0^*, L, A), \tag{C.2}$$

$$\Pi(p_1^*, p_0^*, H, A) \geq \Pi(p_1^*, p_0^*, H, N), \tag{C.3}$$

$$\Pi(p_1^*, p_0^*, H, A) \geq \Pi(p_1^*, p_0^*, L, N), \tag{C.4}$$

where

$$\Pi(p_1^*, p_0^*, H, A) = f_1(H, A)\pi(p_1^*, H, A, 1; \boldsymbol{\mu}^*) + f_0(H, A)\pi(p_0^*, H, A, 0; \boldsymbol{\mu}^*), \tag{C.5}$$

$$\Pi(p_1^*, p_0^*, H, N) = p_0^* - (1 - \theta_H)G - c_H, \tag{C.6}$$

$$\Pi(p_1^*, p_0^*, L, A) = \gamma_L(p_1^* - \psi(L, A, 1)G - c_A) + (1 - \gamma_L)(p_0^* - G - c_A), \tag{C.7}$$

$$\Pi(p_1^*, p_0^*, L, N) = p_0^* - (1 - \theta_L)G. \tag{C.8}$$

Here, constraint (C.1) ensures that p_S^* is an optimal price to the firm conditional on each certification status S. At the same time, the constraint coincides with consumers' purchase constraints, requiring $p_S \leq \overline{p}_S$ so that, upon observing $(p_1, 1)$ or $(p_0, 0)$, consumers optimally choose to purchase the product, i.e., $d(p_S, S) = 1$ for $S \in \{0, 1\}$. Constraints (C.2)–(C.4) ensure that the strategy (H, A) being evaluated is optimal relative to the other strategies, thus consistent with the given belief $\boldsymbol{\mu}^*$.

Since the firm's profit is increasing in price conditional on inducing purchase, the upper bounds bind at the optimum. Hence,

$$p_1^* = \overline{p}_1 \equiv v + \frac{\theta_H}{1 - (1 - \theta_H)t} r, \qquad p_0^* = \overline{p}_0 \equiv v.$$

We next verify whether the strategy (H, A) remains optimal relative to the alternative strategies under the induced prices. Substituting $(p_1, p_0) = (\overline{p}_1, \overline{p}_1)$ into the constraints (C.2)–(C.4) yields the following conditions:

$$\Pi(\overline{p}_1, \overline{p}_0, H, A) \geq \Pi(\overline{p}_1, \overline{p}_0, H, N) \iff c_A \leq \theta_H r,$$
$$\Pi(\overline{p}_1, \overline{p}_0, H, A) \geq \Pi(\overline{p}_1, \overline{p}_1, L, A) \iff c_H \leq (\theta_H - \theta_L)(\frac{\theta_H r t}{1 - (1 - \theta_H)t} + G),$$
$$\Pi(\overline{p}_1, \overline{p}_0, H, A) \geq \Pi(\overline{p}_1, \overline{p}_0, L, N) \iff c_A \leq (\theta_H - \theta_L)G + \theta_H r - c_H.$$

These constraints define two parameter regions, denoted by Ψ_1 and Ψ_2, under which the candidate equilibrium can be sustained. Specifically,

$$\Psi_1 = \left\{ \begin{aligned} & c_A \leq \frac{\theta_H r \big(1 - (1 - \theta_L)t\big)}{1 - (1 - \theta_H)t}, \\ & c_H \leq \frac{(\theta_H - \theta_L)\big((1 - t)G + \theta_H t(r + G)\big)}{1 - (1 - \theta_H)t} \end{aligned} \right\},$$

and

$$\Psi_2 = \left\{ \begin{array}{l} \dfrac{\theta_H r\big(1-(1-\theta_L)t\big)}{1-(1-\theta_H)t} < c_A \le \theta_H r, \\ c_H \le (\theta_H - \theta_L)G + \theta_H r - c_A \end{array} \right\}.$$

In addition, the candidate purchase decision $d(p_S, S) = 1$ for all $S \in \{0,1\}$, which requires that the firm prefers remaining active in the market to exiting under each realized certification status. The firm's profit from selling in the market is

$$\Pi_{\text{sell}} = \begin{cases} v + \frac{\theta_H r}{1-(1-\theta_H)t} - c_H - c_A - \frac{(1-\theta_H)(1-t)}{\theta_H + (1-\theta_H)(1-t)} G, & \text{if } S = 1, \\ v - c_H - c_A - G, & \text{if } S = 0. \end{cases}$$

while exiting the market yields the sunk-cost profit

$$\Pi_{\text{exit}} = -c_H - c_A.$$

A rational firm chooses to sell if $\Pi_{\text{sell}} \ge \Pi_{\text{exit}}$, and otherwise effectively exits by setting a prohibitively high price. Comparing Π_{sell} and Π_{exit} yields the following conditions for remaining active:

$$G \le \eta_0 \equiv \frac{v - tv + \theta_H(r + tv)}{(1-\theta_H)(1-t)} \text{ if } S = 1, \quad G \le \eta_1 \equiv v \text{ if } S = 0.$$

Hence, the candidate purchase decision $d(p_S, S) = 1$ for all $S \in \{0,1\}$ can be sustained only if both conditions are satisfied. Otherwise, the firm prefers to exit in at least one certification state.

Therefore, a PBE

$$\Big((p_1^* = v + \frac{\theta_H}{1-(1-\theta_H)t} r,\, p_0^* = v, H, A),\, (1,1);\, \mu_{HA} = 1 \Big)$$

exists whenever $(c_A, c_H) \in \Psi_1 \cup \Psi_2$ and $G \le \eta_0$ when $S = 1$ and $G \le \eta_1$ when $S = 0$.

(ii) PBE $((p_1^*, p_0^*, H, A), (1,0); \mu_{HA} = 1)$ **.**

We next consider the candidate PBE in which the firm invests in sustainability and applies for certification, sets a price that induces consumer purchase following certification success, and sets a sufficiently high price that deters purchase following certification failure, i.e., $d(p_1, 1) = 1$ and $d(p_0, 0) = 0$.

Conditional on the candidate strategy (H, A), the belief $\mu_{HA} = 1$, and the purchase decisions $d(p_1, 1) = 1$ and $d(p_0, 0) = 0$, the equilibrium prices satisfy

$$p_S^* \in \arg\max_{p_1 \le \bar{p}_1, p_0 \ge \bar{p}_0} \pi(p_S, H, A, S; \boldsymbol{\mu}^*), \quad \forall S \in \{0,1\}, \tag{C.9}$$

$$\Pi(p_1^*, p_0^*, H, A) \ge \Pi(p_1^*, p_0^*, L, A), \tag{C.10}$$

$$\Pi(p_1^*, p_0^*, H, A) \ge \Pi(p_1^*, p_0^*, H, N), \tag{C.11}$$

$$\Pi(p_1^*, p_0^*, H, A) \ge \Pi(p_1^*, p_0^*, L, N), \tag{C.12}$$

where

$$\Pi(p_1^*, p_0^*, H, A) = \gamma_H[p_1^* - c_H - c_A - \frac{(1-\theta_H)(1-t)}{\theta_H + (1-\theta_H)(1-t)}G] + (1-\gamma_H)(-c_H - c_A), \tag{C.13}$$

$$\Pi(p_1^*, p_0^*, H, N) = -c_H, \tag{C.14}$$

$$\Pi(p_1^*, p_0^*, L, A) = \gamma_L(p_1^* - c_A - \psi(L, A, 1)G) + (1-\gamma_L)(-c_A), \tag{C.15}$$

$$\Pi(p_1^*, p_0^*, L, N) = 0. \tag{C.16}$$

Constraint (C.9) characterizes the pricing restrictions required to support the candidate purchase decision. Specifically, the condition $p_1 \le \overline{p}_1$ ensures that consumers purchase the product following certification success, whereas $p_0 > \overline{p}_0$ deters purchase following certification failure. Therefore, the firm remains active in the market when $S = 1$ and effectively exits the market when $S = 0$. Constraints (C.10)–(C.12) ensure that the strategy (H, A) under evaluation is optimal relative to the other feasible strategies and therefore consistent with the belief $\mu_{HA} = 1$.

Since the firm's profit is increasing in price conditional on inducing purchase, the upper bound binds when $S = 1$. Hence,

$$p_1^* = \overline{p}_1 \equiv v + \frac{\theta_H}{1-(1-\theta_H)t}r.$$

When $S = 0$, any price satisfying $p_0 > \overline{p}_0 = v$, which deters purchase and yields the same payoff to the firm. Therefore,

$$p_0^* = p_E \in (v, \infty).$$

We next verify whether the strategy (H, A) remains optimal relative to the alternative strategies under the induced prices. Substituting $(p_1, p_0) = (\overline{p}_1, p_E)$ into (C.10)–(C.12) yields:

$$\Pi(\overline{p}_1, p_E, H, A) \ge \Pi(\overline{p}_1, p_E, L, A) \iff c_A \le (\theta_H - \theta_L)(v + \frac{\theta_H r}{1-(1-\theta_H)t} + (1-t)G),$$

$$\Pi(\overline{p}_1, p_E, H, A) \ge \Pi(\overline{p}_1, p_E, H, N) \iff c_H \le \theta_H(r + tv) + v(1-t) - (1-\theta_H)(1-t)G,$$

$$\Pi(\overline{p}_1, p_E, H, A) \ge \Pi(\overline{p}_1, p_E, L, N) \iff c_A \le -c_H + \theta_H(r + tv) + v(1-t) - (1-\theta_H)(1-t)G.$$

These conditions define the parameter regions define a parameter region, denoted by Ψ_3, under which the candidate equilibrium can be sustained. Specifically,

$$\Psi_3 = \left\{ \begin{array}{l} c_A \le (\theta_H - \theta_L)(v + \frac{\theta_H r}{1-(1-\theta_H)t} + (1-t)G), \\ c_H \le -c_A + \theta_H(r + tv) + v(1-t) - (1-\theta_H)(1-t)G \end{array} \right\}.$$

In addition, the candidate purchase decision $d(p_1, 1) = 1$ and $d(p_0, 0) = 0$, which requires that the firm prefers remaining active in the market when $S = 1$ and prefers exiting the market when $S = 0$. Comparing the payoff from selling with the payoff from exiting yields $G \le \eta_0$ for $S = 1$ and $G > \eta_1$ for $S = 0$. When $S = 0$.

Therefore, a PBE

$$\Big((H, A, p_1^* = v + \frac{\theta_H}{1-(1-\theta_H)t} r,\ p_0^* = p_E),\ (1,0);\ \mu(H, A \mid p_S, S) = 1\Big)$$

exists whenever $(c_A, c_H) \in \Psi_3$, $G > \eta_1$ and $G \leq \eta_0$.

(iii) PBE $((p_1^*, p_0^*, H, A), (0,0); \mu_{HA} = 1)$.

Finally, we consider the candidate PBE under which the firm invests in sustainability and applies for certification, but sets prices that deter consumer purchase under both certification outcomes, that is, $d(p_S, S) = 0$ for all $S \in \{0,1\}$. Under this candidate equilibrium, the firm effectively exits the market regardless of the realized certification status. Therefore, its expected payoff under each strategy is given by

$$\Pi(p_1^*, p_0^*, H, A) = -c_H - c_A, \tag{C.17}$$

$$\Pi(p_1^*, p_0^*, H, N) = -c_H, \tag{C.18}$$

$$\Pi(p_1^*, p_0^*, L, A)) = -c_A, \tag{C.19}$$

$$\Pi(p_1^*, p_0^*, L, N) = 0. \tag{C.20}$$

Comparing these payoffs, we obtain $\Pi(p_1^*, p_0^*, H, A) < \Pi(p_1^*, p_0^*, H, N)$, $\Pi(p_1^*, p_0^*, H, A) < \Pi(p_1^*, p_0^*, L, A)$ and $\Pi(p_1^*, p_0^*, p_0 * H, A) < \Pi(p_1^*, p_0^*, L, N)$. Hence, the strategy (H, A) is not optimal relative to the alternative strategies and therefore cannot be sustained in equilibrium. This implies that the if $G > \eta_0$, the PBE $((p_1^*, p_0^*, H, A), (0,0); \mu_{HA} = 1)$ for $S \in \{0,1\}$ cannot exist. Therefore, in the remainder of the paper, we focus on the parameter region where the penalty G remains below a critical threshold. This condition ensures the firm's ex-post participation when $S = 1$, thereby sustaining the (H, A) equilibrium. Formally, $\boldsymbol{G \leq \frac{v - tv + \theta_H(r + tv)}{(1-\theta_H)(1-t)}}$.

Case 2: (H, N) with $\mu_{HN} = 1$.

In this case, we consider the candidate strategy (H, N), under which the firm invests in sustainability but does not apply for certification. Consumers' on-path beliefs satisfy $\mu_{HN} = 1$. Since certification is not adopted, consumers observe only the uncertified status $(p_0, 0)$. According to (4), consumers' purchase decision is given by $d(p_0, 0) = \mathbb{I}\big(p_0 \leq \overline{p}_0(\boldsymbol{\mu}^*)\big)$, where $\overline{p}_0$ denotes the maximum willingness-to-pay. Under the belief $\mu_{HN} = 1$, this threshold is given by

$$\overline{p}_0 = v + \theta_H r.$$

Off-path belief specification. Under the candidate equilibria (H,N) and (L,N), certification success $S=1$ does not arise on the equilibrium path. Consequently, Bayes' rule does not determine consumers' beliefs following the off-path observation $(p_1,1)$. Following the Intuitive Criterion (Cho and Kreps 1987), consumers attribute an off-equilibrium certified outcome to the firm type with the stronger incentive to invest in sustainability. Consistent with the off-path belief specification in (C3), consumers therefore assign probability one to an investing and certifying firm, i.e., $\hat{\mu}_{HA}=1$. Under this belief, the corresponding off-path price is $\hat{p}_1^*=v+\frac{\theta_H r}{1-(1-\theta_H)t}$. Moreover, recall from **Case 1 (iii)** that $G\leq\frac{v-tv+\theta_H(r+tv)}{(1-\theta_H)(1-t)}$.

Under this condition, a firm obtaining certification success prefers remaining active in the market. Hence, consumers optimally purchase upon observing $(\hat{p}_1^*,1)$, implying $d(\hat{p}_1^*,1)=1$.

We now analyze the firm's pricing strategy under the candidate equilibrium (H,N). Since certification is not adopted, only the uncertified status $(p_0,0)$ is observed on the equilibrium path. Depending on the firm's pricing decision, consumers may either purchase the product or refrain from purchasing it. We analyze these two cases in turn.

(i) PBE $((p_0^*,H,N),1;\mu_{HN}=1)$**.**

We begin by considering the candidate PBE under which the firm invests in sustainability, does not apply for certification, and sets a price that induces consumer purchase, i.e., $d(p_0,0)=1$. Conditional on the candidate strategy (H,N), the belief $\mu_{HN}=1$, and the purchase decision $d(p_0,0)=1$, the equilibrium price satisfies

$$p_0^*\in\arg\max_{p_0\in[0,\overline{p}_0)}\pi(p_0,H,N,0;\boldsymbol{\mu}^*), \tag{C.21}$$

$$\Pi(p_1^*,p_0^*,H,N)\geq\Pi(p_1^*,p_0^*,L,N), \tag{C.22}$$

$$\Pi(p_1^*,p_0^*,H,N)\geq\Pi(p_1^*,p_0^*,H,A), \tag{C.23}$$

$$\Pi(p_1^*,p_0^*,H,N)\geq\Pi(p_1^*,p_0^*,L,A), \tag{C.24}$$

where

$$\Pi(p_1^*,p_0^*,H,N)=p_0^*-c_H-(1-\theta_H)G, \tag{C.25}$$

$$\Pi(p_1^*,p_0^*,H,A)=\gamma_H(p_1^*-c_H-c_A-\psi(H,A,1)G)+(1-\gamma_H)(p_0^*-c_H-c_A-G), \tag{C.26}$$

$$\Pi(p_1^*,p_0^*,L,A)=\gamma_L(p_1^*-c_A-\psi(L,A,1)G)+(1-\gamma_L)(p_0^*-c_A-G), \tag{C.27}$$

$$\Pi(p_1^*,p_0^*,L,N)=p_0^*-(1-\theta_L)G. \tag{C.28}$$

Constraint (C.21) ensures that p_0^* is an optimal price conditional on the uncertified status. The candidate purchase decision $d(p_0,0)=1$ requires that $p_0\leq\overline{p}_0$. Constraints (C.22)–(C.24) ensure that the strategy (H,N) under evaluation is optimal relative to the other feasible strategies and therefore consistent with the belief $\mu_{HN}=1$.

Combining the strategy-optimality constraints with the consumer purchase requirement yields

$$p_0 \le p_1 - \Delta,$$

where

$$\Delta \equiv \max\{\Delta_1, \Delta_2\},$$

with

$$\triangle_1 = \frac{c_A}{1-(1-\theta_H)t}, \quad \Delta_2 = \frac{c_A - c_H + (\theta_H - \theta_L)G}{1-(1-\theta_L)t}.$$

Since the firm's profit is increasing in price conditional on inducing purchase, the upper bound binds au the optimum. Together with the off-path belief specification,

$$p_0^* = \overline{p}_0 \equiv v + \theta_H r, \qquad p_1^* = \overline{p}_1 \equiv v + \frac{\theta_H}{1-(1-\theta_H)t} r.$$

We next verify whether the strategy (H, N) remains optimal relative to the alternative strategies under the induced prices. Substituting $(\overline{p}_1, \overline{p}_0)$ into constraints (C.22)–(C.24) yields the parameter region Ψ_4,

$$\Psi_4 = \begin{Bmatrix} c_A > \theta_H r t(1-\theta_H), \\ c_H \le (\theta_H - \theta_L)G \end{Bmatrix}.$$

In addition, the candidate purchase decision $d(p_0, 0) = 1$ requires that the firm prefers remaining active in the market rather than exiting. The payoff from selling is

$$\Pi_{\text{sell}} = v + \theta_H r - c_H - (1-\theta_H)G,$$

while exiting yields the sunk-cost payoff $\Pi_{\text{exit}} = -c_H$. The firm sells if and only if $\Pi_{\text{sell}} \ge \Pi_{\text{exit}}$, which defines the threshold

$$G \le \eta_2 \equiv \frac{v + \theta_H r}{1-\theta_H}.$$

We get that if $G \le \eta_2$, the firm invests in sustainability, does not apply for certification, and sells in the market; otherwise, it exits by setting a prohibitively high price.

Therefore, a PBE

$$\Big((p_0^* = v + \theta_H r, H, N),\ 1;\ \mu_{HN} = 1\Big)$$

exists whenever $(c_A, c_H) \in \Psi_4$ and $G \le \eta_2$.

(ii) PBE $((p_0^*, H, N), 0; \mu_{HN} = 1)$**.**

We then analyze the PBE under which under which the firm invests in sustainability, does not apply for certification, and sets a price that deters consumer purchase, i.e., $d(p_0, 0) = 0$. Under this candidate equilibrium, the firm effectively exits the market. Together with the off-path belief specification, the firm's expected payoffs under different strategies are

$$\Pi(p_1^*, p_0^*, H, A) = \gamma_H(p_1^* - c_H - c_A - \psi(L, A, 1)G) + (1-\gamma_H)(-c_H - c_A), \tag{C.29}$$

$$\Pi(p_1^*, p_0^*, H, N) = -c_H, \tag{C.30}$$

$$\Pi(p_1^*, p_0^*, L, A) = \gamma_L(p_1^* - c_A - \psi(L, A, 1)G) + (1-\gamma_L)(-c_A), \tag{C.31}$$

$$\Pi(p_1^*, p_0^*, L, N) = 0. \tag{C.32}$$

Comparing these profits, we obtain $\Pi(p_1^*, p_0^*, H, N) < \Pi(p_1^*, p_0^*, L, N)$. Hence, the strategy (H, N) is not optimal relative to the alternative strategies and therefore cannot be sustained in equilibrium. This implies that the PBE $((p_0^*, H, N), 0; \mu_{HN} = 1)$ cannot be sustained if $G > \eta_2$. Consequently, for the remainder of this paper, we restrict our analysis to the scenario where the penalty remains below a critical threshold to ensure the existence of the (H, N) strategy. Formally, we assume $G \leq \frac{v+\theta_H r}{1-\theta_H}$.

In Case 1, we obtain that our subsequent analysis should be conducted under the condition $G \leq \eta_0$. In the current setting, the relevant bound becomes $G \leq \eta_2$, where $\eta_2 < \eta_0$. Therefore, the subsequent analysis is conducted under the assumption $G \leq \eta_2$.

Case 3: (L, A) with $\mu_{LA} = 1$.

In this case, the firm optimally chooses not to invest in sustainability but apply for certification. Consumers' on-path beliefs satisfy $\mu_{LA} = 1$ and $\mu_{HA} = \mu_{HN} = \mu_{LN} = 0$. According to (4), consumers' optimal purchase decision is given by the indicator function $d(p_S, S) = \mathbb{I}(p_S \leq \overline{p}_S(\boldsymbol{\mu})$, where $\overline{p}_S$ denotes the threshold price (i.e., the maximum willingness-to-pay). When consumers believe the firm has adopted the (L, A) strategy, i.e., $\mu_{LA} = 1$, the threshold prices for each certification status are given by

$$\overline{p}_1 = v + \frac{\theta_L r}{1-(1-\theta_L)t} \quad (\text{if } S = 1), \quad \overline{p}_0 = v \quad (\text{if } S = 0).$$

Given the candidate strategy (L, A) and the belief $\mu_{LA} = 1$, the firm's pricing strategy may induce different consumer purchase decisions across certification states. We therefore analyze three candidate PBEs, corresponding respectively to purchase in both states, purchase only following certification success, and no purchase in either state.

(i) PBE $((p_1^*, p_0^*, L, A), (1, 1); \mu_{LA} = 1)$**.**

We begin by considering the candidate PBE under which the firm does not invest in sustainability but apply for certification, sells in the market under both certification status $S \in \{0, 1\}$, and sets prices that induce consumer purchase in both states, i.e., $d(p_S, S) = 1$ for all $S \in \{0, 1\}$.

Conditional on the candidate strategy (L, A), the belief $\mu_{LA} = 1$, and the purchase decision $d(p_S, S) = 1$ for all $S \in \{0, 1\}$, the equilibrium prices satisfy

$$p_S^* \in \arg\max_{p_S \in [0, \overline{p}_S)} \pi(p_S, L, A, S; \boldsymbol{\mu}^*), \quad \forall S \in \{0, 1\}, \tag{C.33}$$

$$s.t. \quad \Pi(p_1^*, p_0^*, L, A) \geq \Pi(p_1^*, p_0^*, L, N), \tag{C.34}$$

$$\Pi(p_1^*, p_0^*, L, A) \geq \Pi(p_1^*, p_0^*, H, A), \tag{C.35}$$

$$\Pi(p_1^*, p_0^*, L, A) \geq \Pi(p_1^*, p_0^*, H, N), \tag{C.36}$$

where

$$\Pi(p_1^*, p_0^*, L, A) = f_1(L,A)\pi(p_1^*, L, A, 1; \boldsymbol{\mu}^*) + f_0(L,A)\pi(p_0^*, H, A, 0; \boldsymbol{\mu}^*), \tag{C.37}$$

$$\Pi(p_1^*, p_0^*, H, N) = p_0^* - c_H - (1-\theta_H)G, \tag{C.38}$$

$$\Pi(p_1^*, p_0^*, H, A) = \gamma_H(p_1^* - c_A - c_H - \psi(H,A,1)G) + (1-\gamma_H)(p_0^* - c_A - c_H - G), \tag{C.39}$$

$$\Pi(p_1^*, p_0^*, L, N) = p_0^* - (1-\theta_L)G. \tag{C.40}$$

Here, constraint (C.33) ensures that p_S^* is an optimal price to the firm conditional on each certification status S. At the same time, the constraint coincides with consumers' purchase constraints, requiring $p_S \leq \overline{p}_S$ so that, upon observing $(p_1, 1)$ or $(p_0, 0)$, consumers optimally choose to purchase the product, i.e., $d(p_S, S) = 1$ for $S \in \{0, 1\}$. Constraints (C.34)–(C.36) ensure that the strategy (L, A) being evaluated is optimal relative to the other strategies, thus consistent with the given belief $\boldsymbol{\mu}^*$.

Since the firm's profit is increasing in price conditional on inducing purchase, the upper bounds bind at the optimum. Hence,

$$p_1^* = \overline{p}_1 \equiv v + \frac{\theta_L}{1-(1-\theta_L)t} r, \qquad p_0^* = \overline{p}_0 \equiv v.$$

We next verify whether the strategy (L, A) remains optimal relative to the alternative strategies under the induced prices. Substituting $(p_1, p_0) = (\overline{p}_1, \overline{p}_1)$ into the constraints (C.34)–(C.36) yields the following conditions:

$$\Pi(\overline{p}_1, \overline{p}_1, L, A) \geq \Pi(\overline{p}_1, \overline{p}_1, L, N) \iff c_A \leq (1-\theta_L)(G-v)t,$$

$$\Pi(\overline{p}_1, \overline{p}_1, L, A) \geq \Pi(\overline{p}_1, \overline{p}_1, H, A) \iff c_H > (\theta_H - \theta_L)(t(\frac{\theta_L r}{1-(1-\theta_L)t} + v) + (1-t)G),$$

$$\Pi(\overline{p}_1, \overline{p}_1, L, A) \geq \Pi(\overline{p}_1, \overline{p}_1, H, N) \iff c_H > (1-\theta_L)(v-G)t + (\theta_H - \theta_L)G + c_A.$$

These constraints define two parameter regions, denoted by Ψ_5, under which the candidate equilibrium can be sustained. Specifically,

$$\Psi_5 = \left\{ \begin{aligned} & c_A \leq \theta_H r, \\ & c_H > \frac{(\theta_H - \theta_L)((1-t)G + \theta_L t(r+G))}{1-(1-\theta_L)t} \end{aligned} \right\}.$$

In addition, the candidate purchase decision $d(p_S, S) = 1$ for all $S \in \{0, 1\}$, which requires that the firm prefers remaining active in the market to exiting under each realized certification status. The firm's profit from selling in the market is

$$\Pi_{\text{sell}} = \begin{cases} v + \frac{\theta_L r}{1-(1-\theta_L)t} - c_A - \frac{(1-\theta_L)(1-t)}{\theta_L + (1-\theta_L)(1-t)} G, & \text{if } S = 1, \\ v - c_A - G, & \text{if } S = 0, \end{cases}$$

while exiting the market yields the sunk-cost profit

$$\Pi_{\text{exit}} = -c_A.$$

A rational firm chooses to sell if $\Pi_{\text{sell}} \geq \Pi_{\text{exit}}$, and otherwise effectively exits by setting a prohibitively high price. Comparing Π_{sell} and Π_{exit} yields the following conditions for remaining active:

$$G \leq \eta_3 \equiv \frac{v - tv + \theta_L(r + tv)}{(1-\theta_L)(1-t)} \text{ if } S = 1, \quad G \leq \eta_1 \equiv v \text{ if } S = 0.$$

Hence, the candidate purchase decision $d(p_S, S) = 1$ for all $S \in \{0,1\}$ can be sustained only if both conditions are satisfied. Otherwise, the firm prefers to exit in at least one certification state.

Therefore, a PBE

$$\Big((L, A, (p_1^* = v + \frac{\theta_L}{1-(1-\theta_L)t} r,\ p_0^* = v\ )),\ (1,1);\ \mu_{LA} = 1\Big)$$

exists whenever $(c_A, c_H) \in \Psi_5$ and $G \leq \eta_3$ when $S = 1$ and $G \leq \eta_1$ when $S = 0$.

(ii) PBE $((p_1^*, p_0^*, L, A), (1,0); \mu_{LA} = 1)$**.**

We next consider the candidate PBE in which the firm does not invest in sustainability but apply for certification, sets a price that induces consumer purchase following certification success, and sets a sufficiently high price that deters purchase following certification failure, i.e., $d(p_1, 1) = 1$ and $d(p_0, 0) = 0$.

Conditional on the candidate strategy (L, A), the belief $\mu_{LA} = 1$, and the purchase decisions $d(p_1, 1) = 1$ and $d(p_0, 0) = 0$, the equilibrium prices satisfy

$$p_S^* \in \arg\max_{p_1 \leq \overline{p}_1, p_0 \geq \overline{p}_0} \pi(p_S, L, A, S; \boldsymbol{\mu}^*), \quad \forall S \in \{0,1\}, \tag{C.41}$$

$$\Pi(p_1^*, p_0^*, L, A) \geq \Pi(p_1^*, p_0^*, L, N), \tag{C.42}$$

$$\Pi(p_1^*, p_0^*, L, A) \geq \Pi(p_1^*, p_0^*, H, A), \tag{C.43}$$

$$\Pi(p_1^*, p_0^*, L, A) \geq \Pi(p_1^*, p_0^*, H, N), \tag{C.44}$$

where

$$\Pi(p_1^*, p_0^*, L, A) = \gamma_L[p_1^* - c_A - \frac{(1-\theta_L)(1-t)}{\theta_L + (1-\theta_L)(1-t)} G] + (1-\gamma_L)(-c_A), \tag{C.45}$$

$$\Pi(p_1^*, p_0^*, H, N) = -c_H, \tag{C.46}$$

$$\Pi(p_1^*, p_0^*, H, A) = \gamma_H(p_1^* - c_A - c_H - \psi(H, A, 1)G) + (1-\gamma_H)(-c_A - c_H), \tag{C.47}$$

$$\Pi(p_1^*, p_0^*, L, N) = 0. \tag{C.48}$$

Constraint (C.41) characterizes the pricing restrictions required to support the candidate purchase decision. Specifically, the condition $p_1 \leq \overline{p}_1$ ensures that consumers purchase the product following certification success, whereas $p_0 > \overline{p}_0$ deters purchase following certification failure. Therefore, the firm remains active in the market when $S = 1$ and effectively exits the market when $S = 0$. Constraints (C.42)–(C.44) ensure that the strategy (L, A) under evaluation is optimal relative to the other feasible strategies and therefore consistent with the belief $\mu_{LA} = 1$.

Since the firm's profit is increasing in price conditional on inducing purchase, the upper bound binds when $S = 1$. Hence,

$$p_1^* = \overline{p}_1 \equiv v + \frac{\theta_L}{1-(1-\theta_L)t} r.$$

When $S = 0$, any price satisfying $p_0 > \overline{p}_0 = v$, which deters purchase and yields the same payoff to the firm. Therefore,

$$p_0^* = p_E \in (v, \infty).$$

We next verify whether the strategy (L, A) remains optimal relative to the alternative strategies under the induced prices. Substituting $(p_1, p_0) = (\overline{p}_1, p_E)$ into (C.42)–(C.44) yields:

$$\Pi(\overline{p}_1, p_E, L, A) \geq \Pi(\overline{p}_1, p_E, L, N) \iff c_A \leq v(1-t) + \theta_L(tv + r) - (1-\theta_L)(1-t)G,$$

$$\Pi(\overline{p}_1, p_E, L, A) \geq \Pi(\overline{p}_1, p_E, H, A) \iff c_H > (\theta_H - \theta_L)((v + \frac{\theta_L r}{1-(1-\theta_L)t})t + (1-t)G),$$

$$\Pi(\overline{p}_1, p_E, L, A) \geq \Pi(\overline{p}_1, p_E, H, N) \iff c_H > c_A - v(1-t) - \theta_L(tv + r) + (1-\theta_L)(1-t)G.$$

These conditions define the parameter regions define a parameter region, denoted by Ψ_6, under which the candidate equilibrium can be sustained. Specifically,

$$\Psi_6 = \left\{ \begin{aligned} & c_A \leq v(1-t) + \theta_L(tv + r) - (1-\theta_L)(1-t)G, \\ & c_H > \max\left\{ (\theta_H - \theta_L)\Big((v + \frac{\theta_L r}{1-(1-\theta_L)t})t + (1-t)G\Big), \right. \\ & \qquad \left. c_A - v(1-t) - \theta_L(tv + r) + (1-\theta_L)(1-t)G \right\} \end{aligned} \right\}.$$

In addition, the candidate purchase decision $d(p_1, 1) = 1$ and $d(p_0, 0) = 0$, which requires that the firm prefers remaining active in the market when $S = 1$ and prefers exiting the market when $S = 0$. Comparing the payoff from selling with the payoff from exiting yields $G \leq \eta_3$ for $S = 1$ and $G > \eta_1$ for $S = 0$. When $S = 0$.

Therefore, a PBE

$$\Big((p_1^* = v + \frac{\theta_L}{1-(1-\theta_L)t} r,\ p_0^* = p_E,\ L, A),\ (1, 0);\ \mu_{LA} = 1\Big)$$

exists whenever $(c_A, c_H) \in \Psi_3$, $G > \eta_0$ and $G \leq \eta_1$.

(iii) PBE $((p_1, p_0, L, A), (0, 0); \mu_{LA} = 1)$**.**

Finally, we consider the candidate PBE under which the firm does not invest in sustainability but applies for certification, but sets prices that deter consumer purchase under both certification outcomes, that is, $d(p_S, S) = 0$ for all $S \in \{0,1\}$. Under this candidate equilibrium, the firm effectively exits the market regardless of the realized certification status. Therefore, its expected payoff under each strategy is given by

$$\Pi(p_1^*, p_0^*, L, A) = -c_A, \tag{C.49}$$

$$\Pi(p_1^*, p_0^*, H, N) = -c_H, \tag{C.50}$$

$$\Pi(p_1^*, p_0^*, H, A)) = -c_H - c_A, \tag{C.51}$$

$$\Pi(p_1^*, p_0^*, L, N) = 0. \tag{C.52}$$

Comparing these payoffs, we find that (L, A) is no longer an optimal strategy. This implies that the if $G > \eta_3$, the PBE $((p_1^*, p_0^*, L, A), (0,0); \mu_{LA} = 1)$ cannot exist. Therefore, in the remainder of the paper, we focus on the parameter region where the penalty G remains below the critical threshold η_2. This ensures the firm's ex-post participation when $S = 1$, thereby sustaining the (L, A) equilibrium. Recall from Case 2 that $\eta_3 > \eta_2$, implying that the firm would optimally choose to sell in the market upon passing the certification regardless of its sustainability investment decision. By restricting attention to $G \le \eta_2$, we focus on the region where the penalty is sufficiently low to keep the firm active without affecting the structure of the (L, A) equilibrium.

Case 4: (L, N) with $\mu_{LN} = 1$.

In this case, we consider the candidate strategy (L, N), under which the firm does not invest in sustainability and does not apply for certification. Consumers' on-path beliefs satisfy $\mu_{LN} = 1$. Since certification is not adopted, consumers observe only the uncertified status $(p_0, 0)$. According to (4), consumers' purchase decision is given by $d(p_0, 0) = \mathbb{I}\big(p_0 \le \overline{p}_0(\boldsymbol{\mu}^*)\big)$, where $\overline{p}_0$ denotes the maximum willingness-to-pay. Under the belief $\mu_{LN} = 1$, the threshold price is

$$\overline{p}_0 = v + \theta_L r.$$

Following the off-path belief specification in Case 2, consumers' off-path beliefs satisfy $\hat{\mu}_{HA} = 1$. Accordingly, $\hat{p}_1^* = v + \frac{\theta_H r}{1-(1-\theta_H)t}$. Furthermore, from Case 1 (iii), we recall the condition $G \le \frac{v - tv + \theta_H(r+tv)}{(1-\theta_H)(1-t)}$. Under this threshold, the firm's preference for selling and the consumers' incentive to purchase are both strictly satisfied, ensuring that the off-path demand is $d(\hat{p}_1^*, 1) = 1$.

We now analyze the firm's pricing strategy under the candidate equilibrium (L, N). Since certification is not adopted, only the uncertified status $(p_0, 0)$ is observed on the equilibrium path. Depending on the firm's pricing decision, consumers may either purchase the product or refrain from purchasing it. We analyze these two cases in turn.

(i) PBE $((p_0^*, L, N), 1; \mu_{LN} = 1)$**.**

We begin by considering the candidate PBE under which the firm does not invest in sustainability nor apply for certification, and sets a price that induces consumer purchase, i.e., $d(p_0, 0) = 1$.

Conditional on the candidate strategy (L, N), the belief $\mu_{LN} = 1$, and the purchase decision $d(p_0, 0) = 1$, the equilibrium price satisfies

$$p_0^* \in \arg\max_{p_0 \in [0, \overline{p}_0)} \pi(p_0, L, N, 0; \boldsymbol{\mu}^*), \tag{C.53}$$

$$\Pi(p_1^*, p_0^*, L, N) \geq \Pi(p_1^*, p_0^*, H, N), \tag{C.54}$$

$$\Pi(p_1^*, p_0^*, L, N) \geq \Pi(p_1^*, p_0^*, L, A), \tag{C.55}$$

$$\Pi(p_1^*, p_0^*, L, N) \geq \Pi(p_1^*, p_0^*, H, A), \tag{C.56}$$

where

$$\Pi(p_1^*, p_0^*, L, N) = p_0^* - (1 - \theta_L)G, \tag{C.57}$$

$$\Pi(p_1^*, p_0^*, H, A) = \gamma_H(p_1^* - c_H - c_A - \psi(H, A, 1)G) + (1 - \gamma_H)(p_0^* - c_H - c_A - G), \tag{C.58}$$

$$\Pi(p_1^*, p_0^*, L, A) = \gamma_L(p_1^* - c_A - \psi(L, A, 1)G) + (1 - \gamma_L)(p_0^* - c_A - G), \tag{C.59}$$

$$\Pi(p_1^*, p_0^*, H, N) = p_0^* - c_H - (1 - \theta_H)G. \tag{C.60}$$

Constraint (C.53) ensures that p_0^* is an optimal price conditional on the uncertified status. The candidate purchase decision $d(p_0, 0) = 1$ requires that $p_0 \leq \overline{p}_0$. Constraints (C.54)–(C.56) ensure that the strategy (L, N) under evaluation is optimal relative to the other feasible strategies and therefore consistent with the belief $\mu_{LN} = 1$.

Combining the strategy-optimality constraints with the consumer purchase requirement yields

$$p_0 \leq p_1 - \Delta,$$

where

$$\Delta \equiv \max\{\Delta_2, \Delta_3\},$$

with

$$\Delta_3 = \frac{c_A - c_H + (\theta_H - \theta_L)G}{1 - (1 - \theta_L)t}.$$

Since the firm's profit is increasing in price conditional on inducing purchase, the upper bound binds at the optimum. Together with the off-path belief specification,

$$p_0^* = \overline{p}_0 \equiv v + \theta_L r, \qquad p_1^* = \overline{p}_1 \equiv v + \frac{\theta_H}{1 - (1 - \theta_H)t} r.$$

We next verify whether the strategy (L, N) remains optimal relative to the alternative strategies under the induced prices. Substituting $(\overline{p}_1, \overline{p}_0)$ into constraints (C.54)–(C.56) yields the parameter regions Ψ_7 and Ψ_8. Specifically,

$$\Psi_7 = \left\{ \begin{array}{l} c_A > \dfrac{\theta_L rt(1 - \theta_L)(1 - (1 - \theta_H)t) + c_H(t - 1 - \theta_L t) + (\theta_H - \theta_L)(1 - (1 - \theta_L)t)G}{1 - (1 - \theta_L)t}, \\ (\theta_H - \theta_L)G < c_H \leq \dfrac{(\theta_H - \theta_L)(G + (1 - \theta_L)(\theta_L rt - G)t)}{1 - (1 - \theta_L)t} \end{array} \right\},$$

and

$$\Psi_8 = \left\{ \begin{aligned} & c_A > \theta_L rt(1-\theta_L), \\ & c_H > \frac{(\theta_H-\theta_L)(G+(1-\theta_L)(\theta_L rt - G)t)}{1-(1-\theta_L)t} \end{aligned} \right\}.$$

In addition, the candidate purchase decision $d(p_0,0)=1$ requires that the firm prefers remaining active in the market rather than exiting. The payoff from selling is

$$\Pi_{\text{sell}} = v + \theta_L r - (1-\theta_L)G,$$

while exiting yields $\Pi_{\text{exit}}=0$. The firm sells if and only if $\Pi_{\text{sell}} \geq \Pi_{\text{exit}}$, which defines the threshold

$$G \leq \eta_4 \equiv \frac{v+\theta_L r}{1-\theta_L}.$$

We get that if $G \leq \eta_4$, the firm without investment and certification sells in the market; otherwise, it exits by setting a prohibitively high price.

Therefore, a PBE

$$\Big((p_0^* = v+\theta_L r, L, N),\ 1;\ \mu_{LN}=1\Big)$$

exists whenever $(c_A, c_H) \in \Psi_7 \cup \Psi_8$ and $G \leq \eta_4$.

(ii) PBE $((p_0^*, L, N), 0; \mu_{LN}=1)$**.**

We then analyze the PBE under which under which the firm does not invest in sustainability nor apply for certification, and sets a price that deters consumer purchase, i.e., $d(p_0,0)=0$. Under this candidate equilibrium, the firm effectively exits the market. Together with the off-path belief specification, the firm's expected payoffs under different strategies are

$$\Pi(p_1^*, p_0^*, L, A) = \gamma_L(p_1^* - c_A - \psi(L,A,1)G) + (1-\gamma_L)(-c_A), \tag{C.61}$$

$$\Pi(p_1^*, p_0^*, H, N) = -c_H, \tag{C.62}$$

$$\Pi(p_1^*, p_0^*, H, A) = \gamma_H(p_1^* - c_A - c_H - \psi(H,A,1)G) + (1-\gamma_H)(-c_A-c_H), \tag{C.63}$$

$$\Pi(p_1^*, p_0^*, L, N) = 0. \tag{C.64}$$

Comparing these profits, we obtain that the strategy (L,N) is not optimal relative to the alternative strategies and therefore cannot be sustained in equilibrium. This implies that the PBE $((p_0^*, L, N), 0; \mu_{LN}=1)$ cannot be sustained if $G > \eta_4$. Consequently, for the remainder of this paper, we restrict our analysis to the scenario where the penalty remains below a critical threshold to ensure the existence of the (L,N) strategy. Formally, we assume $G \leq \frac{v+\theta_H r}{1-\theta_H}$.

In Case 2, we obtain that our subsequent analysis should be conducted under the condition $G \leq \eta_2$. In the current setting, the relevant bound becomes $G \leq \eta_4$, where $\eta_4 < \eta_2$. Therefore, the subsequent analysis is conducted under the assumption $G \leq \eta_4$.

Assumption 1. $G \leq \frac{v+\theta_L r}{1-\theta_L}$.

In summary, we show that different equilibrium outcomes arise in different ranges of the penalty level.

• When $G \leq v$, the firm continues to sell in the market even if it fails the certification, and the conditions under which PBEs exist are as follows:

if $(c_H, c_A) \in \Psi_1 \cup \Psi_2$, there exists a PBE $((p_1^* = v + \frac{\theta_H r}{1-(1-\theta_H)t}, p_0^* = v, H, A), (1,1); \mu_{HA} = 1)$;

if $(c_H, c_A) \in \Psi_4$, there exists a PBE $((p_0^* = v + \theta_H r, H, N), 1; \mu_{HN} = 1)$;

if $(c_H, c_A) \in \Psi_5$, there exists a PBE $((p_1^* = v + \frac{\theta_L r}{1-(1-\theta_L)t}, p_0^* = v, L, A), (1,1); \mu_{LA} = 1)$;

if $(c_H, c_A) \in \Psi_7 \cup \Psi_8$, there exists a PBE $((p_0^* = v + \theta_L r, L, N), 1; \mu_{LN} = 1)$.

• When $v < G \leq \frac{v+\theta_L r}{1-\theta_L}$, the firm chooses not to sell in the market if it fails the certification, and the conditions under which the PBEs exist are as follows:

if $(c_H, c_A) \in \Psi_3$, $p_1^* = p_P^H = v + \frac{\theta_H}{1-(1-\theta_H)t} r$ and $p_0^* = p_E = v + \epsilon$, there exists a PBE $((p_1^* = v + \frac{\theta_H r}{1-(1-\theta_H)t}, p_0^* = p_E, H, A), (1,0); \mu_{HA} = 1)$;

if $(c_H, c_A) \in \Psi_4$, there exists a PBE $((p_0^* = v + \theta_H r, H, N), 1; \mu_{HN} = 1)$;

if $(c_H, c_A) \in \Psi_6$, there exists a PBE $((p_1^* = v + \frac{\theta_L}{1-(1-\theta_L)t} r, p_0^* = p_E, L, A), (1,0); \mu_{LA} = 1)$;

if $(c_H, c_A) \in \Psi_7 \cup \Psi_8$, there exists a PBE $((p_0^* = v + \theta_L r, L, N), 1; \mu_{LN} = 1)$.

Step 2: PBE refinement. It is important to note that the regions sustaining different equilibria may partially overlap; that is, for a given pair (c_A, c_H), multiple equilibria can coexist. To resolve this multiplicity, we adopt a refinement based on sender-optimality and focus on the equilibrium that yields the highest payoff for the firm. This selection criterion ensures a unique prediction and is commonly employed in the analysis of signaling games (Kamenica and Gentzkow 2011, Bimpikis and Mantegazza 2023).

We first analyze the scenario where $G \leq v$. Based on the sender-optimal refinement, we select the equilibrium that yields the highest payoff for the firm among all feasible PBE, as follows:

(i) The PBE $((p_1^*, p_0^*, H, A), (1,1); \mu_{HA} = 1)$, and $((p_0^*, H, N), 1; \mu_{HN} = 1)$ may both be sustained in an overlapping region of (c_A, c_H). Recall that we have defined the parameter regions Ψ_1, Ψ_2, and Ψ_4 as the respective conditions under which each PBE is valid. The overlapping region is given by:

$$\{\theta_H r t(1-\theta_H) < c_A \leq \frac{\theta_H r(1-(1-\theta_L)t)}{1-(1-\theta_H)t}, c_H \leq (\theta_H - \theta_L) G\}.$$

Within this region, we compare the firm's equilibrium payoffs under each strategy to determine which PBE is selected. Specifically, if

$$\Pi(p_1^*, p_0^*, H, A) \geq \Pi(p_0^*, H, N),$$

we select the PBE

$$((p_1^*, p_0^*, H, A), (1,1); \mu_{HA} = 1),$$

and otherwise we select the PBE

$$((p_0^*, H, N), 1; \mu_{HN} = 1).$$

(ii) The PBE $((p_1^*, p_0^*, H, A), (1,1); \mu_{HA} = 1)$, and $((p_1^*, p_0^*, L, A), (1,1); \mu_{LA} = 1)$, may both be sustained in an overlapping region of (c_A, c_H). Recall that we have defined Ψ_1, Ψ_2, and Ψ_5 as the respective conditions under which each PBE is valid. The overlap occurs in the region:

$$\{\frac{(\theta_H - \theta_L)\theta_L rt}{1-(1-\theta_L)t} < c_H \leq (\theta_H - \theta_L)(\frac{\theta_H rt}{1-(1-\theta_H)t} + G)\}.$$

Within this region, we compare the firm's equilibrium payoffs under each strategy to determine which equilibrium is selected. Specifically, within the overlapping region, we select the PBE $((p_1^*, p_0^*, H, A), (1,1); \mu_{HA} = 1)$ if $\Pi(p_1^*, p_0^*, H, A) \geq \Pi(p_1^*, p_0^*, L, A)$, and the PBE $((p_1^*, p_0^*, L, A), (1,1); \mu_{LA} = 1)$ otherwise.

(iii) The PBE $((p_1^*, p_0^*, H, A), (1,1); \mu_{HA} = 1)$ and $((p_0^*, L, N), 1; \mu_{LN} = 1)$ may both be sustained in an overlapping region of (c_A, c_H). Recall that we have defined Ψ_1, Ψ_2, Ψ_7 and Ψ_8 as the respective conditions under which each PBE is valid. The overlap occurs in the region:

$$\Psi_7 = \left\{ \begin{array}{l} \dfrac{\theta_L rt(1-\theta_L)(1-(1-\theta_H)t) - c_H(1+\theta_L t - t) + (\theta_H - \theta_L)(1-(1-\theta_L)t)G}{1-(1-\theta_L)t} < c_A \leq (\theta_H - \theta_L)G + \theta_H r - c_H, \\ (\theta_H - \theta_L)G < c_H \leq \dfrac{(\theta_H - \theta_L)\big(G + (1-\theta_L)(\theta_L rt - G)t\big)}{1-(1-\theta_L)t} \end{array} \right\}.$$

Within this region, we compare the firm's equilibrium payoffs under each strategy to determine which equilibrium is selected. Specifically, within the overlapping region, we select the PBE $((p_1^*, p_0^*, H, A), (1,1); \mu_{HA} = 1)$ if $\Pi(p_1^*, p_0^*, H, A) \geq \Pi(p_0^*, L, N)$, and the PBE $((p_0^*, L, N), 1; \mu_{LN} = 1)$ otherwise. The same method applies to other cases of equilibrium multiplicity, and we omit further details for brevity.

In summary, the equilibrium outcomes vary across different ranges of the penalty level G.

i. When $G \leq v$, the firm chooses to sell in the market even if it fails the certification. There are four potential PBEs that can arise depending on the values of (c_A, c_H) and the belief system induced by the observed prices:

if $(c_A, c_H) \in \Gamma_1 \cup \Gamma_2 \cup \Gamma_3$, there exists a PBE $((p_1^* = v + \frac{\theta_H r}{1-(1-\theta_H)t}, p_0^* = v, H, A), (1,1); \mu_{HA} = 1)$;

if $(c_A, c_H) \in \Gamma_4$, there exists a PBE $((p_0^* = v + \theta_H r, H, N), 1; \mu_{HN} = 1)$;

if $(c_A, c_H) \in \Gamma_5$, there exists a PBE $((p_1^* = v + \frac{\theta_L r}{1-(1-\theta_L)t}, p_0^* = v, L, A), (1,1); \mu_{LA}) = 1)$;

if $(c_A, c_H) \in \Gamma_6 \cup \Gamma_7$, there exists a PBE $((p_0^* = v + \theta_L r, L, N), 1; \mu_{LN} = 1)$,

where $\Gamma_1 = \{c_A \leq \theta_H rt(1-\theta_H), c_H \leq (\theta_H - \theta_L)G\}$, $\Gamma_2 = \{c_A \leq \frac{\theta_L rt(1-\theta_L)(1-(1-\theta_H)t) + G(\theta_H - \theta_L)(1-(1-\theta_L)t)}{1-(1-\theta_L)t} - c_H, (\theta_H - \theta_L)G < c_H \leq \frac{(\theta_H - \theta_L)(G+(1-\theta_L)(\theta_L rt - G)t)}{1-(1-\theta_L)t}\}$, $\Gamma_3 = \{c_A \leq \theta_L rt(1-\theta_L), \frac{(\theta_H - \theta_L)(G+(1-\theta_L)(\theta_L rt - G)t)}{1-(1-\theta_L)t} < c_H \leq (\theta_H - \theta_L)(\frac{\theta_H rtr}{1-(1-\theta_H)t} + G)\}$, $\Gamma_4 = \{c_A >$

$\theta_H rt(1-\theta_H), c_H \le (\theta_H-\theta_L)G\}$, $\Gamma_5 = \{c_A \le \theta_L rt(1-\theta_L), c_H > (\theta_H-\theta_L)(\frac{\theta_H rtr}{1-(1-\theta_H)t}+G)\}$, $\Gamma_6 = \{c_A > \theta_L rt(1-\theta_L), c_H > \frac{(\theta_H-\theta_L)(G+(1-\theta_L)(\theta_L rt-G)t)}{1-(1-\theta_L)t}\}$, and $\Gamma_7 = \{\frac{\theta_L rt(1-\theta_L)(1-(1-\theta_H)t)+G(\theta_H-\theta_L)(1-(1-\theta_L)t)}{1-(1-\theta_L)t} - c_H < c_A, (\theta_H-\theta_L)G < c_H \le \frac{(\theta_H-\theta_L)(G+(1-\theta_L)(\theta_L rt-G)t)}{1-(1-\theta_L)t}\}$.

ii. For $v < G \le v+\theta_L r$, the firm chooses not to sell in the market if it fails the certification. There are four potential PBEs that can arise depending on the values of (c_A, c_H) and the belief system induced by the observed prices:

if $(c_H, c_A) \in \Gamma_8 \cup \Gamma_9 \cup \Gamma_{10}$, there exists a PBE $((p_1^* = v+\frac{\theta_H r}{1-(1-\theta_H)t}, p_0^* = p_E, H, A), (1,0); \mu_{HA}=1)$;

if $(c_H, c_A) \in \Gamma_{11}$, there exists a PBE $((p_0^* = v+\theta_H r, H, N), 1; \mu_{HN}=1)$;

if $(c_H, c_A) \in \Gamma_{12}$, there exists a PBE $((p_1^* = v+\frac{\theta_L r}{1-(1-\theta_L)t}, p_0^* = p_E, L, A), (1,0); \mu_{LA}=1)$;

if $(c_H, c_A) \in \Gamma_{13} \cup \Gamma_{14}$, there exists a PBE $((p_0^* = v+\theta_L r, L, N), 1; \mu_{LN}=1)$, where $\Gamma_8 = \{c_A \le \theta_H rt(1-\theta_H), c_H \le (\theta_H-\theta_L)G\}$, $\Gamma_9 = \{c_A \le (\theta_H-\theta_L)(G+r)+\theta_L rt(1-\theta_H) - c_H, (\theta_H-\theta_L)G < c_H \le (\theta_H-\theta_L)(G+r)+(1-\theta_H)\theta_L rt+(1-(1-\theta_L)t)(\theta_L r - \frac{\theta_H r}{1-(1-\theta_H)t})\}$, $\Gamma_{10} = \{c_A \le (1-(1-\theta_L)t)(\frac{\theta_H r}{1-(1-\theta_H)t}-\theta_L r), (\theta_H-\theta_L)G < c_H \le (\theta_H-\theta_L)((v+\frac{\theta_H r}{1-(1-\theta_H)t})t+(1-t)G)\}$, $\Gamma_{11} = \{c_A > \theta_H rt(1-\theta_H), c_H \le (\theta_H-\theta_L)G\}$, $\Gamma_{12} = \{c_A \le (1-(1-\theta_L)t)(\frac{\theta_H r}{1-(1-\theta_H)t}-\theta_L r), c_H > (\theta_H-\theta_L)((v+\frac{\theta_H r}{1-(1-\theta_H)t})t+(1-t)G)\}$, $\Gamma_{13} = \{c_A > (\theta_H-\theta_L)(G+r)+\theta_L rt(1-\theta_H) - c_H, (\theta_H-\theta_L)G < c_H \le (\theta_H-\theta_L)(G+r)+(1-\theta_H)\theta_L rt+(1-(1-\theta_L)t)(\theta_L r-\frac{\theta_H r}{1-(1-\theta_H)t})\}$, $\Gamma_{14} = \{c_A > (1-(1-\theta_L)t)(\frac{\theta_H r}{1-(1-\theta_H)t}-\theta_L r), c_H > (\theta_H-\theta_L)(G+r)+(1-\theta_H)\theta_L rt+(1-(1-\theta_L)t)(\theta_L r-\frac{\theta_H r}{1-(1-\theta_H)t})\}$.

iii. For $v+\theta_L r < G \le v+\frac{(1-(1-\theta_L)t)(\frac{\theta_H r}{1-(1-\theta_H)t}-\theta_L r)}{1-(1-\theta_L)t}$, the firm chooses not to sell in the market if it fails the certification. There are four potential PBEs that can arise depending on the values of (c_A, c_H) and the belief system induced by the observed prices:

if $(c_H, c_A) \in \Gamma_8 \cup \Gamma_{15}$, there exists a PBE $((p_1^* = v+\frac{\theta_H r}{1-(1-\theta_H)t}, p_0^* = p_E, H, A), (1,0); \mu_{HA}=1)$;

if $(c_H, c_A) \in \Gamma_{11}$, there exists a PBE $((p_0^* = v+\theta_H r, H, N), 1; \mu_{HN}=1)$;

if $(c_H, c_A) \in \Gamma_{12}$, there exists a PBE $((p_1^* = v+\frac{\theta_L r}{1-(1-\theta_L)t}, p_0^* = p_E, L, A), (1,0); \mu_{LA}=1)$;

if $(c_H, c_A) \in \Gamma_{13} \cup \Gamma_{14}$, there exists a PBE $((p_0^* = v+\theta_L r, L, N), 1; \mu_{LN}=1)$, where $\Gamma_{15} = \{c_A \le (\theta_H-\theta_L)r+G(\theta_H-\theta_L+t-\theta_H t) - tv(1-\theta_H) - c_H, (\theta_H-\theta_L)G < c_H \le (\theta_H-\theta_L)((v+\frac{\theta_H r}{1-(1-\theta_H)t})t+(1-t)G)\}$.

iv. For $v+\frac{(1-(1-\theta_L)t)(\frac{\theta_H r}{1-(1-\theta_H)t}-\theta_L r)}{1-(1-\theta_L)t} < G \le v+\theta_H r$, the firm chooses not to sell in the market if it fails certification. There are four potential PBEs that can arise depending on the values of (c_A, c_H) and the belief system induced by the observed prices:

if $(c_H, c_A) \in \Gamma_8 \cup \Gamma_{15}$, there exists a PBE $((p_1^* = v+\frac{\theta_H r}{1-(1-\theta_H)t}, p_0^* = p_E, H, A), (1,0); \mu_{HA}=1)$;

if $(c_H, c_A) \in \Gamma_{11}$, there exists a PBE $((p_0^* = v+\theta_H r, H, N), 1; \mu_{HN}=1)$;

if $(c_H, c_A) \in \Gamma_{16}$, there exists a PBE $((p_1^* = v+\frac{\theta_L r}{1-(1-\theta_L)t}, p_0^* = p_E, L, A), (1,0); \mu_{LA}=1)$;

if $(c_H, c_A) \in \Gamma_{17} \cup \Gamma_{18}$, there exists a PBE $((p_0^* = v+\theta_L r, L, N), 1; \mu_{LN}=1)$, where $\Gamma_{16} = \{c_A \le (1-\theta_L)(G-v)t, c_H > (\theta_H-\theta_L)((v+\frac{\theta_H r}{1-(1-\theta_H)t})t+(1-t)G)\}$ $\Gamma_{17} = \{c_A > (\theta_H-\theta_L)r+G(\theta_H-\theta_L+$

$t-\theta_H t) - tv(1-\theta_H) - c_H, (\theta_H-\theta_L)G < c_H \le (\theta_H-\theta_L)((v+\frac{\theta_H r}{1-(1-\theta_H)t})t+(1-t)G)\}$, $\Gamma_{18} = \{c_A > (1-\theta_L)(G-v)t, c_H > (\theta_H-\theta_L)((v+\frac{\theta_H r}{1-(1-\theta_H)t})t+(1-t)G)\}$.

v. For $v+\theta_H r < G \le v+\frac{\theta_H r}{1-(1-\theta_H)t}$**,** the firm chooses not to sell in the market if it fails certification. There are four potential PBEs that can arise depending on the values of (c_A, c_H) and the belief system induced by the observed prices:

if $(c_H, c_A) \in \Gamma_{19} \cup \Gamma_{15}$, there exists a PBE $((p_1^* = v+\frac{\theta_H r}{1-(1-\theta_H)t}, p_0^* = p_E, H, A), (1,0); \mu_{HA}=1)$;

if $(c_H, c_A) \in \Gamma_{20}$, there exists a PBE $((p_0^* = v+\theta_H r, H, N), 1; \mu_{HN}=1)$;

if $(c_H, c_A) \in \Gamma_{16}$, there exists a PBE $((p_1^* = v+\frac{\theta_L r}{1-(1-\theta_L)t}, p_0^* = p_E, L, A), (1,0); \mu_{LA}=1$;

if $(c_H, c_A) \in \Gamma_{17} \cup \Gamma_{18}$, there exists a PBE $((p_0^* = v+\theta_L r, L, N), 1; \mu_{LN}=1)$, where $\Gamma_{19} = \{c_A \le (1-\theta_H)(G-v)t, c_H \le (\theta_H-\theta_L)((v+\frac{\theta_H r}{1-(1-\theta_H)t})t+(1-t)G)\}$ and $\Gamma_{20} = \{c_A > (1-\theta_H)(G-v)t, c_H \le (\theta_H-\theta_L)G\}$.

vi. For $v+\frac{\theta_H r}{1-(1-\theta_H)t} < G \le v+\frac{\theta_H rt(1-\theta_H)+(\theta_H-\theta_L)r}{(1-\theta_L)t}$**,** the firm chooses not to sell in the market if it fails certification. There are four potential PBEs that can arise depending on the values of (c_A, c_H) and the belief system induced by the observed prices:

if $(c_H, c_A) \in \Gamma_{19}$, there exists a PBE $((p_1^* = v+\frac{\theta_H r}{1-(1-\theta_H)t}, p_0^* = p_E, H, A), (1,0); \mu_{HA}=1)$;

if $(c_H, c_A) \in \Gamma_{20} \cup \Gamma_{21}$, there exists a PBE $((p_0^* = v+\theta_H r, H, N), 1; \mu_{HN}=1)$;

if $(c_H, c_A) \in \Gamma_{16} \cup \Gamma_{22}$, there exists a PBE $((p_1^* = v+\frac{\theta_L r}{1-(1-\theta_L)t}, p_0^* = p_E, L, A), (1,0); \mu_{LA}=1)$;

if $(c_H, c_A) \in \Gamma_{23}$, there exists a PBE $((p_0^* = v+\theta_L r, L, N), 1; \mu_{LN}=1)$,

where $\Gamma_{21} = \{c_A > \theta_H rt(1-\theta_H), (\theta_H-\theta_L)((v+\frac{\theta_H r}{1-(1-\theta_H)t})t+(1-t)G) < c_H \le (\theta_H-\theta_L)G\}$, and $\Gamma_{22} = \{c_A \le \theta_H rt(1-\theta_H), (\theta_H-\theta_L)((v+\frac{\theta_H r}{1-(1-\theta_H)t})t+(1-t)G) < c_H \le (\theta_H-\theta_L)G\}$, $\Gamma_{23} = \{c_A > (1-\theta_L)(G-v)t, c_H > (\theta_H-\theta_L)G\}$.

vii. For $v+\frac{\theta_H rt(1-\theta_H)+(\theta_H-\theta_L)r}{(1-\theta_L)t} < G \le \frac{v+\theta_L r}{1-\theta_L}$**,** the firm chooses not to sell in the market if it fails certification. There are four potential PBEs that can arise depending on the values of (c_A, c_H) and the belief system induced by the observed prices:

if $(c_H, c_A) \in \Gamma_{19}$, there exists a PBE $((p_1^* = v+\frac{\theta_H r}{1-(1-\theta_H)t}, p_0^* = p_E, H, A), (1,0); \mu_{HA}=1)$;

if $(c_H, c_A) \in \Gamma_{20} \cup \Gamma_{24}$, there exists a PBE $((p_0^* = v+\theta_H r, H, N), 1; \mu_{HN}=1)$;

if $(c_H, c_A) \in \Gamma_{16} \cup \Gamma_{25}$, there exists a PBE $((p_1^* = v+\frac{\theta_L r}{1-(1-\theta_L)t}, p_0^* = p_E, L, A), (1,0); \mu_{LA}=1)$;

if $(c_H, c_A) \in \Gamma_{23}$, there exists a PBE $((p_0^* = v+\theta_L r, L, N), 1; \mu_{LN}=1)$,

where $\Gamma_{24} = \{c_A > \theta_L r - \theta_H r - (\theta_H-\theta_L-t+\theta_L t)G - (1-\theta_L)tv + c_H, (\theta_H-\theta_L)((v+\frac{\theta_H r}{1-(1-\theta_H)t})t+(1-t)G) < c_H \le (\theta_H-\theta_L)G\}$ and $\Gamma_{25} = \{c_A \le \theta_L r - \theta_H r - (\theta_H-\theta_L-t+\theta_L t)G - (1-\theta_L)tv + c_H, (\theta_H-\theta_L)((v+\frac{\theta_H r}{1-(1-\theta_H)t})t+(1-t)G) < c_H \le (\theta_H-\theta_L)G\}$.

Consequently, we derive the cutoff investment cost $\hat{c}$, and demonstrate that its value depends on the range of the penalty level G, as summarized in Table 3. Here, $\xi^{(1)}$ and $\xi^{(2)}$ denote the two endogenous cutoff points of the piecewise function, corresponding to different φ_i under different penalty regimes. When summarizing the results under the two broader penalty regimes, these cutoff points are written as (ρ_L, ρ_1) under low penalty and (ρ_H, ρ_2) under high penalty.

Table 3 The cutoff investment cost $\hat{c}$ under different levels of G.

		$c_A \le \xi^1(G)$	$\xi^1(G) < c_A \le \xi^2(G)$	$c_A > \xi^2(G)$
$G \le \eta$	$G \le v$	$\hat{c}_1$	$\hat{c}_2 - c_A$	$\hat{c}_3$
	$v < G \le v + \theta_L r$	$\hat{c}_4$	$\hat{c}_2 - c_A$	$\hat{c}_3$
	$v + \theta_L r < G \le v + \dfrac{(1-\theta_L)t\left(\frac{\theta_H r}{(1-\theta_H)t} - \theta_L t\right)}{1-(1-\theta_L)t}$	$\hat{c}_4$	$\hat{c}_5 - c_A$	$\hat{c}_3$
	$v + (1-\theta_L)t\left(\dfrac{\theta_H r}{(1-\theta_H)t} - \theta_L t\right) < G \le v + \theta_H r$	$\hat{c}_4$	$\hat{c}_5 - c_A$	$\hat{c}_3$
	$v + \theta_H r < G \le v + \dfrac{\theta_H r}{1-(1-\theta_H)t}$	$\hat{c}_4$	$\hat{c}_5 - c_A$	$\hat{c}_3$
$G > \eta$	$v + \dfrac{\theta_H r}{1-(1-\theta_H)t} < G \le v + \dfrac{(1-\theta_H)\theta_L r + (\theta_H - \theta_L)r}{(1-\theta_L)t}$	$\hat{c}_4$	$\hat{c}_4$	$\hat{c}_3$
	$v + \dfrac{(1-\theta_H)\theta_L r + (\theta_H - \theta_L)r}{(1-\theta_L)t} < G \le \dfrac{v + \theta_L r}{1-\theta_L}$	$\hat{c}_4$	$\hat{c}_6 + c_A$	$\hat{c}_3$

For clarity, the explicit expressions of $\hat{c}$ are presented as follows.

• Under low penalty, we have the following five cases.

i. When $G \le v$, the cutoff investment cost is given by

$$\hat{c} = \begin{cases} \hat{c}_1 = (\theta_H - \theta_L)(G + \frac{\theta_H r t}{1-(1-\theta_H)t}), & c_A \le \varphi_1, \\ \hat{c}_2 - c_A = \theta_L r t(1-\theta_H) + (\theta_H - \theta_L)(G + r) - c_A & \varphi_1 < c_A \le \varphi_2, \\ \hat{c}_3 = (\theta_H - \theta_L)G, & c_A > \varphi_2, \end{cases} \tag{C.65}$$

where $\varphi_1 = (1-(1-\theta_L)t)(\frac{\theta_H}{1-(1-\theta_H)t} - \theta_L)r$ and $\varphi_2 = r(\theta_H - (1-t)\theta_L - \theta_H\theta_L t)$.

ii. When $v < G \le v + \theta_L r$, the cutoff investment is given by

$$\hat{c} = \begin{cases} \hat{c}_4 = (\theta_H - \theta_L)(t(v + \frac{\theta_H r}{1-(1-\theta_H)t}) + (1-t)G), & c_A \le \varphi_3, \\ \hat{c}_2 - c_A = \theta_L r t(1-\theta_H) + (\theta_H - \theta_L)(G + r) - c_A & \varphi_3 < c_A \le \varphi_4, \\ \hat{c}_3 = (\theta_H - \theta_L)G, & c_A > \varphi_4, \end{cases} \tag{C.66}$$

where $\varphi_3 = (\theta_H - \theta_L)G + (\theta_H - (1-t)\theta_L - \theta_H - \theta_L t)r - (\theta_H - \theta_L)(G - Gt + t(v + \frac{\theta_H r}{1-(1-\theta_H)t}))$ and $\varphi_4 = (\theta_H(1-t)\theta_L - \theta_H\theta_L t)r$.

iii. When $v+\theta_L r < G \le v + \frac{(1-(1-\theta_L)t)(\frac{\theta_H r}{1-(1-\theta_H)t}-\theta_L r)}{1-(1-\theta_L)t}$, the cutoff investment is given by

$$\hat{c} = \begin{cases} \hat{c}_4 = (\theta_H-\theta_L)(t(v+\frac{\theta_H r}{1-(1-\theta_H)t})+(1-t)G), & c_A \le \varphi_5, \\ \hat{c}_5 - c_A = (\theta_H-\theta_L)r + (\theta_H-\theta_L+(1-\theta_H)t)G-(1-\theta_H)tv - c_A & \varphi_5 < c_A \le \varphi_6, \\ \hat{c}_3 = (\theta_H-\theta_L)G, & c_A > \varphi_6, \end{cases} \tag{C.67}$$

where $\varphi_5 = \frac{\theta_H(r-rt+(1-\theta_L)(G-v)t^2)+(1-t)(t(G-v)-\theta_L(Gt+r-tv))}{1-(1-\theta_H)t}$ and $\varphi_6 = (\theta_H-\theta_L)r+(1-\theta_H)Gt-(1-\theta_H)tv$.

iv. When $v + \frac{(1-(1-\theta_L)t)(\frac{\theta_H r}{1-(1-\theta_H)t}-\theta_L r)}{1-(1-\theta_L)t} < G \le v+\theta_H r$, the cutoff investment is given by

$$\hat{c} = \begin{cases} \hat{c}_4 = (\theta_H-\theta_L)(t(v+\frac{\theta_H r}{1-(1-\theta_H)t})+(1-t)G), & c_A \le \varphi_5, \\ \hat{c}_5 - c_A = (\theta_H-\theta_L)r + (\theta_H-\theta_L+(1-\theta_H)t)G-(1-\theta_H)tv - c_A & \varphi_5 < c_A \le \varphi_6, \\ \hat{c}_3 = (\theta_H-\theta_L)G, & c_A > \varphi_6. \end{cases} \tag{C.68}$$

v. When $v+\theta_H r < G \le v + \frac{\theta_H r}{1-(1-\theta_H)t}$, , the cutoff investment is given by

$$\hat{c} = \begin{cases} \hat{c}_4 = (\theta_H-\theta_L)(t(v+\frac{\theta_H r}{1-(1-\theta_H)t})+(1-t)G), & c_A \le \varphi_5, \\ \hat{c}_5 - c_A = (\theta_H-\theta_L)r + (\theta_H-\theta_L+(1-\theta_H)t)G-(1-\theta_H)tv - c_A & \varphi_5 < c_A \le \varphi_6, \\ \hat{c}_3 = (\theta_H-\theta_L)G, & c_A > \varphi_6. \end{cases} \tag{C.69}$$

vi. $v + \frac{\theta_H r}{1-(1-\theta_H)t} < G \le v + \frac{\theta_H rt(1-\theta_H)+(\theta_H-\theta_L)r}{(1-\theta_L)t}$, the cutoff investment is given by

$$\hat{c} = \begin{cases} \hat{c}_4 = (\theta_H-\theta_L)(t(v+\frac{\theta_H r}{1-(1-\theta_H)t})+(1-t)G), & c_A \le \varphi_7, \\ \hat{c}_3 = (\theta_H-\theta_L)G, & c_A > \varphi_7, \end{cases} \tag{C.70}$$

where $\varphi_7 = \theta_H rt(1-\theta_H)$.

vii. When $v + \frac{\theta_H rt(1-\theta_H)+(\theta_H-\theta_L)r}{(1-\theta_L)t} < G \le \frac{v+\theta_L r}{1-\theta_L}$, the cutoff investment cost is given by

$$\hat{c} = \begin{cases} \hat{c}_4 = (\theta_H-\theta_L)(t(v+\frac{\theta_H r}{1-(1-\theta_H)t})+(1-t)G), & c_A \le \varphi_8, \\ \hat{c}_6 + c_A = (\theta_H-\theta_L)r + (1-\theta_L)tv + (\theta_H-(1-t)\theta_L - t)G + c_A & \varphi_8 < c_A \le \varphi_9, \\ \hat{c}_3 = (\theta_H-\theta_L)G, & c_A > \varphi_9. \end{cases} \tag{C.71}$$

In summary, we derive the cutoff investment cost $\hat{c}$.

- Under low penalty, the cutoff investment cost is given by

$$\hat{c} = \begin{cases} (\theta_H-\theta_L)\left[t\left(\min\{G,v\}+\frac{\theta_H r}{1-(1-\theta_H)t}\right)+(1-t)G\right], & c_A \le \rho_L, \\ (\theta_H-\theta_L+t-\theta_H t)G - (1-\theta_H)t - c_A - (1-\theta_H)(G-v)t\,\mathbb{I}(G\le v) \\ \quad + \theta_H r\,\mathbb{I}(v < G \le v+\theta_L r), & \rho_L < c_A \le \rho_1, \\ (\theta_H-\theta_L)G, & c_A > \rho_1. \end{cases} \tag{C.72}$$

- Under high penalty, the cutoff investment cost is given by

$$\hat{c}=\begin{cases}(\theta_H-\theta_L)(v+\frac{\theta_H rt}{1-(1-\theta_H)t}+(1-t)G), & \text{if } c_A\le\min\{\rho_2,\rho_H\},\\ (\theta_H-\theta_L)G+[(\theta_H-\theta_L)r+(1-\theta_L)tv+(\theta_H-(1-t)\theta_L-t)G+c_A]\mathbb{I}(G>\sigma), & \text{if } \min\{\rho_2,\rho_H\}<c_A\le\rho_3,\\ (\theta_H-\theta_L)G, & \text{if } c_A>\rho_3,\end{cases} \tag{C.73}$$

where

$$\rho_H=\begin{cases}\theta_H rt(1-\theta_H), & \text{if } G\le v+\theta_H r,\\ (1-\theta_H)(G-v)t, & \text{if } v+\theta_H r<G\le\frac{v+\theta_L r}{1-\theta_L},\end{cases} \tag{C.74}$$

$$\rho_L=\begin{cases}(1-(1-\theta_L)t)(\frac{\theta_H}{1-(1-\theta_H)t}-\theta_L)r, & \text{if } G\le v+\frac{(1-(1-\theta_L)t)(\frac{\theta_H r}{1-(1-\theta_H)t}-\theta_L r)}{1-(1-\theta_L)t},\\ (1-\theta_L)(G-v)t, & \text{if } v+\frac{(1-(1-\theta_L)t)(\frac{\theta_H r}{1-(1-\theta_H)t}-\theta_L r)}{1-(1-\theta_L)t}<G\le\frac{v+\theta_L r}{1-\theta_L},\end{cases} \tag{C.75}$$

$$\rho_1=\begin{cases}r(\theta_H-(1-t)\theta_L-\theta_H\theta_L t), & \text{if } G\le v,\\ (\theta_H(1-t)\theta_L-\theta_H\theta_L t)r, & \text{if } v<G\le v+\theta_L r,\\ (\theta_H-\theta_L)r+(1-\theta_H)Gt-(1-\theta_H)tv, & \text{if } v+\theta_L r<G\le\eta,\end{cases} \tag{C.76}$$

$\eta = v + \frac{\theta_H r}{1-(1-\theta_H)t}$, $\rho_2 = \frac{(\theta_L r+t(G-v))(1-t)+\theta_H((v-G)(1-2t)t-(1-t)r)-\theta_H t^2(G-v)}{1-(1-\theta_H)t}$, $\rho_3 = \frac{t((1-\theta_L)(1-t)(G-v)+\theta_H(tG-tv+\theta_L(r+tv-tG))-\theta_H{}^2 r)}{1-(1-\theta_H)t}$ and $\sigma = v + \frac{((1-\theta_H)\theta_H t+\theta_H-\theta_L)r}{(1-\theta_L)t}$. And, $\Phi_l = (\theta_H - \theta_L + t - \theta_H t)G - (1-\theta_H)t - c_A - (1-\theta_H)(G-v)t\,\mathbb{I}(G\le v) + \theta_H r\,\mathbb{I}(v<G\le v+\theta_L r)$, $\Phi_h = (\theta_H-\theta_L)G+[(\theta_H-\theta_L)r+(1-\theta_L)tv+(\theta_H-(1-t)\theta_L-t)G+c_A]\mathbb{I}(G>\sigma)$. $\square$

D. Proof of Proposition 6

Proof. When certification is unavailable (equivalently, when the certification cost is prohibitively high), the equilibrium cutoff investment cost is $\hat{c}=(\theta_H-\theta_L)G$.

We compare the equilibrium strategies and the resulting expected social damage under certification and without certification.

Case 1: $G\le v$. In this regime, certification failure does not induce market exit.

For $c_H\le\Delta_N$, introducing certification changes the equilibrium strategy from (H,N) to (H,A). Since both equilibria involve the same investment decision, expected social damage remains unchanged at $1-\theta_H$.

For $\Delta_N<c_H\le\Delta_A$, the equilibrium strategy changes from (L,N) without certification to (H,A) with certification. Consequently, expected social damage decreases from $1-\theta_L$ to $1-\theta_H$. Since $\theta_H>\theta_L$, certification strictly reduces social damage in this region.

For $c_H>\Delta_A$, the equilibrium strategy changes from (L,N) to (L,A). Because the firm's investment decision remains unchanged, expected social damage remains $1-\theta_L$.

Therefore, when $G\le v$, certification weakly reduces social damage and strictly reduces it for $\Delta_N<c_H\le\Delta_A$.

Case 2: $v < G \leq \eta$. In this regime, firms exit the market following certification failure. Therefore, social damage is incurred only when certification succeeds.

For $c_H \leq \Delta_N$, introducing certification changes the equilibrium strategy from (H, N) to (H, A). Expected social damage decreases from $1 - \theta_H$ to $(1 - \theta_H)(1 - t)$.

For $\Delta_N < c_H \leq \Delta_A$, the equilibrium strategy changes from (L, N) to (H, A). Expected social damage decreases from $1 - \theta_L$ to $(1 - \theta_H)(1 - t)$. Because $(1 - \theta_H)(1 - t) < 1 - \theta_L$, certification strictly reduces social damage.

For $c_H > \Delta_A$, the equilibrium strategy changes from (L, N) to (L, A). Expected social damage decreases from $1 - \theta_L$ to $(1 - \theta_L)(1 - t)$.

Therefore, certification unambiguously reduces social damage when $v < G \leq \eta$.

Case 3: $G > \eta$. In this regime, certification failure also results in market exit.

For $c_H \leq \Delta_A$, introducing certification changes the equilibrium strategy from (H, N) to (H, A). Expected social damage decreases from $1 - \theta_H$ to $(1 - \theta_H)(1 - t)$.

For $\Delta_A < c_H \leq \Delta_N$, the equilibrium strategy changes from (H, N) to (L, A). Expected social damage changes from $1 - \theta_H$ to $(1 - \theta_L)(1 - t)$. Comparing the two expressions, $(1 - \theta_L)(1 - t) > 1 - \theta_H$ if and only if $t < \frac{\theta_H - \theta_L}{1 - \theta_L}$. Hence, certification increases social damage in this region when $t < \frac{\theta_H - \theta_L}{1 - \theta_L}$, and decreases social damage otherwise.

For $c_H > \Delta_N$, the equilibrium strategy changes from (L, N) to (L, A). Expected social damage decreases from $1 - \theta_L$ to $(1 - \theta_L)(1 - t)$.

Combining the above results, when $G > \eta$, certification increases social damage only if $\Delta_A < c_H \leq \Delta_N$ and $t < \frac{\theta_H - \theta_L}{1 - \theta_L}$. In all other cases, certification reduces social damage. Therefore, certification weakly reduces social damage when $G \leq \eta$. When $G > \eta$, certification may increase social damage only in the intermediate investment-cost region $\Delta_A < c_H \leq \Delta_N$ when certification precision is sufficiently low. $\square$

E. Proof of Proposition 7

Proof. We extend the baseline model by allowing a fraction δ of consumers to be sustainability conscious and the remaining $(1 - \delta)$ to be non-sustainability conscious. We first derive the firm's optimal pricing strategies, which are presented as follows.

$$p_1^* = \begin{cases} v, & \text{if } \delta \leq 1 - \frac{\theta_x r}{\theta_x r + \theta_x tv + v - tv}, \\ v + \frac{\theta_x r}{1 - (1 - \theta_x)t}, & \text{if } \delta > 1 - \frac{\theta_x r}{\theta_x r + \theta_x tv + v - tv}, \end{cases} \tag{E.1}$$

$$p_0^* = \begin{cases} v, & \text{if } \delta \leq \frac{v}{\theta_x r + v}, \\ v + \theta_x r, & \text{if } \delta > \frac{v}{\theta_x r + v}. \end{cases} \tag{E.2}$$

Based on the above analysis, we define the following threshold values: $\delta_{LN} = \frac{v}{\theta_L r + v}$, $\delta_{HN} = \frac{v}{\theta_H r + v}$, $\delta_{LA} = 1 - \frac{\theta_L r}{\theta_L r + \theta_L tv + v - tv}$ and $\delta_{HA} = 1 - \frac{\theta_H r}{\theta_H r + \theta_H tv + v - tv}$. Comparing these thresholds yields the relationship $\delta_{HA} < \delta_{LA} < \delta_{HN} < \delta_{LN}$. These threshold values divide the range of δ into five regions that correspond to different market compositions. We then analyze the firm's equilibrium strategies in each region to demonstrate that the main results are robust to consumer heterogeneity.

Before proceeding to the case-by-case analysis, we note that this relationship holds for all five regions. If $G \leq v$, the firm chooses to sell in the market if it fails certification; otherwise, the firm chooses not to sell in the market if it fails certification. For convenience, we define $\tilde{\eta} = \frac{\delta(\theta_H(r+tv)+v-tv)}{1-(1-\theta_H)t}$.

Case 1: $\delta > \max\{\delta_{HA}, \delta_{LA}, \delta_{HN}, \delta_{LN}\}$.

In this case, δ is sufficiently high, the firm adopts the highest possible price under all strategy combinations, rather than setting the baseline price v. Following the same procedure as in the previous analysis, we obtain the corresponding equilibria and the associated ranges of (c_H, c_A).

• Under low penalty ($G \leq \tilde{\eta}$), the cutoff investment cost depends on whether G is below or above v.

i. When $G \leq v$, the cutoff investment cost is given by

$$\hat{c} = \begin{cases} \frac{(\theta_H-\theta_L)(G-Gt(1-\theta_H)-tv(1-(1-\theta_H)t)(1-\delta)+\theta_H rt\delta)}{1-(1-\theta_H)t}, & \text{if } c_A \leq \omega_1, \\ (\theta_H-\theta_L)G + (\theta_H - \theta_H\theta_L t - \theta_L(1-t))\delta r - c_A, & \text{if } \omega_1 < c_A \leq \omega_2, \\ (\theta_H-\theta_L)G, & \text{if } c_A > \omega_2. \end{cases} \tag{E.3}$$

ii. When $v < G \leq \tilde{\eta}$, the cutoff investment cost can be written as

$$\hat{c} = \begin{cases} (\theta_H-\theta_L)\left(G - Gt + \dfrac{(\theta_H(r+tv)+v-tv)t\delta}{1-(1-\theta_H)t}\right), & c_A \leq \omega_3, \\ (\theta_H-\theta_L)G + \delta r\big(\theta_H - (1-t)\theta_L - \theta_H\theta_L t\big) - c_A & \\ \quad + \Big[\delta r(\theta_H-\theta_L) - (1-\theta_H)\delta tv + (\theta_H - \theta_L + t - \theta_H t)G\Big]\,\mathbf{I}(G > \delta(v+\theta_L r)) & \omega_3 < c_A \leq \omega_4, \\ (\theta_H-\theta_L)G, & c_A > \omega_4. \end{cases} \tag{E.4}$$

• Under high penalty ($G > \eta$), the cutoff investment cost can be written as

$$\hat{c} = \begin{cases} (\theta_H-\theta_L)(G - Gt + \frac{(\theta_H(r+tv)+v-tv)t\delta}{1-(1-\theta_H)t}), & \text{if } c_A \leq \omega_5, \\ (\theta_H - \theta_L - (1-\theta_L)t)G + \delta\,(\theta_H-\theta_L)(r+tv) + c_A & \\ \quad + [(1-\theta_L)t\,G - \delta(\theta_H-\theta_L)(r+tv)]\,\mathbb{I}\big(G \leq \delta(v + \dfrac{(\theta_H - \theta_L + (1-\theta_H)\theta_H t)r}{(1-\theta_L)t})\big), & \text{if } \omega_5 < c_A \leq \omega_6, \\ (\theta_H-\theta_L)G, & \text{if } c_A > \omega_6, \end{cases} \tag{E.5}$$

where $\omega_1 = \frac{\delta r(1-(1-\theta_L)t)(\theta_H-\theta_L+(1-\theta_H)\theta_L t)}{1-(1-\theta_H)t}$, $\omega_2 = \delta r(\theta_H - (1-t)\theta_L - \theta_H\theta_L t)$,

$$\omega_3 = \begin{cases} Gt(\theta_H-\theta_L) - \frac{\delta r\left((\theta_H-1)^2\theta_L t^2 + t((\theta_H-2)\theta_L+\theta_H) - \theta_H + \theta_L\right)}{(\theta_H-1)t+1} - \delta tv(\theta_H-\theta_L), & \text{if } v < G \leq \delta(v+\theta_L r), \\ (t-\theta_L t)G + \frac{\delta(\theta_H-\theta_L)(1-t)}{1-(1-\theta_H)t} - (1-\theta_L)\delta tv, & \text{if } \delta(v+\theta_L r) < G \leq \tilde{\eta}, \end{cases} \tag{E.6}$$

$$\omega_4 = \begin{cases} \delta r(\theta_H - (1-t)\theta_L - \theta_H\theta_L t), & \text{if } v < G \le \delta(v + \theta_L r), \\ (\theta_H - \theta_L)\delta r + (t - \theta_H t)G - (1-\theta_H)\delta t v, & \text{if } \delta(v+\theta_L r) < G \le \tilde{\eta}, \end{cases} \tag{E.7}$$

$$\omega_5 = \begin{cases} \delta\theta_H r t(1-\theta_H), & \text{if } \tilde{\eta} < G \le \delta(v + \frac{(\theta_H - \theta_L + (1-\theta_H)\theta_H t)r}{(1-\theta_L)t}), \\ G(t - \theta_H t) + \frac{\delta r(t-1)(\theta_H - L)}{1-(1-\theta_H)t} - \delta(1-\theta_H)tv, & \text{if } \delta(v + \frac{(\theta_H - \theta_L + (1-\theta_H)\theta_H t)r}{(1-\theta_L)t}) < G \le \frac{v+\theta_L r}{1-\theta_L}, \end{cases} \tag{E.8}$$

and $\omega_6 = G(t - \theta_L t) + \delta(\theta_L r - \theta_H r - (1-\theta_L)tv)$.

We find that when $\delta > \delta_{LN}$, the equilibrium structure and comparative statics remain qualitatively the same as in the baseline model.

Case 2: $\max\{\delta_{HA}, \delta_{LA}, \delta_{HN}\} < \delta \le \delta_{LN}$.

In this case,when the equilibrium strategy is (L, N), the firm sets the baseline price v; under all other equilibrium strategies, the firm charges the highest feasible price. Following the same procedure as in the previous analysis, we obtain the corresponding equilibria and the associated ranges of (c_H, c_A).

Under low penalty ($G \le \tilde{\eta}$), the cutoff investment cost depends on whether G is below or above v.

- When $G \le v$, the cutoff investment cost is given by

$$\hat{c} = \begin{cases} \frac{(\theta_H - \theta_L)(G - Gt(1-\theta_H) - tv(1-(1-\theta_H)t)(1-\delta) + \theta_H r t\delta)}{1-(1-\theta_H)t}, & \text{if } c_A \le \omega_7, \\ (\theta_H - \theta_L)G + \delta\theta_H r - (1-\delta)(1-(1-\theta_H)t)v - c_A, & \text{if } \omega_8 < c_A \le \omega_9, \\ (\theta_H - \theta_L)G, & \text{if } c_A > \omega_9. \end{cases} \tag{E.9}$$

- When $v < G \le \tilde{\eta}$, the cutoff investment cost can be written as

$$\hat{c} = \begin{cases} (\theta_H - \theta_L)(G - Gt + \frac{(\theta_H(r+tv) + v - tv)t\delta}{1-(1-\theta_H)t}), & \text{if } c_A \le \omega_{10}, \\ G(\theta_H(1-t) - \theta_L + t) + \delta\theta_H r + v(\delta - \delta(1-\theta_H)t - 1) - c_A, & \text{if } \omega_{10} < c_A \le \omega_{11}, \\ (\theta_H - \theta_L)G, & \text{if } c_A > \omega_{11}. \end{cases} \tag{E.10}$$

Under high penalty ($G > \eta$), the cutoff investment cost can be written as

$$\hat{c} = \begin{cases} (\theta_H - \theta_L)(G - Gt + \frac{(\theta_H(r+tv) + v - tv)t\delta}{1-(1-\theta_H)t}), & \text{if } c_A \le \omega_5, \\ (\theta_H - \theta_L - (1-\theta_L)t)G + \delta\,(\theta_H - \theta_L)(r + tv) + c_A & \\ \quad + [(1-\theta_L)t\,G - \delta(\theta_H - \theta_L)(r+tv) - c_A]\,\mathbb{I}\big(G \le \delta(v + \dfrac{(\theta_H - \theta_L + (1-\theta_H)\theta_H t)r}{(1-\theta_L)t})\big), & \text{if } \omega_5 < c_A \le \omega_6, \\ (\theta_H - \theta_L)G, & \text{if } c_A > \omega_6, \end{cases} \tag{E.11}$$

where $\omega_7 = \delta\theta_L r - (1-\delta)(1-(1-\theta_L)t)v$, $\omega_8 = \frac{(1-(1-\theta_L)t)(\delta\theta_H r - (1-\delta)(1-(1-\theta_H)t)v)}{1-(1-\theta_H)t}$, $\omega_9 = \delta\theta_H r - (1-\delta)(1-(1-\theta_H)t)v$, $\omega_{10} = (\theta_H - \theta_L)Gt + \frac{\delta\theta_H r(1-(1-\theta_L)t)}{1-(1-\theta_H)t} - (1 - \delta - t(1 - \theta_H - \delta + \theta_L\delta))v$, $\omega_{11} = \delta\theta_H r - (1-(1-\theta_H)t)(1-\delta)v$,

$$\omega_{12} = \begin{cases} \delta\theta_H r t(1-\theta_H), & \text{if } v < G \le \delta(v + \frac{(\theta_H - \theta_L + (1-\theta_H)\theta_H t)r}{(1-\theta_L)t}), \\ G(1-\theta_H)(\delta + t - 1) + \dfrac{\delta\theta_H r(t - \theta_L t - 1)}{1-(1-\theta_H)t} & \text{if } \delta(v + \frac{(\theta_H - \theta_L + (1-\theta_H)\theta_H t)r}{(1-\theta_L)t}) < G \le \tilde{\eta}. \\ \quad + tv(\delta\theta_H - \delta\theta_L + \theta_L - 1) - \delta v + v, & \end{cases} \tag{E.12}$$

and $\omega_{13} = v - (1-\theta_L)tv - (\theta_H r + v)\delta - (1-\theta_H - t + \theta_L t + \delta\theta_H - \delta)G$.

We find that when $\delta_{HN} < \delta \leq \delta_{LN}$, the firm's equilibrium behavior varies with the penalty G. Specifically, when $G \leq v$, as δ decreases, the equilibrium region associated with strategy (L, A) becomes smaller. When $G \leq v$ and the certification cost lies within an intermediate range (i.e., $\omega_7 < c_A \leq \omega_8$), no pure-strategy equilibrium exists. When $G > \tilde{\eta}$, the qualitative trends and main insights remain consistent with those in the baseline model.

Case 3: $\{\delta_{HA}, \delta_{LA}\} < \delta \leq \{\delta_{HN}, \delta_{LN}\}$.

In this case, when the equilibrium strategy is either (L, N) or (H, N), the firm sets the baseline price v; under all other equilibrium strategies, the firm charges the highest feasible price. Following the same procedure as in the previous analysis, we obtain the corresponding equilibria and the associated ranges of (c_H, c_A).

• Under low penalty ($G \leq \tilde{\eta}$), the cutoff investment cost depends on whether G is below or above v.

i. When $G \leq v$, the cutoff investment cost is given by

$$\hat{c} = \begin{cases} \frac{(\theta_H-\theta_L)(G-Gt(1-\theta_H)-tv(1-(1-\theta_H)t)(1-\delta)+\theta_H rt\delta)}{1-(1-\theta_H)t}, & \text{if } c_A \leq \omega_7, \\ (\theta_H-\theta_L)G + \delta\theta_H r - (1-\delta)(1-(1-\theta_H)t)v - c_A, & \text{if } \omega_8 < c_A \leq \omega_9, \\ (\theta_H-\theta_L)G, & \text{if } c_A > \omega_9. \end{cases} \tag{E.13}$$

ii. When $v < G \leq \tilde{\eta}$, the cutoff investment cost can be written as

$$\hat{c} = \begin{cases} (\theta_H-\theta_L)(G - Gt + \frac{(\theta_H(r+tv)+v-tv)t\delta}{1-(1-\theta_H)t}), & \text{if } c_A \leq \omega_{10}, \\ (\theta_H-\theta_L)G + \delta\theta_H r - (1-\delta)(1-(1-\theta_H)t)v - c_A, & \text{if } \omega_{10} < c_A \leq \omega_{11}, \\ (\theta_H-\theta_L)G, & \text{if } c_A > \omega_{11}. \end{cases} \tag{E.14}$$

• Under high penalty ($G > \eta$), the cutoff investment cost can be written as

$$\hat{c} = \begin{cases} (\theta_H-\theta_L)(G - Gt + \frac{(\theta_H(r+tv)+v-tv)t\delta}{1-(1-\theta_H)t}), & \text{if } c_A \leq \omega_{12}, \\ c_A + G(\theta_H + (\theta_L-1)t - \theta_L) - \delta\theta_L r + \delta v(-\theta_L t + t - 1) + v & \\ \quad + \mathbf{I}(G \leq \delta(v + \frac{r}{t}))\Big[G(-(\theta_L-1)t - \theta_L) + \delta\theta_L r - \delta v(-\theta_L t + t - 1) - v - c_A\Big] & \text{if } \omega_{12} < c_A \leq \omega_{13}, \\ (\theta_H-\theta_L)G, & \text{if } c_A > \omega_{13}, \end{cases} \tag{E.15}$$

where

$$\omega_{12} = \begin{cases} G(t-\theta_H t) + \frac{\delta r\left({\theta_H}^2 t - \theta_L t + \theta_L\right)}{1-(1-\theta_H)t} + v(\delta + \delta(\theta_H-1)t - 1), & \text{if } v < G \leq \delta(v+\frac{r}{t}), \\ \delta\theta_H r - (1-\delta)v(1-(1-\theta_H)t), & \text{if } \delta(v+\frac{r}{t}) < G \leq \frac{v+\theta_L r}{1-\theta_L}, \end{cases} \tag{E.16}$$

and $\omega_{13} = G(t-\theta_L t) + \delta\theta_L r + v(\delta + \delta(\theta_L-1)t - 1)$.

For $\delta_{LA} < \delta \leq \delta_{HN}$, the results are qualitatively similar to those in Case 2; hence, we omit the detailed discussion. As δ decreases further, the feasible region of the (L, A) strategy continues to shrink.

Case 4: $\delta_{HA} < \delta \leq \min\{\delta_{LA}, \delta_{HN}, \delta_{LN}\}$.

In this case, the firm sets the baseline price v under all equilibrium strategies. Following the same procedure as in the previous analysis, we obtain the corresponding equilibria and the associated ranges of (c_H, c_A).

• Under low penalty ($G \leq \tilde{\eta}$), the cutoff investment cost depends on whether G is below or above v.

i. When $G \leq v$, the cutoff investment cost is $\hat{c} = (\theta_H - \theta_L)G$.

ii. When $v < G \leq \tilde{\eta}$, the cutoff investment cost can be written as

$$\hat{c} = \begin{cases} (\theta_H - \theta_L)(G - Gt + \frac{(\theta_H(r+tv)+v-tv)t\delta}{1-(1-\theta_H)t}), & \text{if } c_A \leq \omega_{10}, \\ (\theta_H - \theta_L + t - \theta_H t)G + \delta\theta_H r - (1 - \delta + (1-\theta_H)\delta t)v - c_A, & \text{if } \omega_{10} < c_A \leq \omega_{11}, \\ (\theta_H - \theta_L)G, & \text{if } c_A > \omega_{11}. \end{cases} \tag{E.17}$$

• Under high penalty ($G > \tilde{\eta}$), the cutoff investment cost can be written as

$$\hat{c} = \begin{cases} (\theta_H - \theta_L)(G - Gt + \frac{(\theta_H(r+tv)+v-tv)t\delta}{1-(1-\theta_H)t}), & \text{if } c_A \leq \omega_{14}, \\ (\theta_H - \theta_L)G + (1-\theta_L)(v-G)t + c_A & \\ \quad + (-(1-\theta_L)(v-G)t - c_A)\mathbb{I}(G \leq \dfrac{v(\delta - t(\delta - \delta H + L - 1) - 1) + \delta H r}{t(H-L)}), & \text{if } \omega_{14} < c_A \leq \omega_{15}, \\ (\theta_H - \theta_L)G, & \text{if } c_A > \omega_{15}, \end{cases} \tag{E.18}$$

where

$$\omega_{14} = \begin{cases} \delta\theta_H r + (\delta - 1)v((\theta_H - 1)t + 1), & \text{if } \tilde{\eta} < G \leq \frac{v(\delta - t(\delta - \delta H + L - 1) - 1) + \delta H r}{t(H-L)}, \\ (G - \theta_H G + \frac{\delta\theta_H r(\theta_- \theta_L)}{1-(1-\theta_H)t} - (1 - \theta_L - \theta_H\delta + \theta_L\delta)v)t, & \text{if } \frac{v(\delta - t(\delta - \delta H + L - 1) - 1) + \delta H r}{t(H-L)} < G \leq \frac{v + \theta_L r}{1-\theta_L}. \end{cases} \tag{E.19}$$

and $\omega_{15} = (1-\theta_L)(G-v)t$.

For $\delta_{HA} < \delta_{LA}$, when $G \leq v$, the firm no longer finds certification profitable and thus chooses not to apply for certification. In all other respects, the main equilibrium structure remains qualitatively consistent with the baseline model.

Case 5: $\delta \leq \{\delta_{HA}, \delta_{LA}, \delta_{HN}, \delta_{LN}\}$.

In this case, the firm adopts the highest possible price under all strategy combinations, rather than setting the baseline price v. Following the same procedure as in the previous analysis, we obtain the corresponding equilibria and the associated ranges of (c_H, c_A).

• When $G \leq v$, the cutoff investment cost is $\hat{c} = (\theta_H - \theta_L)G$.

• When $G > v$, the cutoff investment cost can be written as

$$\hat{c} = \begin{cases} (\theta_H - \theta_L)(G + (v-G)t), & \text{if } c_A \leq \omega_{16}, \\ (\theta_H - \theta_L)G + (1-\theta_L)(v-G)t + c_A, & \text{if } \omega_{16} < c_A \leq \omega_{15}, \\ (\theta_H - \theta_L)G, & \text{if } c_A > \omega_{15}, \end{cases} \tag{E.20}$$

where $\omega_{16} = (1-\theta_H)t(G-v)$.

Finally, for $\delta < \delta_{HA}$, when $G \le v$, the firm will not apply for certification; when $G > v$, the inequality $(\theta_H - \theta_L)(G + (v-G)t) < (\theta_H - \theta_L)G$, indicating that certification always weakens the incentive to invest in sustainability. Hence, no certification-induced investment improvement exists in this case.

Overall, these results confirm that while the analytical thresholds differ, the qualitative equilibrium structure and comparative statics remain robust to consumer heterogeneity. $\square$

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