Shipment Monitoring, Allocation and the Impact on Food Waste

Tao Lu

School of Business, University of Connecticut, Storrs, CT 06269

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Abstract

Often advocated for the potential to reduce waste in fresh produce supply chains, sensor technologies enable firms to monitor the condition of each product unit and allocate them to markets at different transportation distances, thereby matching units with shorter shelf lives with closer markets. In this paper, we consider a firm purchasing and transporting a fresh produce product for sale in two destination markets. Post-harvest processes cause a proportion of the product to be in risky condition, potentially becoming unsalable ("spoiled") due to shelf-life deterioration during transportation. With monitoring technology, the firm first determines a purchase quantity and then decides how to allocate each unit (in either good or risky condition) between markets. We characterize the optimal allocation policies, which can be used to efficiently determine the optimal purchase quantity. Our analysis shows that such technology may inadvertently increase total waste by inducing larger purchase quantities, while this increase is often, but not always, accompanied by a lower waste rate and improved distribution efficiency. As firms increasingly adopt monitoring technology for shipment allocation, additional policy and managerial attention may be required. Improvements in post-harvest processes can strengthen the waste-reduction benefits of monitoring technology. Firms seeking to maintain their sustainability goals can capture most of the profit gains from monitoring by carefully moderating increases in purchase quantities.

Keywords: sensor technology, food waste, allocation, newsvendor, supply uncertainty.

1 Introduction

According to the Food and Agriculture Organization (FAO) of the United Nations, 30% of the food produced for human consumption is lost or wasted somewhere along the food supply chain (Rezaei and Liu 2017). Due to their short shelf-life and high perishability, losses of fruits and vegetables are more significant: 10% – 20% lost during production, 14% – 15% lost during the post-harvest, processing and distribution stages in industrialized regions (e.g., Europe and North America), and even larger losses in developing regions (FAO 2011, cf. fig. 6). The lack of effective cold chain infrastructure to maintain the food quality during the post-harvest and transportation stages is one of the major causes (World Economic Forum 2022). Field data show that freshness deterioration during transportation – such as weight loss, bruise and decay – can significantly reduce the appearance and other quality attributes of fruits, serving as a major cause of fruit rejection at retail stores (Lai et al. 2011). On average, 12.6% of the fresh vegetables and fruits shipped to major supermarkets in the United States (US) fail to sell to end consumers, with shrinkage rates as high as 43.1% for vulnerable fruits such as papayas (Buzby et al. 2016). Erwin (2024, para. 3) indicates that "due to their short shelf life, around 25% of strawberries are discarded at the distribution and retail level." The loss of fresh produce products has critical environmental consequences. Fruits often have high carbon footprints due to the resources used in their growth and distribution, including high irrigation, pesticide use, and the energy required for refrigerated storage and transportation (Mersereau n.d.). Food waste resulting from inefficient cold chain facilities accounts for 2% of total global greenhouse gas emissions (World Economic Forum 2022).

Recently, emerging sensor technologies have created opportunities to combat fresh produce waste by monitoring ambient variables (e.g., time, temperature, humidity) along the cold chain starting from the farm. For example, Zest Labs, a California-based agri-tech company, has developed a pallet-level monitoring system that provides visibility into shipment freshness and enables firms to make more informed decisions in managing inventory and distribution. As noted by Fresh Fruit Portal (2018), variability in time and temperature during post-harvest processes (e.g., pre-cooling, processing, handling) can cause strawberries leaving the supplier to differ dramatically in their remaining shelf life. Because these differences cannot be detected by visual inspection or tracked by traditional date labels, firms without pallet-level monitoring technology must allocate pallets with varying freshness blindly across destinations. With monitoring technology, firms can identify pallets with different remaining shelf lives and allocate them accordingly, such that "pallets with lesser shelf life can be sent to closer stores while those with more shelf life can be sent further away [...and] the technology can reduce retailers' food waste by 50%" (Fresh Fruit Portal 2018, para. 3–15). As another example, Walmart has developed its own food monitoring system to manage fruit shipments and expects to eliminate $2 billion in food waste over five years (Walmart 2018).

In this paper, we develop an analytical model to study the impact of monitoring technology on a firm's purchase and allocation decisions, as well as its implications for waste reduction. Specifically, we consider a firm purchasing and transporting a fresh produce product for sale in two destination markets with uncertain demand. Post-harvest processes make some of the product in risky condition, possibly becoming unsalable (or "spoiled") when reaching a market due to shelf-life deterioration during transportation. Monitoring technology enables the firm to identify those risky units and allocate each shipment unit based on its condition and the distance to each market. Units directed to a nearer market are exposed to a lower risk of spoilage. The firm solves a two-stage optimization: It first chooses a purchase quantity, and then determines how to allocate each shipment unit (in either good or risky condition) between the two markets.

We fully characterize the structure of the optimal allocation policies, which helps efficiently determine the optimal purchase quantity. Unlike typical newsvendor problems with supply uncertainty, monitoring technology creates a cross-market dependence such that the total quantity supplied to one market can either increase or decrease with the profitability of the other market. Then, we examine the implications of monitoring for waste reduction. Through analytical results for special cases and a numerical study calibrated using California strawberries, we show that, despite its advocated potential to reduce food waste, monitoring technology may inadvertently increase total waste. At the same time, this increase in total waste is often accompanied by a lower waste rate (i.e., less waste per unit ordered), reflecting improved distribution efficiency; however, when post-harvest processes yield a low fraction of good units, monitoring may increase both total waste and the waste rate. These unintended consequences arise because monitoring technology can lead to a larger purchase quantity, resulting in more leftovers and waste.

Our results offer important policy and managerial implications. As firms increasingly adopt monitoring technology, greater policy and managerial attention, not less, is needed. Specifically, improvements in post-harvest processes can strengthen the waste-reduction benefit of monitoring technology. Moreover, by moderating increases in order quantities and using monitoring primarily for allocation, waste-conscious firms can capture most of the profit gains from monitoring while preserving their sustainability goals. Improved post-harvest processes can also reinforce firms' incentives to moderate order quantities for waste reduction. Thus, policymakers and non-governmental organizations (NGOs) should continue and expand efforts to support producers in strengthening post-harvest cold chain facilities.

2 Related Literature

Our paper adds to the growing literature on the impact of sensor technologies in supply chain settings such as spare part inventory management (Li and Ryan 2011, Song and Zhang 2024), replenishment of perishable inventory (Ketzenberg et al. 2015), and after-sales service contracting (Li and Tomlin 2022). Haass et al. (2015) document a simulation study on shipping bananas to multiple destinations with intelligent containers monitoring the ripening degree of bananas. Lu and Tomlin (2023) study a firm's sourcing strategies with competing suppliers when sensor technology provides updated information on the shipment from an unreliable supplier. Motivated by blockchain technologies, some papers explore the implications of supply chain traceability on, e.g., food safety (Dong et al. 2022b) and product quality (Cui et al. 2022). Keskin et al. (2024) analyze a model in which a fresh produce retailer, facing freshness-dependent random demand, uses a blockchain-enabled smart contract to ensure the transparency of product freshness from a supplier. Different from the above papers, we study how monitoring technology can be leveraged to allocate perishable shipments across markets according to their conditions. To the best of our knowledge, we are the first to develop an analytical model of this novel monitoring-enabled condition-based shipment allocation and examine its implications for reducing food waste.

Our results shed light on the impact of monitoring technology on food waste, thus related to the growing sustainable operations literature concerning waste (e.g., Belavina et al. 2017, Ata et al. 2019, Belavina 2021, Akkaş and Gaur 2022). Several studies investigate the implications of retail operations on food waste. Various aspects of grocery store operations have been considered, e.g., shelf space selection (Akkaş 2019), salesforce compensation (Akkaş and Sahoo 2020), the issuing policies for premade food (Park et al. 2022), and sales promotion strategies (Wu and Honhon 2023). Lee and Tongarlak (2017) consider a retail grocer that sells a perishable product under uncertain demand and may choose to convert excess units to a secondary product. Motivated by consumer packaged goods with fixed shelf lives, Akkaş and Honhon (2022) study the issuing policies from a warehouse to a retail store with a given order-up-level maintained in the warehouse. Akkaş and Honhon (2023) further analyze a class of issuing policies based on an shipping-age threshold. Different from the above papers, our model is focused on the sourcing and transportation stages of a fresh-produce supply chain. We take into account waste arising from not only excess inventory but also the shelf-life deterioration during transportation. Unlike most consumer packaged goods, the shelf life of fresh produce is more sensitive to random factors during post-harvest processes (e.g., temperature, transportation time) and is thus hard to track without monitoring technology.[1]

Because we model spoilage as a random yield loss from the total shipment, our work is thus relevant to the literature on supply chain management with supply uncertainty (Yano and Lee

[1]There are papers examining waste-related sustainable operations in different contexts such as by-product synergy in agricultural and manufacturing industries (Lee 2012), converting municipal waste to energy (Ata et al. 2012), biomass commercialization of agricultural processing firms (Li et al. 2022), and quick response in the fashion industry (Long and Nasiry 2022, Long and Gui 2024, Tuna and Swinney 2021).

1995). This literature is typically focused on a firm's sourcing decisions when the delivered order quantities by suppliers suffer from random losses (e.g., Dada et al. 2007, Tomlin 2009, Tang and Kouvelis 2011, Dai et al. 2016, Ang et al. 2017, Dong et al. 2022a). For supply uncertainty due to transportation, Lu et al. (2017) examine a sourcing problem involving multiple transportation carriers with uncertain transit times. Some papers study process improvement to mitigate supply uncertainty. Wang et al. (2010) compares the value of process improvement with that of dual sourcing. Tang et al. (2014) study process improvement in a decentralized supply chain. Wang et al. (2014) and Lee and Lu (2015) examine yield improvement in competitive settings. Compared to this literature, a unique feature of our model is that monitoring technology enables the firm to allocate each shipment unit based on its condition at an interim point in time, before supply uncertainty is fully realized. This condition-based allocation, as an operational means of yield improvement, leads to several novel properties of the optimal solution that has not been documented in the literature; see detailed discussions in §4.1.3.

Finally, from the modeling perspective, our problem is related to the literature on coproduction operations where a single production run simultaneously generates multiple types of outputs (e.g., Tomlin and Wang 2008, Chen et al. 2017, Lu et al. 2019, Lin et al. 2020). Motivated by the agriculture industry, Boyabatlı (2015) considers a coproduction process that yields two horizontally different products in fixed proportions to the quantity of a primary input. In our model, monitoring technology enables the firm to identify two types of units differing in spoilage risk from the total quantity, thus mirroring a coproduction system to some extent. However, our problem is vastly different. Among other differences, a unique feature of our problem is that the firm can decide how to allocate the two types of units between markets at an interim stage.

3 The Model

We consider a firm purchasing and shipping a perishable product for sale in two markets, indexed by $i \in \mathcal{M} = \{1, 2\}$, located in different geographic regions. Demand at each market $i \in \mathcal{M}$ is a random variable D_i with a cumulative distribution function (cdf) denoted by $F_i(x)$ and $F_i(0) = 0$. We will use shorthand notation $\bar{F}(x) = 1 - F(x)$ frequently in the paper.

The supply process is divided into two phases, as illustrated in Figure 1. The first phase consists of all post-harvest processes before the product is ready for distribution between the two markets.[2] In the second phase, the product is distributed to the two destination markets through separate transportation lanes. The firm pays a cost c_o for each unit purchased and

[2]For products sourced overseas (e.g., bananas from Latin America), the first phase includes import transportation from the origin country to a transshipment location where the product is sorted for distribution.

processed in the first phase. The transportation cost per unit to each market i is denoted by c_i. Without loss of generality, we assume that market 1 requires a shorter transportation distance than market 2 and consequently $c_1 < c_2$. For example, for California strawberries, markets 1 and 2 can represent, for instance, the West Coast and East Coast markets, respectively.

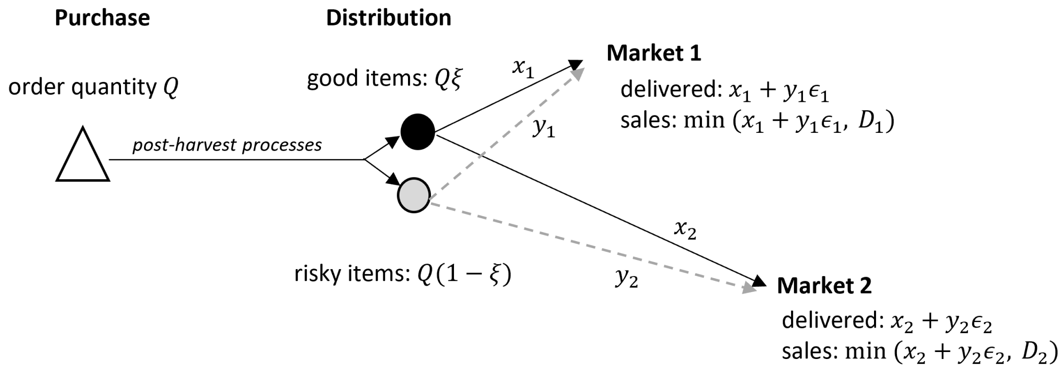


Figure 1: Purchase and allocation with monitoring

Denote by Q the purchase quantity of the firm. Post-harvest processes in the first phase impact the shelf-life conditions of the Q units. A (possibly random) proportion of the product, denoted by ξ, is kept in good condition, whereas the $(1-\xi)$ proportion of the product suffers some loss in shelf life due to the variability of post-harvest processes (e.g., being exposed to sunlight for too long).[3] As such, at the beginning of the second phase, $Q\xi$ units of the shipment are in good condition (denoted by the black circle in Figure 1) while $Q(1-\xi)$ units are in risky condition (denoted by the gray circle in Figure 1). We assume that the units in good condition have a sufficiently long shelf life and will be successfully delivered to both markets with probability one. However, units in risky condition may become unsalable after reaching the markets due to shelf life deterioration during transportation. Evidence has shown that quality deterioration, such as bruise and decay, during transportation can significantly worsen the appearance and other quality attributes of fruits, which is a major cause for fruit rejection at retail stores (Lai et al. 2011). For brevity, we will refer to units that become unsalable at the retail level due to deterioration during transportation as "spoilage." If y_i risky units are shipped to market i, $\epsilon_i y_i$ units will be delivered successfully, i.e., in salable condition, whereas $(1-\epsilon_i)y_i$ units will arrive spoiled, where $\epsilon_i \in [0,1]$ is a random yield factor. The randomness of ϵ_i is a result of the variability in the transportation process to market i (e.g., delay, temperature excursion inside a refrigerated truck). To reflect the fact that a greater risk of spoilage is involved if risky units are directed to a more distant market, we assume that ϵ_1 is first-order stochastically larger than ϵ_2. Moreover, the ϵ_i's are independent since they are determined by random events with separate

[3]In the literature, it is common to assume two product types when analyzing yield processes that simultaneously generate varying qualities; see, e.g., Tomlin and Wang (2008), Chen et al. (2017).

fleets on different transportation lanes. A more detailed explanation of the proposed random yield model is provided in Appendix A. In the base model, we will focus on the case in which each ϵ_i follows a Bernoulli distribution: $\epsilon_i = 1$ with probability β_i while $\epsilon_i = 0$ with probability $1 - \beta_i$ where $0 \leq \beta_2 < \beta_1 \leq 1$ so ϵ_1 is first-order stochastically larger. The all-or-nothing yield risk is a good approximation of our problem if shipment units directed to each market are carried by a single truck so all the units in the truck experience the same delay and temperature excursion (if any).[4] Nevertheless, in §5.3 we show that our results can be generalized to the case where the ϵ_i's follow general probability distributions.

In each market, the successfully delivered product will sell at a retail price p per unit. The product, if spoiled or if left over, will be disposed of at a cost of c_d per unit.[5] In practice, c_d may encompass the waste disposal fee charged at landfill facilities, together with any administrative surcharges and logistics costs for arranging the disposal. To focus on the primary research questions regarding the impact of monitoring on tactical supply chain decisions (i.e., purchase and allocation) and food waste, we assume that the product sells at a fixed price as long as it reaches a market in salable condition. The assumption of exogenous selling price and demand allows us to evaluate the impact of monitoring-enabled shipment allocation on total waste while abstracting away from the impact of price adjustment; such an assumption is common in the literature as to food waste (e.g., Lee and Tongarlak 2017, Akkaş and Honhon 2022). Operational-level decisions such as markdown pricing at grocery stores are beyond the scope of our research. Additionally, to rule out the trivial case where the firm orders nothing, we assume $p - c_1 - c_o > 0$, i.e., the profit margin is positive at least for the nearer market.

3.1 Two-stage Optimization with Monitoring

Sensors are deployed to monitor the condition of each shipment unit such that the firm can identify whether each unit is in good or risky condition before the product is distributed. Therefore, the firm can determine how to allocate the two types of shipments (in good and risky conditions) between two markets. The problem is therefore divided into two stages. In the first stage, the firm determines an order quantity Q. In the second stage, the firm determines how to allocate each shipment unit based on its condition.

We first formulate the second-stage problem for any given order quantity Q and product split ξ. We consider the purchase cost c_o as a sunk cost in the second stage and will incorporate it

[4] Also, the all-or-nothing yield risk is commonly assumed in the literature (e.g., Ang et al. 2017, Lu and Tomlin 2023) for analytical convenience.

[5] According to ReFED (n.d.), only a very small portion of unsold food is effectively donated or recycled in the US while the vast majority of unsold product becomes waste. We therefore do not consider food donation or recycling in our model, but our primary insights will remain valid if a fixed proportion of unsold inventory is effectively donated with and without monitoring.

in the first-stage problem. Let $R(Q,\xi)$ represent the firm's maximum profit in the second stage. Denote by x_i (y_i) the units in good (risky) condition allocated to each market $i \in \mathcal{M}$. The value of $R(Q,\xi)$ is determined by the following optimization problem with decision variables $\mathbf{x} = [x_1, x_2]$ and $\mathbf{y} = [y_1, y_2]$.

$$R(Q,\xi) = \max_{\mathbf{x},\mathbf{y}} \sum_{i\in\mathcal{M}} \left\{E_{D_i,\epsilon_i}\left[p\min(D_i, x_i + y_i\epsilon_i) - c_d((1-\epsilon_i)y_i + (x_i + y_i\epsilon_i - D_i)^+)\right] - c_i(x_i + y_i)\right\} \tag{1}$$

$$\sum_{i\in\mathcal{M}} x_i = Q\xi \tag{2}$$

$$\sum_{i\in\mathcal{M}} y_i = Q(1-\xi) \tag{3}$$

$$x_i \geq 0,\ y_i \geq 0 \text{ for all } i \in \mathcal{M} \tag{4}$$

The objective function (1) is the sum of the profits from the two markets. The term inside the expectation of (1) is given by the sales revenue minus the disposal cost. Note that the quantity successfully delivered to each market is given by $x_i + y_i\epsilon_i$, while the unsold quantity, i.e., the waste, consists of two parts: $(1-\epsilon_i)y_i$ units arriving spoiled, and $(x_i + y_i\epsilon_i - D_i)^+$ units left over. Constraints (2) and (3) ensure that the total allocated quantity is equal to the amount of shipments available in either good or risky conditions. Thus, our base model implicitly assumes that discarding units in transit is not optimal. In §5.2, we will discuss the case in which the firm may dispose of some risky units in transit, instead of sending them to any destination market. Constraint (4) ensures that all quantities are nonnegative.

In the first stage, the firm determines an order quantity $Q \geq 0$ to maximize the expected profit

$$\Pi(Q) = E_\xi\left[R(Q,\xi)\right] - c_o Q, \tag{5}$$

which is equal to the expected profit in the second stage minus the purchase cost. The expectation of $R(Q,\xi)$ is taken over ξ, so we allow a random split between good and risky units to capture the uncertainty in the post-harvest process. See Figure 1 for a graphic illustration of the decision variables and uncertain parameters in the model.

3.2 Benchmark Without Monitoring

As a benchmark, we consider a scenario without monitoring technology, and so the firm has no information to distinguish good and risky units when determining the shipment allocation.[6]

[6]Recall that our focus is on temperature-sensitive products (such as fresh produce and meat products) for which it is hard to determine the remaining shelf life without monitoring the full history of temperature or other relevant ambient parameters. This is different from consumer packaged goods for which one can mostly infer the

Consequently, the firm determines an order quantity to be sent to each market i, denoted by q_i, in a single-stage problem. We assume that every unit will equally likely be sent to each market. Thus, the fraction of the shipment that will be successfully delivered to each market i is given by $z_i = \xi + (1-\xi)\epsilon_i$, a composition of random variables ξ and ϵ_i. The amount of product unsold in market i consists of the units arriving in unsalable condition, $(1-z_i)q_i$, and the excess units, $(q_i z_i - D_i)^+$. See Figure 2 for an illustration of the model without monitoring.

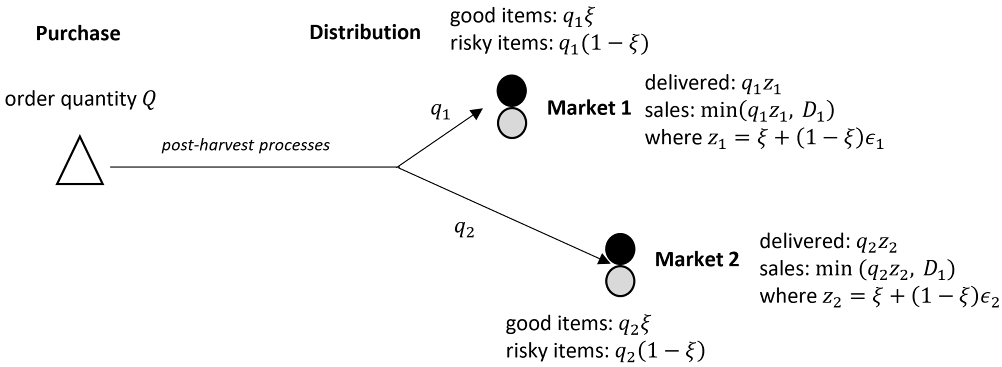


Figure 2: Purchase and allocation without monitoring

Absent any information about the condition of each shipment unit, the two order quantities q_1 and q_2 can be determined independently. The firm solves two separate newsvendor problems subject to random yield losses $(1-z_1)$ and $(1-z_2)$, respectively. Formally, for each market i, the firm maximizes the following expected profit by choosing $q_i \geq 0$:

$$\Pi_i^N(q_i) = E[p\min(D_i, q_i z_i) - c_d((1-z_i)q_i + (q_i z_i - D_i)^+)] - (c_o + c_i)q_i. \tag{6}$$

The above problem has been well studied in the supply uncertainty literature (Yano and Lee 1995), and its optimal solution is characterized in Proposition B.1 of Appendix B. In this paper, we will use it as a benchmark to discuss the impact of shipment monitoring.

4 Results

4.1 Optimal Solution with Monitoring

4.1.1 The Second-Stage Allocation Problem

We start by analyzing the second-stage allocation problem (1)-(4) for any given order quantity Q and proportion ξ. Clearly, the objective function (1) is concave in $(\mathbf{x}, \mathbf{y})$ and all the constraints are linear. Therefore, the optimal allocation $\mathbf{x}^* = [x_1^*, x_2^*]$ and $\mathbf{y}^* = [y_1^*, y_2^*]$ can be characterized

remaining shelf life from expiry dates.

through analyzing the Karush–Kuhn–Tucker (KKT) conditions. The general expression of KKT conditions can be found in Appendix C.1. An appealing property of our problem is that the markets can be sorted according to the spoilage risk and transportation cost (i.e., the spoilage risk and transportation cost of market 2 are both higher than those of market 1). By exploiting this property, we are able to characterize the structure of the optimal allocation $\mathbf{x}^*$ and $\mathbf{y}^*$. Define the following thresholds as functions of Q and ξ which will be useful for characterizing the optimal allocation: $C_1(Q,\xi) = 1 - \beta_1\bar{F}_1(Q) - (1-\beta_1)\bar{F}_1(Q\xi)$, $C_2(Q,\xi) = \beta_1 F_1(Q(1-\xi)) - F_2(Q\xi)$, $C_3(Q,\xi) = \beta_2\bar{F}_2(Q\xi) - \beta_1\bar{F}_1(Q(1-\xi))$, $C_4(Q,\xi) = (1-\beta_2)\bar{F}_2(Q\xi) - (1-\beta_1)$ and

$$C_5(Q,\xi) = \begin{cases} \beta_2\bar{F}_2(Q\xi - x^o) - \beta_1\bar{F}_1(Q(1-\xi)+x^o) & \text{if } C_4(Q,\xi) < 0 \\ 0 & \text{if } C_4(Q,\xi) \geq 0. \end{cases}$$

where x^o is a solution to $(1-\beta_1)\bar{F}_1(x^o) = (1-\beta_2)\bar{F}_2(Q\xi - x^o)$.[7] Furthermore, define $\Phi = \frac{c_2 - c_1}{p + c_d}$ which reflects the difference in transportation costs of two markets. As stated in Proposition 1, the optimal allocation depends on the value of Φ and the five thresholds defined above. The proofs of all the results in the paper can be found in Appendix C.

Proposition 1. *Given any $Q \geq 0$ and $\xi \in [0,1]$, the optimal allocation of shipment $\mathbf{x}^*$ and $\mathbf{y}^*$ is characterized by one of the following five cases:*

(i) If $C_1(Q,\xi) \leq \Phi$, then $\mathbf{x}^* = [Q\xi, 0]$, $\mathbf{y}^* = [Q(1-\xi), 0]$, i.e., allocate all units to market 1.

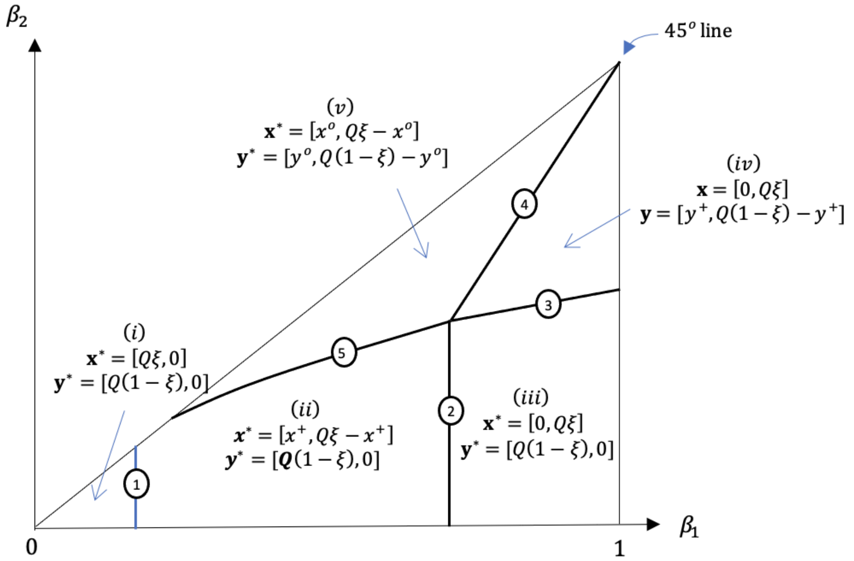
(ii) If $C_2(Q,\xi) < \Phi < C_1(Q,\xi)$ and $C_5(Q,\xi) \leq \Phi$, then $\mathbf{x}^* = [x^\dagger, Q\xi - x^\dagger]$ and $\underline{y}^* = [Q(1-\xi), 0]$ where $x^\dagger$ solves $\bar{F}_2(Q\xi - x^\dagger) - \beta_1\bar{F}_1(x^\dagger + Q(1-\xi)) - (1-\beta_1)\bar{F}_1(x^\dagger) = \Phi$. That is, allocate all risky units to market 1 and split good units between two markets.

(iii) If $C_3(Q,\xi) \leq \Phi \leq C_2(Q,\xi)$, then $\mathbf{x}^* = [0, Q\xi]$ and $\mathbf{y}^* = [Q(1-\xi), 0]$, i.e., allocate all risky units to market 1 and all good units to market 2.

(iv) If $C_3(Q,\xi) > \Phi$ and $C_4(Q,\xi) \geq 0$, then $\mathbf{x}^* = [0, Q\xi], \mathbf{y}^* = [y^\dagger, Q(1-\xi) - y^\dagger]$ where $y^\dagger$ solves $\beta_2\bar{F}_2(Q - y^\dagger) - \beta_1\bar{F}_1(y^\dagger) = \Phi$. That is, allocate all good units to market 2 but split risky units between two markets.

(v) Otherwise (if $C_4(Q,\xi) < 0$ and $C_5(Q,\xi) > \Phi$), then $\mathbf{x}^* = [x^o, Q\xi - x^o]$ and $\mathbf{y}^* = [y^o, Q(1-\xi) - y^o]$. where x^o is determined by $(1-\beta_1)\bar{F}_1(x^o) = (1-\beta_2)\bar{F}_2(Q\xi - x^o)$ and given x^o, y^o is determined by $\beta_2\bar{F}_2(Q - x^o - y^o) - \beta_1\bar{F}_1(x^o + y^o) = \Phi$. That is, split both good and risky units between two markets.

[7] There exists an $x_o \in (0, Q\xi)$ only if $C_4(Q,\xi) < 0$, so $C_5(Q,\xi)$ is defined with two different cases.



$$1{:}\ C_1(Q,\xi) = 1 - \beta_1 \bar{F}_1(Q) - (1-\beta_1)\bar{F}_1(Q\xi) = \Phi$$
$$2{:}\ C_2(Q,\xi) = \beta_1 F_1\big(Q(1-\xi)\big) - F_2(Q\xi) = \Phi$$
$$3{:}\ C_3(Q,\xi) = \beta_2 \overline{F_2}(Q\xi) - \beta_1 \bar{F}_1\big(Q(1-\xi)\big) = \Phi$$
$$4{:}\ C_4(Q,\xi) = (1-\beta_2)\overline{F_2}(Q\xi) - (1-\beta_1) = 0$$
$$5{:}\ C_5(Q,\xi) = \beta_2 \bar{F}_2(Q\xi - x^o) - \beta_1 \bar{F}_1\big(x^o + Q(1-\xi)\big) = \Phi \text{ for } C_4(Q,\xi) < 0$$

Figure 3: Illustration of the second-stage optimal allocation for any given Q and ξ; for illustrative purposes, the graph shows all possible cases, some of which may vanish as Q and ξ vary.

Depending on the success probabilities of sending risky units to each market (i.e., β_1 and β_2), it can be optimal to split the units of a certain type between two markets or send all of them to a single market. Let us graphically illustrate the five cases of Proposition 1 on a β_1-β_2 coordinate plane; see Figure 3. Recall that $\beta_1 > \beta_2$ and so only the triangular area below the 45^o line is relevant. For cases (i)-(iii), β_2 is low and so all risky units are allocated to market 1, i.e., $\mathbf{y}^* = [Q(1-\xi), 0]$, whereas the allocation of good units depends on β_1, i.e., how likely those risky units are to reach market 1 successfully. In case (i) where β_1 is also low (so risky units are not likely to reach market 1), all good units are sent to market 1 too (i.e., $\mathbf{x}^* = [Q\xi, 0]$) whereas market 2 is shut down; in case (ii) where β_1 is moderate, good units are split between markets 1 and 2; in case (iii) where β_1 is high enough (so risky units are highly like to reach market 1 unspoiled), all good units are used to serve market 2 (i.e., $\mathbf{x}^* = [0, Q\xi]$). There are other two cases when β_2 is relatively high. In case (iv) where both β_1 and β_2 are high (so risky units are highly likely to reach both markets), risky units are split between markets whereas all the good units are allocated to market 2. In case (v) where β_1 is not very high but close to β_2, it is optimal to split both good and risky units between markets.

It is worth highlighting that case (v) may arise as an optimal solution in which *both* shipment types are split. One might anticipate that the optimal allocation follows a greedy policy: Assign risky units first to the closer market until the amount of risky units is depleted or the closer market receives a sufficient allocation, because sending risky units to the more distant market involves a greater spoilage risk and a higher transportation cost. Interestingly, in case (v) of Proposition 1, such a greedy policy is not optimal; instead, the firm should split *both* good and risky units between markets. The optimality of splitting both shipment types can be explained by the notion of risk diversification. There is value to splitting risky units between different transportation lanes so as to spread the chance of all risky units ending up spoiled. When β_1 and β_2 are moderate and close to each other, the benefit of shipment diversification can outweigh the high transportation cost and risk associated with market 2. As stated in Corollary 1, if β_1 is sufficiently large relative to β_2, case (v) can be eliminated and hence the greedy allocation rule is optimal.

Corollary 1. *If $\beta_2(1-\beta_1)/(1-\beta_2) \leq \Phi$, case (v) of Proposition 1 will not occur, and it is thus optimal to always allocate risky items first to the closer market until all risky units are depleted or the closer market receives sufficient units.*

4.1.2 The First-Stage Purchase Problem

Because concavity is preserved under maximization and expectation, it is easy to show that the first-stage objective function $\Pi(Q)$ is concave. Define $r(Q) = E_\xi\left[\frac{\partial R(Q,\xi)}{\partial Q}\right]$. The optimal order quantity Q^* is attained at the point such that $r(Q) = c_o$, i.e., the marginal second-stage profit is equal to the unit purchase cost. Let λ_x^* and λ_y^* denote the Lagrangian multiplier associated with constraints (2) and (3), respectively, in the optimal solution to the allocation problem. By the Envelope Theorem for constrained optimization, it follows that

$$\frac{\partial R(Q,\xi)}{\partial Q} = \xi\lambda_x^* + (1-\xi)\lambda_y^*. \tag{7}$$

The explicit characterization of the optimal allocation in Proposition 1 enables us to calculate λ_x^*, λ_y^* and hence $\frac{\partial R(Q,\xi)}{\partial Q}$ in closed form for any given Q and ξ, thereby efficiently evaluating $r(Q)$, as summarized in Proposition 2.

Proposition 2. *The first-order derivative of $R(Q,\xi)$ with respect to Q is given below:*

(i) *If $C_1(Q,\xi) \leq \Phi$, $\frac{\partial R(Q,\xi)}{\partial Q} = (p-c_1) - (p+c_d)[1-\beta_1\bar{F}_1(Q) - \xi(1-\beta_1)\bar{F}_1(Q\xi)]$;*

(ii) *if $C_2(Q,\xi) < \Phi < C_1(Q,\xi)$ and $C_5(Q,\xi) \leq \Phi$, then $\frac{\partial R(Q,\xi)}{\partial Q} = (p-c_1) - (p+c_d)[1 - \beta_1\bar{F}_1(x^\dagger + Q(1-\xi)) - \xi(1-\beta_1)\bar{F}_1(x^\dagger)]$;*

(iii) *if $C_3(Q,\xi) \leq \Phi \leq C_2(Q,\xi)$, then $\frac{\partial R(Q,\xi)}{\partial Q} = [p - \xi c_2 - (1-\xi)c_1] - (p+c_d)[1 - \xi\bar{F}_2(Q\xi) - (1-\xi)\beta_1\bar{F}_1(Q(1-\xi))]$;*

(iv) if $C_3(Q,\xi) > \Phi$ *and* $C_4(Q,\xi) \geq 0$*, then* $\frac{\partial R(Q,\xi)}{\partial Q} = p - c_2 - (p+c_d)[1 - \beta_2 \bar{F}_2(Q - y^\dagger) - \xi(1-\beta_2)\bar{F}_2(Q\xi)]$;

(v) Otherwise (if $C_4(Q,\xi) < 0$ *and* $C_5(Q,\xi) > \Phi$*),* $\frac{\partial R(Q,\xi)}{\partial Q} = p - c_1 - (p+c_d)[1 - \beta_1 \bar{F}_1(x^o + y^o) - \xi(1-\beta_1)\bar{F}_1(x^o)]$

where the values of $x^\dagger$*,* $y^\dagger$ *and* x^o, y^o *are as defined in Proposition 1. Furthermore,* $r(Q) = E_\xi\left[\frac{\partial R(Q,\xi)}{\partial Q}\right]$ *is a decreasing function of* Q*. The optimal* Q^* *can be found by solving the equation* $r(Q) = c_o$*.*

Because $r(Q)$ is decreasing, to solve a general problem, we can perform a bisection search to find the optimal Q^*. As shown below, for some special cases, the optimal order quantity Q^* and the resulting allocation quantities can be derived in closed form.

4.1.3 Impact on Allocation Quantities

To gain deeper insights into the impact of monitoring on shipment allocation, we consider a special case where ξ is deterministic and demand D_i follows a uniform distribution $U(0, 2\mu)$ with mean μ for both $i = 1, 2$. We further assume $\beta_1 = 1$ and $\beta_2 < 1$. As such, two markets have identical demand distributions, and market 1 is free from spoilage risk because risky units can reach market 1 with certainty. Under these conditions, we will derive the closed-form optimal solutions for the scenarios with and without monitoring, denoted by superscripts I and N, respectively.

Define $\phi_i = \frac{p - c_i - c_o}{p + c_d}$ for each market $i \in \mathcal{M}$. Let q_i^N represent the quantity sent to market i absent monitoring, and Q^I denote the optimal order quantity for the first-stage purchase problem with monitoring. As characterized in Proposition 3, these optimal quantities depend crucially on the values of ϕ_1 and ϕ_2. As in the standard newsvendor problem, ϕ_i is the critical fractile associated with market i, which increases with the profit of selling one unit in market i. For simplicity, we will hereafter refer to $p - c_i - c_o$ as the "marginal profit," with the understanding that it does not incorporate demand or supply uncertainty.

Proposition 3. *Assume that* $D_i \sim U(0, 2\mu)$ *for all* $i \in \mathcal{M}$*,* ξ *is deterministic,* $\beta_1 = 1$ *and* $\beta_2 < 1$*. Without monitoring, the optimal quantities shipped to each market are given by*

$$q_1^N = 2\mu\phi_1 \text{ and } q_2^N = \begin{cases} 0 & \text{if } \phi_2 \leq (1-\xi)(1-\beta_2), \\ \frac{2\mu(\phi_2 - (1-\xi)(1-\beta_2))}{\beta_2 + \xi^2(1-\beta_2)} & \text{if } (1-\xi)(1-\beta_2) < \phi_2 \leq 1 - \xi(1-\xi)(1-\beta_2), \\ \frac{2\mu(\phi_2 + \xi(1-\beta_2) - 1)}{\xi^2(1-\beta_2)} & \text{otherwise.} \end{cases} \tag{8}$$

With monitoring, the optimal solution is characterized as follows.

- *For $\xi \geq \frac{1}{2}$, the optimal solution is given by one of the two cases:*

  (i) *if $\phi_2 \leq 0$, $Q^* = Q^i$ with all units to market 1;*

  (ii) *if $\phi_2 > 0$, $Q^* = Q^{ii}$ with all risky units to market 1 and good units split between markets.*

- *For $\xi < \frac{1}{2}$, the optimal solution is given by one of the six cases below:*

  (i) *if $\phi_2 \leq 0$, $Q^I = Q^i$ and send all units to market 1;*

  (ii) *if $0 < \phi_2 < A_1(\phi_1)$, $Q^I = Q^{ii}$ with all risky units to market 1 and good units split between markets;*

  (iii-a) *if $A_1(\phi_1) \leq \phi_2 \leq \min(A_2(\phi_1), A_3(\phi_1))$, $Q^I = Q^{iiia}$ with all risky units to market 1 and all good units to market 2;*

  (iii-b) *if $A_2(\phi_1) < \phi_2 \leq A_5(\phi_1)$, $Q^I = Q^{iiib}$ with all risky units to market 1 and all good units to market 2;*

  (iv-a) *if $\phi_2 > \max(A_3(\phi_1), A_4(\phi_1))$, $Q^I = Q^{iva}$ with all good units to market 2 and risky units split between markets;*

  (iv-b) *if $A_5(\phi_1) < \phi_2 \leq A_4(\phi_1)$, $Q^I = Q^{ivb}$, with all good units to market 2 and risky units split between markets.*

The closed-form expressions of the thresholds $A_i(\phi_1)$, the optimal order and allocation quantities for the above cases are summarized in Table 1.

Without monitoring, as discussed in §3.2, the firm's problem amounts to choosing an order quantity independently for each market, subject to a random yield factor z_i, where $z_i = 1$ with probability β_i and $z_i = \xi$ with probability $1 - \beta_i$. Because market 1 is free from spoilage risk ($\beta_1 = 1$), it receives the standard newsvendor order quantity $q_1^N = 2\mu\phi_1$. However, when deciding on q_2^N, the firm has to entertain the spoilage risk. With probability $1 - \beta_2$, a $(1 - \xi)$ proportion of the shipment will reach market 2 in unsalable condition. Thus, the expression of q_2^N depends on ϕ_2 as well as β_2 and ξ. Note that q_2^N can be either smaller or greater than the standard newsvendor quantity, $2\mu\phi_2$, as a way to avoid spoilage costs or compensate for yield losses, respectively.

Now consider the case with monitoring. Because the firm can allocate the product between markets based on the condition of each unit, the two markets can no longer be treated separately. Depending on the ϕ_i's, the optimal solution may fall into various cases, as detailed in Proposition 3 and illustrated in Figure 4.

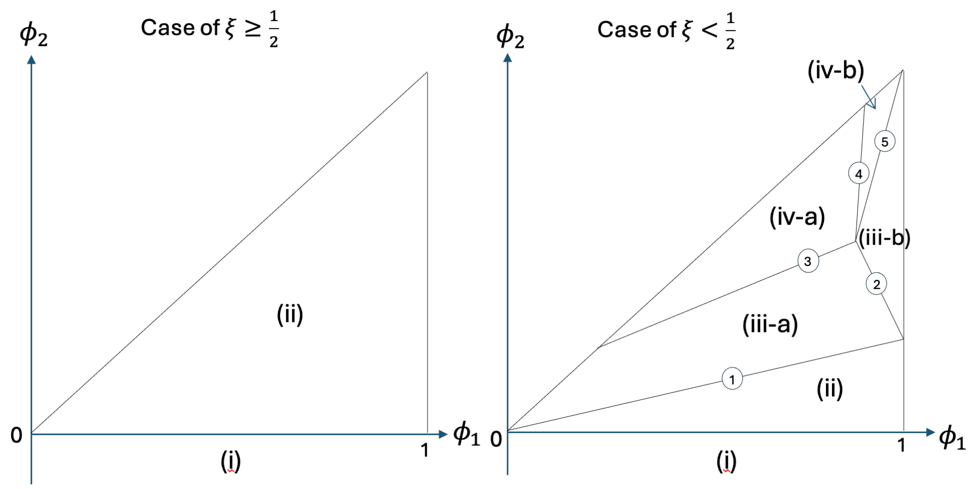
Table 1: Closed-form expressions of the optimal solution under different cases in Proposition 3 where $y^{iva}(Q)=\frac{\beta_2 Q+2\mu(1+\phi_1-\phi_2-\beta_2)}{1+\beta_2}$ and $y^{ivb}(Q)=\frac{\beta_2 Q+2\mu(\phi_1-\phi_2-\beta_2)}{\beta_2}$.

case	Q^I	$\begin{bmatrix} x_1^* \\ x_2^* \end{bmatrix}$	$\begin{bmatrix} y_1^* \\ y_2^* \end{bmatrix}$
i	$Q^i=2\mu\phi_1$	$\begin{bmatrix} Q^i\xi \\ 0 \end{bmatrix}$	$\begin{bmatrix} Q^i(1-\xi) \\ 0 \end{bmatrix}$
ii	$Q^{ii}=2\mu(\phi_1+\phi_2)$	$\begin{bmatrix} 2\mu(\phi_1+\phi_2)\xi-2\mu\phi_2 \\ 2\mu\phi_2 \end{bmatrix}$	$\begin{bmatrix} 2\mu(\phi_1+\phi_2)(1-\xi) \\ 0 \end{bmatrix}$
iii-a	$Q^{iiia}=\frac{2\mu((1-\xi)\phi_1+\xi\phi_2)}{1-2\xi+2\xi^2}$	$\begin{bmatrix} 0 \\ Q^{iiia}\xi \end{bmatrix}$	$\begin{bmatrix} Q^{iiia}(1-\xi) \\ 0 \end{bmatrix}$
iii-b	$Q^{iiib}=\frac{2\mu((1-\xi)\phi_1+\xi\phi_2+\xi-1)}{\xi^2}$	$\begin{bmatrix} 0 \\ Q^{iiib}\xi \end{bmatrix}$	$\begin{bmatrix} Q^{iiib}(1-\xi) \\ 0 \end{bmatrix}$
iv-a	$Q^{iva}=\frac{2\mu(\beta_2\phi_1+\phi_2+(1-\beta_2)(\xi(1+\beta_2)-1)}{\beta_2+(1-\beta_2)^2\xi^2}$	$\begin{bmatrix} 0 \\ Q^{iva}\xi \end{bmatrix}$	$\begin{bmatrix} y^{iva}(Q^{iva}) \\ Q^{iva}(1-\xi)-y^{iva}(Q^{iva}) \end{bmatrix}$
iv-b	$Q^{ivb}=\frac{2\mu(\phi_1+(1-\beta_2)\xi-1)}{(1-\beta_2)\xi^2}$	$\begin{bmatrix} 0 \\ Q^{ivb}\xi \end{bmatrix}$	$\begin{bmatrix} y^{ivb}(Q^{ivb}) \\ Q^{ivb}(1-\xi)-y^{ivb}(Q^{ivb}) \end{bmatrix}$

Thresholds: $A_1(\phi_1)=\frac{\xi}{1-\xi}\phi_1$, $A_2(\phi_1)=-\frac{1-\xi}{\xi}\phi_1+\frac{1-2\xi+2\xi^2}{\xi-\xi^2}$, $A_3(\phi_1)=\frac{(\beta_2\xi+(1-\beta_2)\xi^2)\phi_1+(1-\beta_2)(1-2\xi+2\xi^2)}{1-\xi+(1-\beta_2)\xi^2}$, $A_4(\phi_1)=\frac{(\beta_2+(1-\beta_2)\xi^2)\phi_1-\beta_2(1-(1-\beta_2)\xi+2(1-\beta_2)\xi^2)}{(1-\beta_2)\xi^2}$, and $A_5(\phi_1)=\frac{(\beta_2+(1-\beta_2)\xi)\phi_1-\beta_2}{(1-\beta_2)\xi}$.

If $\xi\ge\frac{1}{2}$, i.e., good units account for at least half of the total quantity, then the optimal solution has a simple structure. If the marginal profit of market 2 is negative ($\phi_2\le 0$), then serve only market 1 with the standard newsvendor quantity $2\mu\phi_1$ (case (i)). Otherwise, if $\phi_2>0$, direct all risky units to market 1 and split good units between the two markets (case(ii)). Notably, with this allocation, monitoring technology fully resolves spoilage risk because risky units can always reach market 1 ($\beta_1=1$). Consequently, in case (ii), the optimal order size is $Q^{ii}=2\mu(\phi_1+\phi_2)$ and each market i will receive its standard newsvendor quantity $2\mu\phi_i$, as if there were no spoilage risk; see $\mathbf{x}^*$ and $\mathbf{y}^*$ in case (ii) of Table 1.

If $\xi<\frac{1}{2}$, the optimal solution will have a much more complex structure. To explain the intuition, let us discuss how the optimal allocation quantities change as ϕ_2 increases while ϕ_1 remains fixed. If ϕ_2 is small enough relative to ϕ_1, the optimal solution is either case (i) or case (ii), as discussed earlier. However, as ϕ_2 increases, market 2 becomes more profitable, and due to the low proportion of good units, the standard newsvendor quantity $Q^{ii}=2\mu(\phi_1+\phi_2)$ for case (ii) may no longer provide sufficient good units for market 2. To obtain more good units, the firm will increase the order quantity beyond Q^{ii}, leading to case (iii-a), (iii-b), (iv-a) or (iv-b). Figure 5 presents an illustrative example, where the solid black curves denote the quantities of good units and the dashed gray curves correspond to the risky units. As ϕ_2 increases, more units are supplied to market 2. Initially, in case (ii), the firm can achieve this by allocating more risky and fewer good units to market 1 while keeping the total fixed at the standard newsvendor quantity, i.e., $x_1^*+y_1^*=2\mu\phi_1$. When $\phi_2>0.39$, no more good units can be shifted from market



Optimal Allocation: (i) all to market 1. (ii) all risky units to market 1 & split good units.
(iii-a, iii-b) all risky to market 1 & all good to market 2.
(iv-a, iv-b) all good to market 2 & split risky units.

Note: Line segments in the right panel: (1) $\phi_2 = \frac{\xi}{1-\xi}\phi_1$; (2) $\phi_2 = -\frac{1-\xi}{\xi}\phi_1 + \frac{1-2\xi+2\xi^2}{\xi-\xi^2}$; (3) $\phi_2 = \frac{(\beta_2\xi+(1-\beta_2)\xi^2)\phi_1+(1-\beta_2)(1-2\xi+2\xi^2)}{1-\xi+\xi^2-\beta_2\xi^2}$; (4) $\phi_2 = \frac{(\beta_2+(1-\beta_2)\xi^2)\phi_1-\beta_2(1-\xi+2\xi^2+\beta_2\xi-2\beta_2\xi^2)}{(1-\beta)\xi^2}$; (5) $\phi_2 = \frac{(\beta_2+(1-\beta_2)\xi)\phi_1-\beta_2}{(1-\beta_2)\xi}$.

Figure 4: Structure of the optimal solution with monitoring as characterized in Proposition 3

1 to market 2 (i.e., $x_1^* = 0$). Thus, the firm increases the total purchase quantity and follows case (iii-a), sending all risky units to market 1 and all good units to market 2. Interestingly, to ensure sufficient good units for market 2, the firm chooses to "over-deliver" in market 1 with risky units, i.e., $y_1^* > 2\mu\phi_1$, even though the risky units can always reach market 1. As ϕ_2 continues to increase, approximately above 0.56, it becomes too costly to further over-deliver in market 1, so the firm follows case (iv-a), reducing the risky units for market 1 and using some risky units, along with all good units, to serve market 2. The intuition for cases (iii-b) and (iv-b) are similar to that for cases (iii-a) and (iv-a), respectively, despite the different expressions of the optimal quantities.[8]

The above discussion indicates that shipment monitoring creates a cross-market dependence, leading to several novel properties of the optimal solution that are absent in typical newsvendor problems with supply uncertainty (such as our benchmark without monitoring). As formalized in Proposition B.2 of Appendix B.3, in our setting, the total quantity allocated to each market can either increase or decrease with the critical fractile of the other market (as illustrated by cases (iii-a) and (iv-a) in the left panel of Figure 5), whereas absent monitoring, the quantity shipped to each market is always independent of the other market's characteristics. Our results imply that, for example, as the transportation cost to a market increases, the optimal quantity supplied to the other market can become either larger or smaller.

[8]The different expressions arise because, in cases (iii-a) and (iv-a), all the shipping quantities are within the support of demand, whereas in cases (iii-b) and (iv-b) the optimal quantities sent to market 1 exceed the support of D_1, which is possible when the ϕ_i's are sufficiently large.

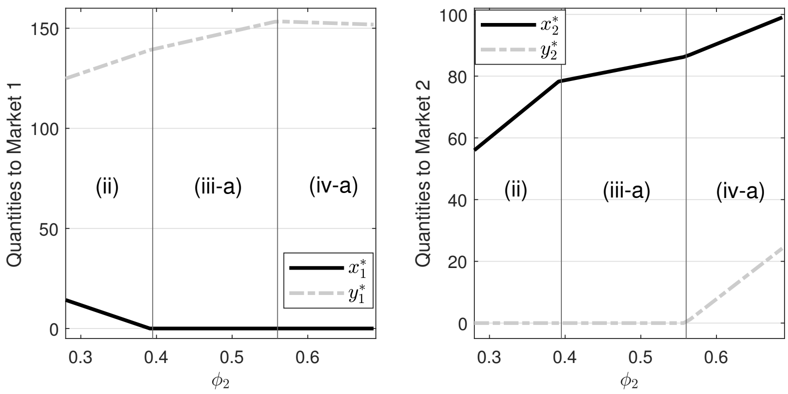


Figure 5: Optimal allocation quantities as a function of ϕ_2. [parameter values: $D_1, D_2 \sim U[0, 100]$, $\xi = 0.36$, $\beta_1 = 1, \beta_2 = 0.65$; p, c_1, c_2, c_o and c_d are chosen such that $\phi_1 = 0.70$ and ϕ_2 varies as shown in the figure.]

4.2 Impact on Food Waste

The total waste consists of units that are spoiled and left over in each market. Because the adoption of shipment monitoring changes the firm's optimal purchase quantity from $Q^N = \sum_{i \in \mathcal{M}} q_i^N$ to Q^I, its impact on food waste is not straightforward. In the benchmark scenario without monitoring, the (expected) total waste is $W^N = \sum_{i \in \mathcal{M}} W_i^N$ where $W_i^N = E[(z_i q_i^N - D_i)^+ + (1 - z_i) q_i^N]$ represents the waste amount in market i. With monitoring, the total waste is $W^I = \sum_{i \in \mathcal{M}} W_i^I$ where $W_i^I = E[(x_i^* + \epsilon_i y_i^* - D_i)^+ + (1 - \epsilon_i) y_i^*]$. We further define $S^I = \sum_{i \in \mathcal{M}} S_i^I$ and $S^N = \sum_{i \in \mathcal{M}} S_i^N$ as the (expected) total sales with and without monitoring, respectively, where $S_i^I = E[\min(D_i, x_i^* + \epsilon_i y_i^*)]$ and $S_i^N = E[\min(D_i, z_i q_i^N)]$ represent the sales in market i. Additionally, we define the waste rate as the waste-to-order ratio, i.e., W^I/Q^I and W^N/Q^N for the cases with and without monitoring, respectively. Because the order quantity equals waste plus sales, a lower waste rate means that a larger share of ordered units is ultimately sold rather than wasted, reflecting greater distributional efficiency. In what follows, we first analyze two special cases to characterize the impact of monitoring on these metrics, and then use a calibrated numerical study to explore more general settings.

4.2.1 Analysis of Two Special Cases

The first special case is in line with that analyzed in §4.1.3 with $\beta_1 = 1$ and $\xi \geq \frac{1}{2}$. As discussed earlier, in this case, all units can reach market 1 in salable condition, and monitoring allows the firm to eliminate spoilage by rerouting risky units to market 1.

Proposition 4. *Suppose that $0 \leq \beta_2 < \beta_1 = 1$, $D_i \sim U[0, 2\mu]$ for both $i \in \mathcal{M}$, and $\xi \geq \frac{1}{2}$ is deterministic. (i) The order quantity with monitoring is larger than that without monitoring* *(i.e., $Q^I > Q^N$) if and only if (iff) $p < \tilde{p} = \frac{c_o + c_2 + c_d\tilde{\phi}}{1-\tilde{\phi}}$, where $\tilde{\phi} = \frac{1}{1+\xi}$. (ii) Monitoring increases the total waste, i.e., $W^I > W^N$, iff $p < \hat{p} = \frac{c_o + c_2 + c_d\hat{\phi}}{1-\hat{\phi}}$, where $\hat{\phi} = \sqrt{\frac{(1-\xi)(1-\beta_2)}{1+\xi}}$. (iii) For each market, $S_1^I = S_1^N$, $W_1^I = W_1^N$, $S_2^I \geq S_2^N$ whereas $W_2^I > W_2^N$ iff $p < \hat{p}$. (iv) Monitoring reduces the waste rate, i.e., $\frac{W^I}{Q^I} \leq \frac{W^N}{Q^N}$.*

Part (i) of Proposition 4 establishes that monitoring will result in a larger order quantity if the selling price p is below a threshold $\tilde{p}$. Consider the case without monitoring. Spoilage risk can have two opposite effects on the optimal order quantity. For a product with a low profit margin, the spoilage risk discourages the firm from ordering a large quantity out of fear of potential spoilage. However, when the profit margin is high enough, the firm will find it profitable to inflate the order size to offset the potential yield loss. With monitoring technology to mitigate spoilage risk, the firm will order more for low-margin products since spoilage is less of a concern, but order less for high-margin products since there is less value to inflating the order size. Consequently, monitoring has twofold effects: It reduces spoilage but may increase the total quantity purchased, leading to more leftover units. As shown in part (ii), when the selling price is lower than a threshold $\hat{p}$ (which is smaller than $\tilde{p}$), the effect of increased order size can dominate, leading to an increase in total waste, even though monitoring fully avoids spoilage in this special case. At the same time, part (iii) indicates that monitoring increases sales in market 2, although it may generate more waste in that market. Sales and waste in market 1 are unaffected because, in this special case, market 1 is free of spoilage risk and always receives the standard newsvendor quantity $2\mu\phi_1$. Moreover, part (iv) indicates that although monitoring may increase the total waste, it always lowers waste per unit ordered, thereby enhancing distribution efficiency.

Next, we examine another special case in which market 1 has $\beta_1 < 1$ and deterministic demand $D_1 = \mu$, whereas market 2 has the same uniform demand $D_2 \sim U[0, 2\mu]$ as before. Compared with the first special case, this special case incorporates two additional considerations: First, allocating risky units to market 1 reduces spoilage but does not eliminate it; second, the more distant market is associated with greater demand uncertainty, for example, due to longer lead times. To focus on the most representative case, we assume $(1-\xi)(1-\beta_i) \leq \phi_i \leq (1-\xi)(1-\beta_i) + \beta_i$ for each market i, which implies that without monitoring, the firm will neither shut down nor over-supply in each market.[9] We identify the conditions under which, similar to the previous case, monitoring enables the firm to allocate all risky units to market 1, thereby characterizing the impact of monitoring with closed-form conditions.

[9]More precisely, under these conditions, the firm will supply exactly $D_1 = \mu$ units to market 1 and a positive quantity that does not exceed the support of D_2.

Proposition 5. *Suppose that* $0 \leq \beta_2 < \beta_1 < 1$, $D_1 = \mu$, $D_2 \sim U(0, 2\mu)$, *and* $(1-\xi)(1-\beta_i) < \phi_i < (1-\xi)(1-\beta_i) + \beta_i$ *for all* $i \in \mathcal{M}$. *There exists an* $\underline{\xi}$ *such that for any deterministic* $\xi > \underline{\xi}$, *the optimal allocation quantities with monitoring are given by* $\mathbf{x}^* = [\mu - Q^I(1-\xi), Q^I - \mu]$ *and* $\mathbf{y}^* = [Q^I(1-\xi), 0]$, *where* $Q^I = 2\mu(\phi_2 - (1-\xi)(1-\beta_1)) + \mu$. *In this case, the following holds true: (i)* $Q^I > Q^N$ *iff* $p < \tilde{p} = \frac{c_o + c_2 + c_d \tilde{\phi}}{1-\tilde{\phi}}$ *where* $\tilde{\phi} = (1-\beta_1)(1-\xi) + \frac{\beta_1 - \beta_2}{(1-\beta_2)(1+\xi)}$. *(ii)* $W^I > W^N$ *iff* $p < \hat{p} = \frac{c_o + c_2 + c_d \hat{\phi}}{1-\hat{\phi}}$ *where* $\hat{\phi} = \frac{\sqrt{(1-\beta_2)(1-\xi^2)((1-\beta_2)^2 - (1-\beta_1)^2(\beta_2 + (1-\beta_2)\xi^2))}}{(1-\beta_2)(1+\xi)}$. *(iii) For each market,* $S_1^I < S_1^N$, $W_1^I > W_1^N$, *and* $S_2^I > S_2^N$, *whereas* $W_2^I > W_2^N$ *iff* p *is smaller than a threshold* $\check{p}$. *(iv) There exists a threshold* $p^\dagger$ *such that* $\frac{W^I}{Q^I} > \frac{W^N}{Q^N}$ *if* $p < p^\dagger$.

Analogous to the previous case, monitoring can increase both the order quantity and total waste when the selling price is low. However, unlike in the previous case, part (iii) of Proposition 5 shows that monitoring leads to more waste and fewer sales in market 1. This is because, when $\beta_1 < 1$, risky units may become spoiled in market 1, and monitoring shifts more risky units to market 1 while the total quantity received by market 1 remains $D_1 = \mu$. Finally, and importantly, part (iv) establishes that monitoring may increase the waste rate. Intuitively, this occurs when monitoring-induced order expansion creates substantially more leftover units in market 2 than the additional sales it generates.

Taken together, the two settings presented in Propositions 4 and 5 demonstrate that monitoring can increase total waste, which may originate from either the nearby or the remote market. Moreover, it may or may not reduce the waste rate and improve distribution efficiency.

4.2.2 Numerical Illustration

We now turn to a more general setting, with model parameters calibrated based on a real-world example of fresh strawberries harvested in California, which account for 85% - 90% strawberry production in the US (Holmes 2024). We consider two representative cities – Los Angeles (LA) and Boston – as the local and remote markets, indexed by markets 1 and 2, respectively. Based on travel distances and publicly available information on refrigerated trucking costs, the transportation costs are estimated as $c_1 = \$0.02$ and $c_2 = \$0.38$ per pound. According to publicly available information on landfill disposal fees, we set $c_d = \$0.06$ per pound. According to USDA Economic Research Service, in 2023, the average retail price for fresh strawberries was \$3.80 per pound, and farmers received 53% of the retail price (Stewart and Hyman 2024). We therefore set $p = \$3.80$ and $c_o = \$2.01$. According to various sources, the loss of fresh strawberries due to shelf-life deterioration ranges from 8% to 25%.[10] In our model, each unit

[10] Stewart and Hyman (2024) estimates that 8% of fresh strawberries shipped from farms are lost due to spoilage or damage. A USDA study estimates that 14.2% of strawberries shipped to supermarkets were lost in 2011-2012. Erwin (2024) documents that "due to their short shelf life, around 25% of strawberries are discarded at the

sent to market i becomes spoiled with probability $(1-\xi)(1-\beta_i)$. The above evidence suggests that, approximately, the average value of $(1-\xi)(1-\beta_i)$ lies between 8% and 25%. Due to the lack of publicly available data, we are unable to separately calibrate ξ and β_i with precision. Since the main purpose of this numerical study is to illustrate the insights derived earlier, rather than to obtain precise predictions, we consider representative scenarios with $\beta_1 = 0.9$, $\beta_2 = 0.1$ and ξ varies over $\{0.8, 0.7, 0.6\}$. Under these three scenarios, $\frac{1}{2}\sum_{i=1}^{2}(1-\xi)(1-\beta_i)$ equals 10%, 15% and 20%, respectively, capturing a plausible range of spoilage probabilities. We note that in developing countries lacking reliable cold chain infrastructure, this probability can be even higher, estimated at 25% – 40% (Hammam and Abouelwafa 2022). We assume that for each market i, D_i follows a Gamma distribution with a mean of μ_i and a coefficient of variation of cv_i. We set $\mu_1 = \mu_2 = 1000$ based on daily shipment volumes of a representative Walmart store, and set $cv_1 = 0.5$ and $cv_2 = 1$ to capture the greater demand uncertainty associated with the remote market as in Proposition 5. More details and references regarding our model calibration can be found in Appendix B.4 and an additional numerical study with different parameter settings can be found in Appendix B.5.

Figure 6 plots the order quantities, waste amounts and waste rates with and without monitoring technology as functions of the selling price p. In all three scenarios, monitoring results in a larger order quantity ($Q^I > Q^N$), and in most instances, it leads to a larger waste amount ($W^I > W^N$), especially when the profit margin is low or when the good-unit fraction is low. On the other hand, monitoring always reduces the waste rate when the good-unit fraction is $\xi = 0.8$, but not necessarily when ξ is lower. As mentioned above, empirical evidence suggests a retail price of $p = 3.80$, as marked by the vertical dotted line in each graph. At this representative selling price, the waste amount increases by 4.37, 28.96, and 82.42 pounds for $\xi = 0.8$, 0.7, and 0.6, respectively. In the first two scenarios, the waste rates are reduced from 19.1% to 17.6% and from 19.4% to 18.2%, respectively. Thus, in these two cases, monitoring presents a trade-off: Increases in waste amount are accompanied by lower waste rates and improved distribution efficiency. However, when $\xi = 0.6$, the waste rate increases from 17.7% to 18.7%; consequently, monitoring simultaneously increases the waste amount and undermines distribution efficiency.

4.3 Policy and Management Implications

Although the potential of monitoring technology to reduce food waste has recently been advocated, our results reveal that it may inadvertently result in more waste because they can entice firms to "over-order." Thus, monitoring technology is *not* a panacea for reducing fresh

distribution and retail level."

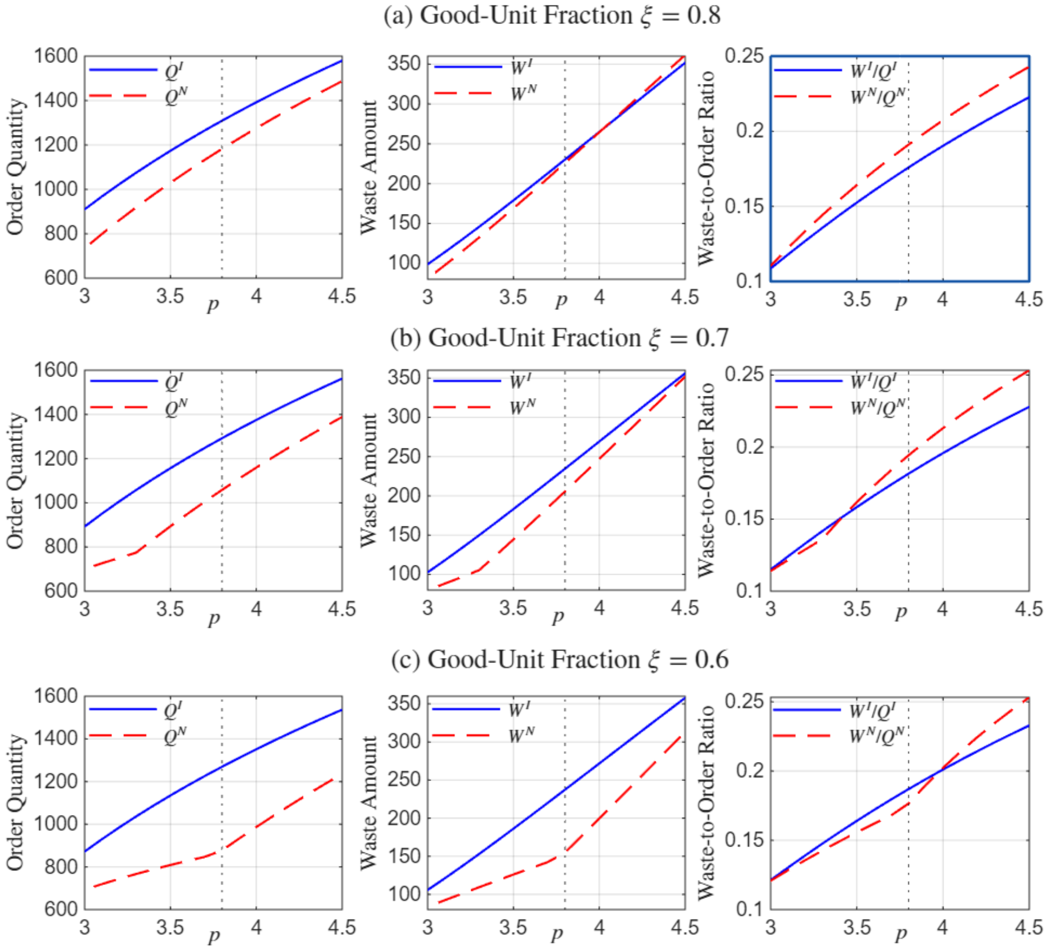


Figure 6: Impact of monitoring technology on order quantity, waste amount and waste rate as functions of selling price p, where the vertical dotted line marks the representative price of $p = 3.80$ suggested by empirical evidence.

produce waste. As more firms adopt such technology for shipment allocation, additional policy and managerial attention are needed.

4.3.1 Post-Harvest Cold Chain Infrastructure

Although monitoring enables condition-based allocation at the distribution stage, an issue remains at the source due to variability in post-harvest processes, such as pre-cooling.[11] As illustrated in §4.2.2, the fraction of good units before distribution, ξ, crucially affects the waste consequences of monitoring technology. This fraction can be improved through more reliable post-harvest cold chain facilities. In practice, various countries and regions have implemented initiatives to enhance cold chain facilities at farms. For example, the USDA's Farm Service Agency offers low-interest financing to help growers upgrade storage facilities.[12] Improving cold

[11] "It is essential to pre-cool the fruits as fast as possible after harvest by removing the field heat. The marketability can be extremely reduced with the cooling delay" (Alcéo 2016, p11).

[12] `https://www.fsa.usda.gov/resources/programs/farm-storage-facility-loan-fsfl-program` accessed Aug 10, 2025.

chain infrastructure is even more critical in developing countries; for instance, initiatives have been implemented in India and Nigeria to help local farmers improve storage facilities (World Economic Forum 2022).

One might anticipate that with the adoption of monitoring technology to aid distribution, less effort would be needed to improve cold chain infrastructure at farms. However, our results reveal that as firms increasingly adopt monitoring technology, greater effort to improve cold chain facilities at the origin may be required.

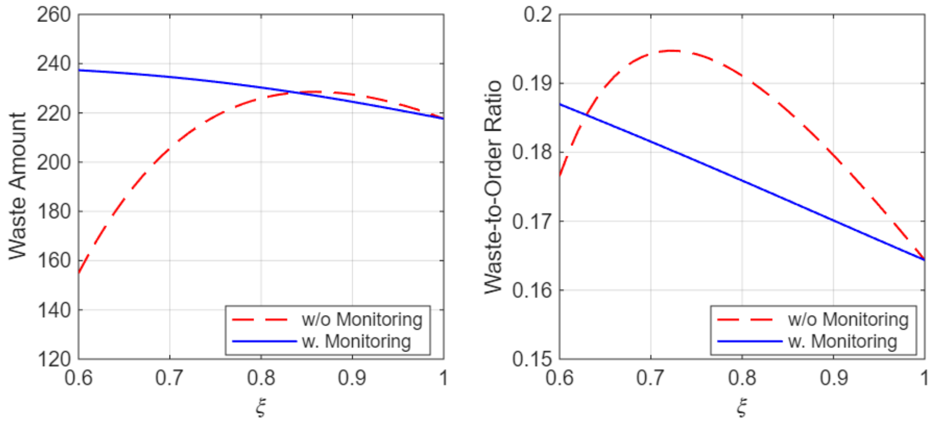


Figure 7: Waste amount and waste rate without and with monitoring as functions of ξ; parameter values are chosen as in §4.2.2 with $p = 3.80$.

Corollary 2. *In both settings of Propositions 4 and 5, shipment monitoring increases the total waste iff ξ is lower than a threshold $\hat{\xi}$. Moreover, in the setting of Proposition 5, monitoring increases the waste rate iff ξ is lower than a threshold $\check{\xi}$.*

In Corollary 2, we show that, under the previously analyzed special cases, shipment monitoring does not necessarily reduce the waste amount or waste rate unless the good-unit fraction ξ is sufficiently high. More generally, under the calibrated setting discussed in §4.2.2, Figure 7 illustrates how the waste amount and waste rate vary with ξ, both with and without monitoring technology. Shipment monitoring reduces the waste amount only when $\xi \geq 0.83$, and reduces the waste rate when $\xi \geq 0.63$. Thus, deploying monitoring technology to enhance shipment allocation alone may not guarantee a reduction in either the waste amount or the waste rate, unless it is accompanied by sufficiently reliable cold-chain facilities at the source. Moreover, within the parameter range considered, both the waste amount and the waste rate decrease in ξ when monitoring technology is adopted, whereas they can increase in ξ absent monitoring. This is because, without monitoring, an improvement in ξ may encourage the firm to order more due to the reduced spoilage risk, which can in turn lead to a higher waste amount and waste rate. By contrast, with monitoring technology and condition-based shipment allocation, the firm's ordering decision becomes less sensitive to spoilage risk associated with ξ. As a result, improvements in ξ translate more directly into reductions in these waste metrics.[13]

Taken together, these findings suggest that monitoring technology for distribution and improvements in post-harvest processes complement each other in reducing waste. Shipment monitoring can generate sufficient waste-reduction benefits only when post-harvest processes yield an adequately high fraction of good units. Conversely, monitoring mitigates the distortion in ordering incentives, allowing improvements in source quality to contribute more effectively to waste reduction. Therefore, despite the rapid development of monitoring technology, government agencies and NGOs should continue to expand, rather than reduce, their efforts to help producers upgrade post-harvest cold-chain infrastructure.

4.3.2 A Waste-Conscious Solution

The USDA and the Environmental Protection Agency (EPA) have launched the US Food Loss and Waste 2030 Champions initiative, inviting leading businesses to commit to cutting food waste by 50% by 2030. Major retailers such as Aldi and Walmart have joined this initiative and regularly publish their progress in waste reduction.[14] Consistent with this practice, wasted food amount is also a key metric regularly tracked and reported by the EPA.[15] As such, waste amount is a common metric for evaluating firms' publicly disclosed progress on waste reduction. Our results show that although monitoring often reduces the waste rate, it can increase the total waste amount in many cases by inducing a larger optimal order quantity, i.e., $Q^I > Q^N$. For firms making public commitments to reducing food waste, this raises a natural question: to what extent can they benefit from monitoring technology while still maintaining their waste-reduction goals?

The value of monitoring arises from two sources: condition-based allocation in the second stage and the adjustment of the order quantity in the first stage. Our analysis suggests that the latter contributes to the increase in waste amount when monitoring leads to a larger optimal order quantity. Inspired by this observation, we define a waste-conscious quantity as $Q^A = \eta Q^I + (1-\eta)\min(Q^N, Q^I)$ where $\eta \in [0,1]$. A practical way to capture some benefit of monitoring while curbing waste is to order Q^A in the first stage and then use monitoring for allocation

[13] Note that we compare the scenarios with and without monitoring, without explicitly modeling the firm's adoption decision for monitoring technology. For completeness, Figure 7 includes values of ξ up to one. However, when ξ is close to one, the firm may have little incentive to adopt monitoring. Given the plethora of real-world applications of monitoring technology in fresh produce shipping, the practically relevant range of ξ should be below the level at which monitoring would become unprofitable to adopt.

[14] `https://www.epa.gov/sustainable-management-food/united-states-food-loss-and-waste-2030-champions` accessed July 11, 2025

[15] See, e.g., `https://www.epa.gov/system/files/documents/2024-04/2019-wasted-food-report_508_opt_ec_4.23correction.pdf` accessed Aug 9, 2025.

in the second stage. If $\eta = 1$, then $Q^A = Q^I$, which corresponds to the profit-maximizing solution. With a lower η, the firm can moderate the increase in its order quantity relative to the profit-maximizing solution and potentially mitigate the increase in waste. If $\eta = 0$, then $Q^A = \min(Q^N, Q^I)$ whereby the firm uses monitoring for allocation without increasing the first-stage order quantity. Let Π^A represent the expected profit achieved by the waste-conscious solution.

Based on the calibrated example in §4.2.2, we solve for the waste-conscious solution with η varied from 0 to 1 and evaluate the resulting firm profit and waste amount. Figure 8 depicts the changes in profit and waste amount when the firm adopts monitoring technology under the waste-conscious solution, relative to the no-monitoring benchmark. Table 2 further reports the optimality gap of the waste-conscious solution, defined as $(\Pi^I - \Pi^A)/\Pi^I$, along with the corresponding changes in waste amount. The optimality gap captures the profit loss associated with restraining the order quantity increase relative to the profit-maximizing solution, and therefore reflects the value of order expansion.

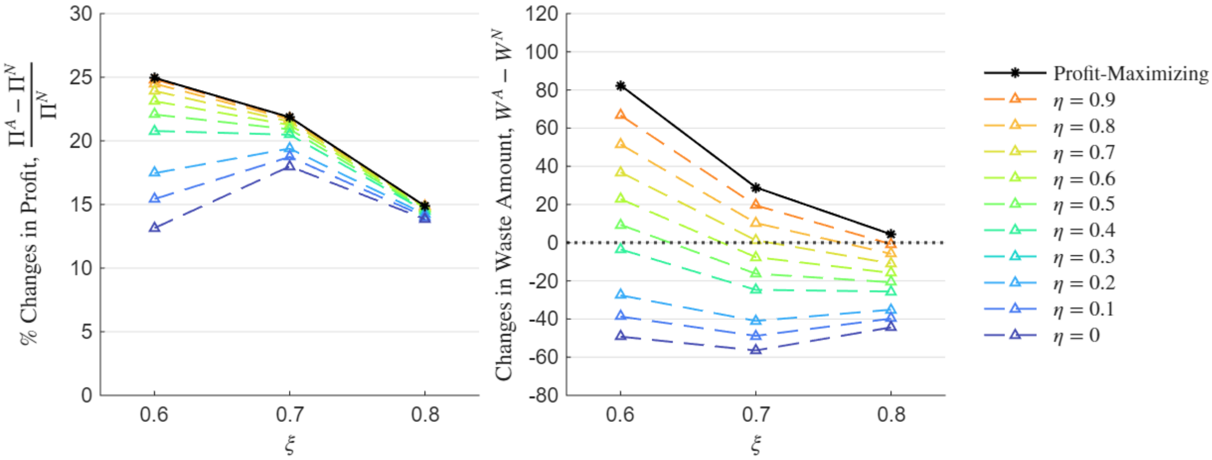


Figure 8: Monitoring-induced changes in profit, waste amount and waste rate if the firm orders $Q^A = \eta Q^I + (1-\eta)\min(Q^I, Q^N)$ and uses monitoring to optimally allocate shipments; parameter values are chosen as in §4.2.2 with $p = 3.80$.

As shown in Figure 8, if the firm pursues the profit-maximizing solution ($\eta = 1$), monitoring technology leads to the highest profit improvement; however, it also increases the waste amount in all three scenarios with $\xi = 0.6$, 0.7, and 0.8. By contrast, the waste-conscious solution makes it possible to capture a significant portion of the benefit from monitoring without increasing waste. As detailed in Table 2, if the firm uses monitoring only for allocation without expanding the first-stage order quantity (by setting $\eta = 0$), it incurs profit losses of only 9.45%, 3.17%, and 0.89% relative to the profit-maximizing solution, while reducing waste by 49.25, 56.46, and 44.39 pounds for the scenarios of $\xi = 0.6, 0.7$ and 0.8, respectively. By setting a higher η and

Table 2: Optimality gap of the waste-conscious solution ($\frac{\Pi^I - \Pi^A}{\Pi^I}$) and its impact on waste reduction; parameter values are chosen as in §4.2.2 with $p = 3.80$.

	$\xi = 0.6$		$\xi = 0.7$		$\xi = 0.8$	
η	Opt Gap (%)	$W^A - W^N$	Opt Gap (%)	$W^A - W^N$	Opt Gap (%)	$W^A - W^N$
0	9.45	-49.25	3.17	-56.46	0.89	-44.39
0.1	7.62	-38.71	2.56	-48.83	0.72	-39.78
0.2	6.00	-27.58	2.02	-40.99	0.57	-35.11
0.3	4.57	-15.85	1.54	-32.95	0.44	-30.38
0.4	3.34	-3.54	1.13	-24.71	0.32	-25.59
0.5	2.31	9.36	0.78	-16.26	0.22	-20.74
0.6	1.47	22.84	0.50	-7.61	0.14	-15.83
0.7	0.82	36.89	0.28	1.24	0.08	-10.87
0.8	0.36	51.51	0.12	10.28	0.04	-5.85
0.9	0.09	66.69	0.03	19.52	0.01	-0.77
1	0	82.42	0	28.96	0	4.37

thus allowing some order expansion, the firm can trade off profit against waste reduction. For example, if the firm opts for greater profit improvement, setting $\eta = 0.4$ enables it to reduce waste in all three scenarios, but with smaller profit losses: 3.34%, 1.13%, and 0.32%, respectively. Furthermore, it is worth noting that the optimality gaps shrink as ξ increases. In other words, the profit gain from increasing the order quantity becomes smaller when post-harvest processes have a better yield. This echoes our earlier discussion: Improvements in post-harvest processes reinforce the waste-reduction benefit of monitoring by making order expansion less valuable, thereby giving firms stronger incentives to follow the waste-conscious solution.

In Appendix B.5.2, we show that our findings are robust through a more extensive numerical study, in which we investigate a wider range of parameter values and also allow ξ to be uncertain at the first-stage ordering decision, as permitted by our general model framework.

5 Extensions

To show the robustness of our main results and obtain additional insights, we have explored several model extensions. In what follows, we provide a summary of our key findings from these extensions, whereas more details can be found in Appendix D.

5.1 More Than Two Markets

In general, the firm can sell to more than two markets. Let $\mathcal{M} = \{1, 2, ..., n\}$ denote the set of n markets. Without loss of generality, the markets are ordered according to their distances from the origin; that is, market i is closer to the origin than market $i+1$ for all $i = 1, 2, ..., n-1$. As before, a nearer market has a lower transportation cost and spoilage risk, i.e., $c_1 < c_2 < ... < c_n$

and $\beta_1 > \beta_2 > ... > \beta_n$. In Proposition D.1 of Appendix D.1, we prove that given any purchase quantity Q, the optimal allocation policy follows a market-partition rule. Specifically, the set of markets is partitioned into four groups according to their distances: $\mathcal{M}_1 = \{i : i \le z_1\}$, $\mathcal{M}_2 = \{i : z_1 + 1 \le i \le z_2\}$, $\mathcal{M}_3 = \{i : z_2 + 1 \le i \le z_3\}$, and $\mathcal{M}_4 = \{i : i \ge z_3 + 1\}$. In the optimal solution, the first group $\mathcal{M}_1$, consisting of the nearest z_1 markets, will receive only risky units. The second group $\mathcal{M}_2$, consisting of the markets farther away than z_1 but closer than $z_2 + 1$, will receive both risky and good units. The third group $\mathcal{M}_3$, including farther markets up to z_3, will receive only good units. The fourth group, any market more distant than z_3, will not be served. (Some of the groups may degenerate as we allow $z_i = z_{i+1}$.) Our numerical results confirm that our main results, derived from the two-market model, continue to hold for n-market problems.

5.2 Disposal Before Allocation

Our analysis has assumed that all units must be sent to the destination markets. In some situations, the firm might discard some risky units before allocating them to different markets from a central warehouse. We refer to those discarded units as in-transit disposal. Let c_t denote the cost incurred for each unit disposed of in transit. To incorporate in-transit disposal, we modify our base model by replacing the equality constraint (3) with inequality $\sum_{i\in\mathcal{M}} y_i \le Q(1-\xi)$, and subtracting the in-transit disposal cost $c_t(Q - \sum_{i\in\mathcal{M}}(x_i + y_i))$ from the objective function (1). In Proposition D.2, we establish that if c_t exceeds a threshold, the firm will not dispose of any risky unit and so the optimal solution remains the same as in our base model. The threshold can be computed based on a Lagrangian dual variable from the base model. Otherwise, the optimal allocation quantities fall into more cases than in the base model, because the firm can now ship none or only part of risky units to markets. Allowing in-transit disposal can affect some of our findings on waste. Without monitoring, the total waste is the same as in the base model, since the firm is unable to identify risky units. However, with monitoring, the total waste now includes not only the leftover and spoilage as discussed earlier but also the units discarded in transit. The possibility of in-transit disposal gives the firm an additional incentive to over-order when the profit margin is exceedingly high. The firm may deliberately purchase an excessive quantity, knowing that some units will be disposed of in transit. Hence, as discussed in Appendix D.2, monitoring technology can induce more waste either when the profit margin is low (as explained by our base-model analysis) or when the profit margin is very high (resulting in extensive in-transit disposal). Our findings suggest that if monitoring induces in-transit disposal, more efforts should be taken by supply chain firms or nonprofit organizations to handle the discarded units in transit.

5.3 ϵ_i's Follow General Distributions

Our results can be extended to the case where random yield factors for risky units, ϵ_1 and ϵ_2, follow general probability distributions on support $[0, 1]$ and ϵ_1 is first-order stochastically larger than ϵ_2. As detailed in Appendix D.3, we can show that the optimal allocation quantities follow similar cases as in the base model, although the conditions for each case to be optimal are derived in a more implicit fashion. We have numerically verified the robustness of our main findings when the ϵ_i's follow more general distributions. In the numerical study, we assume each ϵ_i to follow a probability distribution with cdf $G_i(x) = 1 - (1 - x)^{b_i}$ over support $[0, 1]$ and $0 < b_1 < b_2$. So, G_i is first-order stochastically larger as b_i becomes smaller, and it reduces to a uniform distribution on $[0, 1]$ when $b_i = 1$. Our numerical results indicate that shipment monitoring is more likely to increase total waste for low margin products; moreover, this adverse outcome is more likely when b_i is larger, or equivalently, ϵ_i is stochastically smaller. As in the base model, the waste-conscious solution proposed in §4.3.2 allows the firm to leverage the value of monitoring while maintaining their waste reduction goal.

5.4 Supplier-Managed Shipping

In the base model, the firm sources from the origin and then manages its distribution to different destinations, thus taking responsibility of the shipping costs and potential spoilage. This is often the case for large retailers with extensive distribution facilities such as Walmart (Lee et al. 2013). In some cases, a retailer may buy from a large wholesaler or distributor, referred to as the supplier hereafter, who manages the transportation from the origin to local markets. To incorporate this situation, we consider a supply chain where the ordering and distribution decisions are decentralized: a supplier managing the distribution of the product and a retailer choosing how much to order for each market (e.g., a grocery chain serving multiple regions). We have a Stackelberg game in which the retailer first chooses an order quantity o_i for each market i, and then the supplier determines how much to procure and allocate to fulfill these orders with or without monitoring technology akin to the base model. In line with the supply uncertainty literature, the supply chain payment depends generally on both the amount ordered and the amount effectively delivered (Tomlin and Wang 2005, Wang et al. 2010). We assume that the retailer pays τw per unit ordered and $(1 - \tau)w$ per unit delivered in salable condition (but up to its order quantity o_i in case the supplier over-delivers) where parameter $\tau \in [0, 1]$ reflects the extent to which the retailer shares the supply risk. Alternatively, the contract agreement can be interpreted as the retailer paying w per unit ordered in advance and receiving a refund of $(1-\tau)w$ per unit undelivered or delivered in unsalable condition. The detailed mathematical formulation and discussions of the game are provided in Appendix D.4.

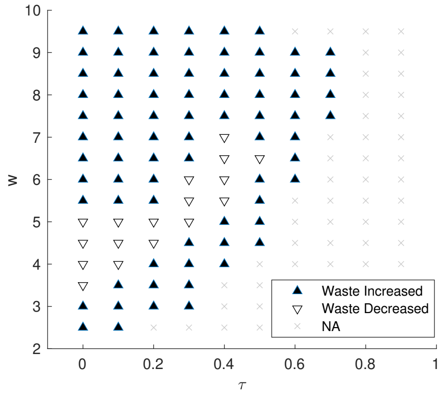


Figure 9: Monitoring-induced changes in total waste in a decentralized supply chain with contract parameters (w, τ) where an instance is marked by "NA" if any player chooses not to trade under the given (w, τ). (Parameter values: $D_1, D_2 \sim U[0, 10]$, $\xi = 0.6$, $\beta_1 = 1$, $\beta_2 = 0.2$, $p = 10$, $c_o = 2$, $(c_1, c_2) = (0.1, 0.2)$, $c_d = 0.05$)

We numerically explore the impact of supply chain decentralization and contract parameters (w, τ) on the waste implications of monitoring technology. First, we find that even when the product's profit margin is high enough such that monitoring reduces waste in a centralized supply chain, monitoring can increase waste when the supply chain is decentralized. This can be explained by the insights derived earlier: Monitoring is more likely to increase waste for low-margin products. Decentralization results in double marginalization, thus shrinking the profit margin for each player and making the increase in waste more probable. Moreover, we observe that monitoring can reduce waste in a decentralized supply chain only if contract parameters (w, τ) appropriately balance each supply chain member's profit margin and risk allocation. As illustrated in Figure 9, for fixed $\tau \leq 0.5$, there exists a medium range of w where monitoring reduces waste. This is because the equilibrium quantities are shaped by both firms' incentives. For lower w, the supplier has a lower profit margin, whereas for higher w, the retailer has a lower margin. As discussed earlier, low-margin firms tend to order or ship less out of fear of spoilage, and monitoring reduces spoilage risk and can thus lead to a larger order quantity and more waste. The parameter τ also affects the firms' order behaviors. The supplier effectively earns $(1-\tau)w$ per unit of successful delivery, so a higher τ leads the supplier to ship less due to spoilage. Similarly, with a higher τ, the retailer bears a larger share of the spoilage cost, and thus prefers to reduce the order quantity to avoid spoilage. Consequently, when τ is high, monitoring can motivate higher production and shipping quantities, resulting in more waste.

Thus, in the example of Figure 9, monitoring increases waste for higher τ values. On the other hand, given a small τ, anticipating the supplier to ship more to compensate for spoilage, the retailer has incentive to strategically order less, especially when w is high. This opposing effect appears to dominate for $w > 5$ in the example; in such cases, monitoring induces the retailer to order more and ultimately causes more waste.

6 Conclusion

Motivated by the emerging use of monitoring technology in fresh produce supply chains, we study its impacts on firms' purchase and distribution decisions, as well as its implications for waste reduction. We characterize the optimal policy for the condition-based shipment allocation enabled by such technology. Despite its presumed benefit in reducing waste, we show that monitoring technology may inadvertently increase total waste by inducing a larger purchase quantity, while often, but not always, enhancing distribution efficiency by lowering the waste rate. Thus, as firms increasingly adopt monitoring technology for shipment allocation, additional regulatory interventions, such as supporting producers in upgrading their post-harvest facilities, and greater managerial attention, such as moderating order quantity expansions, are needed.

As a first step toward exploring shipment monitoring and allocation, we make several model simplifications. To focus on the role of monitoring in allocation, we assume fixed selling prices. An important direction for future research is to incorporate pricing decisions. Additionally, we consider two product types that differ in spoilage risk. It would be worthwhile to explore the solution structure when there are more than two product types. In the model extensions, we analyze a decentralized supply chain in which a single retailer manages multiple markets. It would be interesting to consider a setting where multiple retailers source from a common supplier who manages the allocation.

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References

Akkaş A (2019) Shelf space selection to control product expiration. *Production and Operations Management* 28(9):2184–2201.

Akkaş A, Gaur V (2022) Om forum—reducing food waste: An operations management research agenda. *Manufacturing & Service Operations Management* 24(3):1261–1275.

Akkaş A, Honhon D (2022) Shipment policies for products with fixed shelf lives: Impact on profits and waste. *Manufacturing & Service Operations Management* 24(3):1611–1629.

Akkaş A, Honhon D (2023) Determining maximum shipping age requirements for shelf life and food waste management. *Production and Operations Management* 32(7):2173–2188.

Akkaş A, Sahoo N (2020) Reducing product expiration by aligning salesforce incentives: A data-driven approach. *Production and Operations Management* 29(8):1992–2009.

Alcéo RGA (2016) *Temperatures and shelf-life duration in the strawberry supply chain.* Master's thesis, Universidade de Lisboa (Portugal).

Ang E, Iancu DA, Swinney R (2017) Disruption risk and optimal sourcing in multitier supply networks. *Management Science* 63(8):2397–2419.

Ata B, Lee D, Sönmez E (2019) Dynamic volunteer staffing in multicrop gleaning operations. *Operations Research* 67(2):295–314.

Ata B, Lee D, Tongarlak MH (2012) Optimizing organic waste to energy operations. *Manufacturing & Service Operations Management* 14(2):231–244.

Belavina E (2021) Grocery store density and food waste. *Manufacturing & Service Operations Management* 23(1):1–18.

Belavina E, Girotra K, Kabra A (2017) Online grocery retail: Revenue models and environmental impact. *Management Science* 63(6):1781–1799.

Boyabatlı O (2015) Supply management in multiproduct firms with fixed proportions technology. *Management Science* 61(12):3013–3031.

Buzby J, Bentley J, Padera B, Campuzano J, Ammon C (2016) Updated supermarket shrink estimates for fresh foods and their implications for ers loss-adjusted food availability data United States Department of Agriculture.

Chen YJ, Tomlin B, Wang Y (2017) Dual coproduct technologies: Implications for process development and adoption. *Manufacturing & Service Operations Management* 19(4):692–712.

Cui Y, Hu M, Liu J (2022) Value and design of traceability-driven blockchains. *Manufacturing & Service Operations Management* Forthcoming.

Dada M, Petruzzi NC, Schwarz LB (2007) A newsvendor's procurement problem when suppliers are unreliable. *Manufacturing & Service Operations Management* 9(1):9–32.

Dai T, Cho SH, Zhang F (2016) Contracting for on-time delivery in the us influenza vaccine supply chain. *Manufacturing & Service Operations Management* 18(3):332–346.

Dong L, Geng X, Xiao G, Yang N (2022a) Procurement strategies with unreliable suppliers under correlated random yields. *Manufacturing & Service Operations Management* 24(1):179–195.

Dong L, Jiang P, Xu F (2022b) Impact of traceability technology adoption in food supply chain networks. *Management Science* .

Erwin J (2024) We are developing a game-changing technique to save strawberries. Available at `https://blogs.ifas.ufl.edu/fshndept/2024/03/04/we-are-developing-a-game-changing-technique-to-save-strawberries` accessed May 23, 2025.

FAO (2011) Global food losses and food waste – extent, causes and prevention. Rome.

Fresh Fruit Portal (2018) Zest Labs fights food waste by routing pallets according to real-time freshness. Available at `https://www.freshfruitportal.com/news/2018/07/19/technology-zest-labs-food-waste-profits-sensors` accessed Aug 1, 2025.

Haass R, Dittmer P, Veigt M, Lütjen M (2015) Reducing food losses and carbon emission by using autonomous control–a simulation study of the intelligent container. *International Journal of Production Economics* 164:400–408.

Hammam N, Abouelwafa S (2022) Technical, economic and marketing assessment to determine the losses of strawberry fruits during harvesting in the west nubaria region. *Middle East Journal of Agriculture Research* 11(03):733–746.

Holmes GJ (2024) The california strawberry industry: Current trends and future prospects. *International Journal of Fruit Science* 24(1):115–129.

Keskin NB, Li C, Song JSJ (2024) The blockchain newsvendor: Value of freshness transparency and smart contracts. *Management Science* Forthcoming.

Ketzenberg M, Bloemhof J, Gaukler G (2015) Managing perishables with time and temperature history. *Production and Operations Management* 24(1):54–70.

Lai Y, Emond J, Nunes M (2011) Environmental conditions encountered during distribution from the field to the store affect the quality of strawberry ('albion'). *Proceedings of Florida State Horticultural Society* 124:213–220.

Lee CY, Lu T (2015) Inventory competition with yield reliability improvement. *Naval Research Logistics (NRL)* 62(2):107–126.

Lee D (2012) Turning waste into by-product. *Manufacturing & Service Operations Management* 14(1):115–127.

Lee D, Tongarlak MH (2017) Converting retail food waste into by-product. *European Journal of Operational Research* 257(3):944–956.

Lee W, Slater G, C K (2013) How walmart distributes strawberries to 200 stores. Available at `https://projects.scpr.org/interactives/from-farm-to-you` accessed July 21, 2025.

Li B, Boyabatlı O, Avcı B (2022) Economic and environmental implications of biomass commercialization in agricultural processing. *Management Science* .

Li C, Tomlin B (2022) After-sales service contracting: Condition monitoring and data ownership. *Manufacturing & Service Operations Management* 24(3):1494–1510.

Li R, Ryan JK (2011) A bayesian inventory model using real-time condition monitoring information. *Production and Operations Management* 20(5):754–771.

Lin YT, Sun H, Wang S (2020) Designing sustainable products under coproduction technology. *Manufacturing & Service Operations Management* 22(6):1181–1198.

Long X, Gui L (2024) Waste not want not? the environmental implications of quick response and upcycling. *Manufacturing & Service Operations Management* 26(2):612–631.

Long X, Nasiry J (2022) Sustainability in the fast fashion industry. *Manufacturing & Service Operations Management* .

Lu T, Chen YJ, Tomlin B, Wang Y (2019) Selling co-products through a distributor: The impact on product line design. *Production and Operations Management* 28(4):1010–1032.

Lu T, Fransoo JC, Lee CY (2017) Carrier portfolio management for shipping seasonal products. *Operations Research* 65(5):1250–1266.

Lu T, Tomlin B (2023) Forewarned is forearmed? contingent sourcing, shipment information, and supplier competition. *Manufacturing & Service Operations Management* 25(5):1796–1813.

Mersereau T (n.d.) What is the carbon footprint of strawberries? a life-cycle analysis. Available at `https://impactful.ninja/what-is-the-carbon-footprint-of-strawberries` accessed Jul 16, 2024.

Park JH, Iancu D, Plambeck E (2022) On the management of premade foods Working paper. Available at SSRN 4148756.

ReFED (n.d.) In the u.s., 35% of all food goes unsold or uneaten – and most of that goes to waste `https://refed.org/food-waste/the-challenge/`, accessed on Sep 20, 2022.

Rezaei M, Liu B (2017) Food loss and waste in the food supply chain. Available at `https://openknowledge.fao.org/server/api/core/bitstreams/36cb45bc-392c-41fb-97f1-90ca1f16ee7f/content` accessed Aug 1, 2025.

Song JS, Zhang Y (2024) Predictive three-dimensional printing of spare parts with internet of things. *Management Science* .

Stewart H, Hyman J (2024) Farmers received about half of what consumers paid for fresh strawberries from 2020 – 23. USDA Economic Research Service.

Tang SY, Gurnani H, Gupta D (2014) Managing disruptions in decentralized supply chains with endogenous supply process reliability. *Production and Operations Management* 23(7):1198–1211.

Tang SY, Kouvelis P (2011) Supplier diversification strategies in the presence of yield uncertainty and buyer competition. *Manufacturing & Service Operations Management* 13(4):439–451.

Tomlin B (2009) Disruption-management strategies for short life-cycle products. *Naval Research Logistics (NRL)* 56(4):318–347.

Tomlin B, Wang Y (2005) On the value of mix flexibility and dual sourcing in unreliable newsvendor networks. *Manufacturing & Service Operations Management* 7(1):37–57.

Tomlin B, Wang Y (2008) Pricing and operational recourse in coproduction systems. *Management Science* 54(3):522–537.

Tuna AK, Swinney R (2021) Are fast supply chains sustainable? Working paper. Available at SSRN https://ssrn.com/abstract=3995549.

Walmart (2018) Eden: The tech that's bringing fresher groceries to you. Available at https://corporate.walmart.com/news/2018/03/01/eden-the-tech-thats-bringing-fresher-groceries-to-you, accessed on Aug 17, 2024.

Wang Y, Gilland W, Tomlin B (2010) Mitigating supply risk: Dual sourcing or process improvement? *Manufacturing & Service Operations Management* 12(3):489–510.

Wang Y, Xiao Y, Yang N (2014) Improving reliability of a shared supplier with competition and spillovers. *European Journal of Operational Research* 236(2):499–510.

World Economic Forum (2022) How can sustainable food cold chains help feed developing countries? Available at https://www.weforum.org/stories/2022/11/sustainable-food-cold-chains-feed-developing-countries accessed July 11, 2025.

Wu Q, Honhon D (2023) Don't waste that free lettuce! impact of bogof promotions on retail profit and food waste. *Production and Operations Management* 32(2):501–523.

Yano CA, Lee HL (1995) Lot sizing with random yields: A review. *Operations research* 43(2):311–334.

Online Appendices for "Shipment Monitoring, Allocation, and the Impact on Food Waste"

Tao Lu

University of Connecticut

A Interpretation of the Random Yield Model

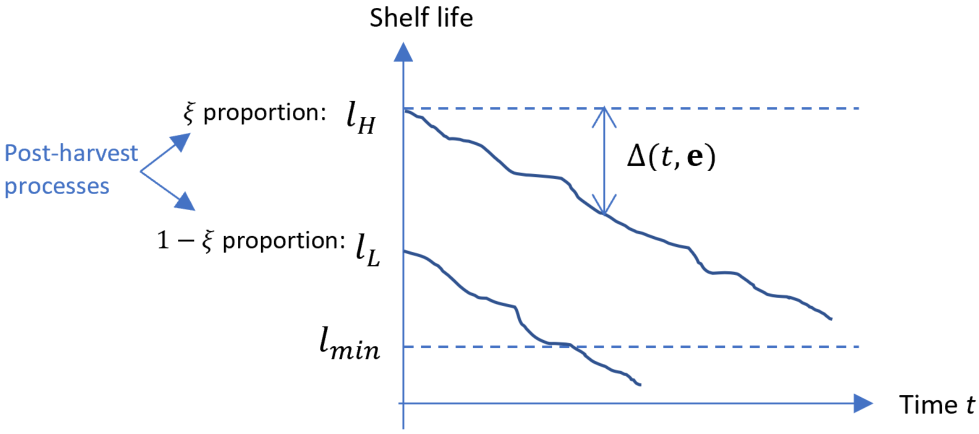


Figure A.1: Shelf life interpretation of the random yield model

Consider a setting where post-harvest processes render a $(1 - \xi)$ proportion of the product with a short shelf life l_L while a proportion ξ is kept in good condition and hence with a long shelf life l_H where $l_H > l_L$. As time elapses, their shelf lives continue to drop. The decrease in shelf life, denoted by $\Delta(t, \mathbf{e})$, is a general function of time t and the history of all relevant environmental variables $\mathbf{e}$ up to t. The remaining shelf life of the good (resp. risky) units upon arrival at market i is given by $l_H - \Delta(t_i, \mathbf{e}_i)$ (resp. $l_L - \Delta(t_i, \mathbf{e}_i)$) where t_i and $\mathbf{e}_i$ are uncertain variables influenced by random shocks (e.g., delay, temperature excursion) occurring along the transportation lane to each market i.

Let l_{min} be the minimum shelf-life level required for sale. The proposed random yield model assumes that l_H is high enough such that $\Pr\{l_H - \Delta(t_i, \mathbf{e}_i) \geq l_{min}\} = 1$ for both $i = 1$ and 2, i.e., units in good condition will reach both markets before spoilage with probability one. The yield factor ϵ_i in the model represents the proportion of risky units sent to market i that experience an event $\{l_L - \Delta(t_i, \mathbf{e}_i) \geq l_{min}\}$. If all the units in the shipment to market i are exposed to the same t_i and $\mathbf{e}_i$ (because, for instance, they are carried in the same refrigerated truck), then we have the base-model setting where ϵ_i is a Bernoulli random variable with success probability $\beta_i = \Pr\{l_L - \Delta(t_i, \mathbf{e}_i) \geq l_{min}\}$. Otherwise, the ϵ_i's follow general probability distributions as considered in §5.3.

Because the transportation distance to market 1 is shorter, it is more likely that t_1 is smaller than t_2 and the history of environmental variables $\mathbf{e}_1$ is more favorable than $\mathbf{e}_2$. So, ϵ_1 is first-order stochastically larger than ϵ_2. Because the $(t_i, \mathbf{e}_i)$'s are due to random shocks on separate transportation lanes, it is reasonable to assume the ϵ_i's to be independent.

B Supplemental Results of the Base Model

B.1 Optimal Order Quantities without Monitoring

The optimal order quantities without monitoring are characterized below.

Proposition B.1. *Without monitoring, the optimal shipping quantity to each market $i \in \mathcal{M}$ is given by q_i^N where (i) $q_i^N = 0$ if $\phi_i \leq 1 - E[z_i]$, and (ii) q_i^N is the unique solution to the following equation: $E\left[z_i \bar{F}_i(q_i z_i)\right] = 1 - \phi_i$ if $\phi_i > 1 - E[z_i]$, where $\phi_i = \frac{p - c_i - c_o}{p + c_d}$.*

Proof of Proposition B.1. Using the relation $min(a, b) = b - (b - a)^+$, we can rewrite the objective function as $\Pi_i^N(q_i) = (p - c_i - c_o)q_i - (p + c_d)E\left[(1 - z_i)q_i + (q_i z_i - D_i)^+\right]$. The first-order derivative is given by $\frac{\partial \Pi^N}{\partial q_i} = (p - c_i - c_o) - (p + c_d)\left(1 - E[z_i \bar{F}_i(q_i z_i)]\right)$. The objective function is clearly concave in q_i. The optimal shipping quantity follows directly from the first-order optimality condition. □

B.2 Effect of Purchase Cost

As the purchase cost c_o increases, there is generally no simple pattern in how the optimal allocation strategy changes. In fact, as c_o increases, the optimal allocation can oscillate between different cases. A key reason is that some of the thresholds, for instance, $C_2(Q, \xi)$ and $C_3(Q, \xi)$, are not necessarily monotone in Q. So, $r(Q)$ can oscillate between different cases as Q increases.[16] Let us illustrate this complexity with a numerical example below.

Example B.1. Assume $\xi = 0.4$ with probability one. Each demand D_i follows a Gamma distribution with a shape parameter 5 and a scale parameter 20. Other parameters are given by $p = 50$, $c_d = 0$, $c_o = 5$, $c_1 = 1$, $c_2 = 4$, $\beta_1 = 0.6$ and $\beta_2 = 0.3$.

Panel (a) of Figure B.1 plots the relevant thresholds that determine which case is active in the expression of $r(Q)$ as presented in Proposition 2. The horizontal dashed line represents

[16] The decreasing property of $r(Q)$ in Proposition 2 is proved using the argument that $R(Q, \xi)$ is concave in Q as concavity is preserved under the maximization of the second-stage problem. We can also show that $\frac{\partial R(Q,\xi)}{\partial Q}$ is decreasing in Q for each case of Proposition 2 and continuous at each boundary between cases. However, it is possible that as Q increases, $\frac{\partial R(Q,\xi)}{\partial Q}$ moves back and forth between different cases of Proposition 2.

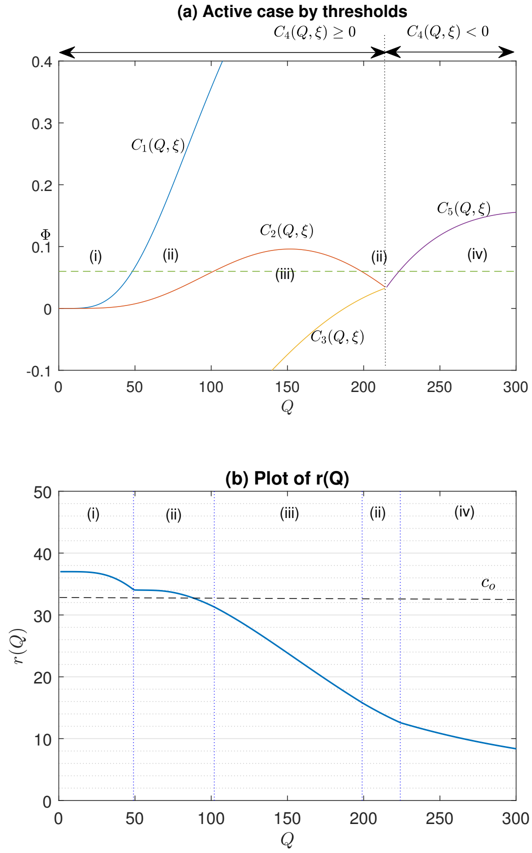


Figure B.1: Nuanced structure of $r(Q)$ for a deterministic ξ.

the value of Φ, which is equal to 0.06 in this example. As Q increases from 0 to 300, the active case follows the sequence (i), (ii), (iii), (ii) and (v). As shown in the graph, $C_2(Q,\xi)$ first increases and then decreases with Q. Consequently, the active case can switch from (ii) to (iii) and then go back to (ii) again. Panel (b) plots the curve of $r(Q)$, and the optimal Q^* occurs at the intersection between $r(Q)$ and c_o. Due to the nuanced behavior of the active case, as the purchase cost c_o decreases, e.g., from 32 to 12 in the graph, the optimal order and allocation strategy will first follow case (ii) and then case (iii), but will switch back to case (ii) again.

B.3 Cross-Market Dependence

From Proposition 3, we can see that, compared to the case without monitoring, shipment monitoring results in a *cross-market dependence*, leading to several novel properties of the optimal solution that have not been documented in the literature. In typical newsvendor problems with supply uncertainty (like our problem without monitoring), the optimal quantity for a market is determined solely by the critical fractile and the supply risk associated with that market, regardless of the characteristics of other markets. If a market is free of supply risk, the optimal service level will equal the corresponding critical fractile ϕ_i. As summarized below, none of these properties necessarily hold in the presence of monitoring technology.

The total quantity allocated to each market can be increasing, decreasing, or constant in the critical fractile of the other market, as summarized in Table B.1. As already illustrated by Figure 5, a higher ϕ_2 makes it more profitable to sell in market 2. To have sufficient units available in market 2, the firm may send either more or fewer units to market 1. For similar reasons, with a higher ϕ_1, the firm will deliver more to market 1, which results in market 2 receiving more units in cases (iii-a) and (iii-b) but fewer in cases (iv-a) and (iv-b). Our results imply that, for example, as the transportation cost to a market increases, the optimal quantity supplied to the other market can become either larger or smaller. As explained earlier, to have sufficient good units for market 2, it can be optimal to over-deliver in market 1, resulting in a service level greater than ϕ_1.

Proposition B.2. *In contrast to the solution without monitoring, monitoring technology gives rise to the following properties for the optimal solution: (1) With all else fixed, the total quantity allocated to each market $i = 1, 2$ can be either increasing or decreasing in ϕ_{3-i}, i.e., the critical fractile of the other market. Table B.1 summarizes the cross-market effect of ϕ_i for each case of Proposition 3. (2) Market 1, even though free of supply uncertainty, can receive an optimal service level greater than ϕ_1.*

Table B.1: Cross-market effect of critical fractile ϕ_i on the optimal allocation quantities

Case	Effect of ϕ_2 on Market 1			Effect of ϕ_1 on Market 2		
	$\frac{\partial x_1^*}{\partial \phi_2}$	$\frac{\partial y_1^*}{\partial \phi_2}$	$\frac{\partial (x_1^*+y_1^*)}{\partial \phi_2}$	$\frac{\partial x_2^*}{\partial \phi_1}$	$\frac{\partial y_2^*}{\partial \phi_1}$	$\frac{\partial (x_2^*+y_2^*)}{\partial \phi_1}$
i	0	0	0	0	0	0
ii	-	+	0	0	0	0
iii-a/iii-b	0	+	+	+	0	+
iv-a/iv-b	0	-	-	+	-	-

Proof of Proposition B.2. Property (1) It suffices to verify the sign of each derivative listed in Table B.1, based on the expressions given in Table 1 of the main paper. For each case, we have the following.

(i) $\frac{\partial x_i^*}{\partial \phi_{3-i}} = \frac{\partial y_i^*}{\partial \phi_{3-i}} = 0$ for both $i = 1, 2$.

(ii) $\frac{\partial x_1^*}{\partial \phi_2} = -2\mu(1-\xi)$, $\frac{\partial y_1^*}{\partial \phi_2} = 2\mu(1-\xi)$, $\frac{\partial (x_1^*+y_1^*)}{\partial \phi_2} = 0$, and $\frac{\partial x_2^*}{\partial \phi_1} = \frac{\partial y_2^*}{\partial \phi_1} = 0$.

(iii-a) $\frac{\partial x_1^*}{\partial \phi_2} = 0$, $\frac{\partial y_1^*}{\partial \phi_2} = \frac{\partial (x_1^*+y_1^*)}{\partial \phi_2} = \frac{2\mu(1-\xi)\xi}{1-2\xi+2\xi^2} > 0$.

$\frac{\partial y_2^*}{\partial \phi_1} = 0$, $\frac{\partial x_2^*}{\partial \phi_1} = \frac{\partial (x_2^*+y_2^*)}{\partial \phi_1} = \frac{2\mu(1-\xi)\xi}{1-2\xi+2\xi^2} > 0$.

(iii-b) $\frac{\partial x_1^*}{\partial \phi_2} = 0$, $\frac{\partial y_1^*}{\partial \phi_2} = \frac{\partial (x_1^*+y_1^*)}{\partial \phi_2} = \frac{2\mu(1-\xi)}{\xi} > 0.$

$\frac{\partial y_2^*}{\partial \phi_1} = 0$, $\frac{\partial x_2^*}{\partial \phi_1} = \frac{\partial (x_2^*+y_2^*)}{\partial \phi_1} = \frac{2\mu(1-\xi)}{\xi} > 0.$

(iv-a) $\frac{\partial x_1^*}{\partial \phi_2} = 0$, $\frac{\partial y_1^*}{\partial \phi_2} = \frac{\partial (x_1^*+y_1^*)}{\partial \phi_2} = -\frac{2\mu(1-\beta)\xi^2}{\beta+\xi^2(1-\beta^2)} < 0.$

$\frac{\partial x_2^*}{\partial \phi_1} = \frac{2\mu\beta}{\beta+\xi^2-\beta^2\xi^2} > 0$, $\frac{\partial y_2^*}{\partial \phi_1} = -\frac{2\mu\xi(\beta+\xi-\beta\xi)}{\beta+\xi^2-\beta^2\xi^2} < 0$, $\frac{\partial (x_2^*+y_2^*)}{\partial \phi_1} = -\frac{2\mu(1-\beta)\xi^2}{\beta+\xi^2-\beta^2\xi^2} < 0$

(iv-b) $\frac{\partial x_1^*}{\partial \phi_2} = 0$, $\frac{\partial y_1^*}{\partial \phi_2} = \frac{\partial (x_1^*+y_1^*)}{\partial \phi_2} = -\frac{2\mu}{\beta} < 0.$

$\frac{\partial x_2^*}{\partial \phi_1} = \frac{2\mu}{\xi(1-\beta)} > 0$, $\frac{\partial y_2^*}{\partial \phi_1} = -\frac{2\mu(\beta+\xi-\beta\xi)}{(1-\beta)\beta\xi} < 0$, $\frac{\partial (x_2^*+y_2^*)}{\partial \phi_1} = -\frac{2\mu}{\beta} < 0.$

Property (2) In case (ii), the optimal quantity shipped to market 1 is equal to $2\mu\phi_1$, resulting in a standard service level ϕ_1. We can verify that the quantity is larger than $2\mu\phi_1$ in, for instance, case (iii-a). To do so, we substitute the boundary condition $\frac{\xi}{1-\xi}\phi_1 \le \phi_2$ into the expression of Q^{iii-a}, and obtain $x_1^* + y_1^* \ge 2\mu\phi_1$. Since market 1 has no spoilage risk, the resulting service level is greater than ϕ_1. □

B.4 Model Parameter Calibration

California makes up 85% – 90% of strawberry production in the US with three major growing districts, Oxnard, Santa Maria and Watsonville-Salinas (Holmes 2024). We consider strawberries sourced from Santa Maria and shipped to two representative markets in the West and East coasts: Los Angeles (LA) and Boston, respectively.

Retail Price and Purchase Cost. According to the USDA Economic Research Service, in 2023, the average retail price for fresh strawberries was \$3.80 per pound, and farmers received 53% of the retail price (Stewart and Hyman 2024). We therefore set $p = \$3.80$ and $c_o = \$2.01$.

Transportation and Disposal Costs. Strawberries are typically shipped using standard 53-foot refrigerated trailers. Each 53-foot trailer generally accommodates 26 stacks of standard pallets.[17] Each pallet of strawberries is packed with 120 boxes, with 9 pounds of strawberries per box.[18] Based on this, the total weight per truckload is 28,080 pounds. The cost of shipping a refrigerated truckload to a major city in the US is about \$3.17 – \$3.65 per mile.[19] We use the average value of \$3.41 per mile. According to Google Maps, the driving distances from Santa Maria to LA and Boston are approximately 160 miles and 3107 miles, respectively. This translates to shipping costs of \$545.6 and \$10,595 per truckload for the two markets, respectively. Dividing by the total weight per truckload, we obtain approximate per-pound transportation costs of $c_1 = \$0.02$ and $c_2 = \$0.38$.

[17] `https://haletrailer.com/blog/how-many-pallets-fit-in-53-trailer/` accessed May 23, 2025.
[18] `https://mainstreetproduce.com/produce/` accessed May 26, 2025.
[19] `https://www.dat.com/blog/sweet-times-for-carriers-hauling-strawberries-to-the-big-apple` accessed May 26, 2025.

The disposal cost c_d includes disposal service fees, surcharges, and logistics costs incurred by the firm when disposing of each unit of waste. Massachusetts has a landfill tipping fee of \$122.63 per ton (BioCycle 2021). Similarly, the landfill tipping fee for food waste in the LA area ranges from \$100 to \$126.22 per ton.[20] Based on these values, we set $c_d = \$0.06$ per pound.

Spoilage Probabilities. In our model, each unit sent to market i will become unsalable with probability $(1 - \xi)(1 - \beta_i)$. Stewart and Hyman (2024) estimate that 8% of fresh strawberries shipped from farms are lost due to spoilage or damage. Buzby et al. (2016) report that 14.2% of strawberries shipped to supermarkets were lost during 2011-2012. Erwin (2024) documents that "due to their short shelf life, around 25% of strawberries are discarded at the distribution and retail level." The above sources suggest that the loss of fresh strawberries due to shelf-life deterioration ranges from 8% to 25%. As discussed in the main paper, we choose the values of ξ, β_1 and β_2 such that the average value of $(1 - \xi)(1 - \beta_i)$ falls within this range. We note that in developing countries lacking reliable cold chain infrastructure, this probability can be even higher, estimated at 25% - 40% (Hammam and Abouelwafa 2022, p. 734). Admittedly, a limitation of our calibration is that, due to the lack of data on interim deterioration during post-harvest processes, we are unable to calibrate ξ and β_i separately. Nevertheless, in Appendix B.5, we report a comprehensive numerical study in which either ξ or β_i varies.

Demand. According to Lee et al. (2013), Walmart distributes 1.5 million to 2.7 million strawberries across 200 stores from Santa Maria per day during peak season. Based on 20 strawberries per pound, on average, each Walmart store receives $(1.5+2.7)*10^6/(2*20*200) = 525$ pounds of strawberries. To estimate demand, we assume this received quantity represents the optimal shipping quantity, Q^N, in the absence of monitoring. If both D_1 and D_2 follow Gamma distributions with mean μ, with $cv_1 = 0.5$ and $cv_2 = 1.0$, respectively, and the other parameters are calibrated as above – that is, $p = 3.80$, $c_o = 2.01$, $c_1 = 0.02$, $c_2 = 0.38$, $c_d = 0.06$, $\beta_1 = 0.9$, $\beta_2 = 0.1$, and $\xi \in \{0.8, 0.7, 0.6\}$ – a numerical search indicates that when $\mu = 1000$, $Q^N/2$ is close to the observed shipping quantity for one store. We therefore set the mean demand to 1000.

B.5 Additional Numerical Results

In this appendix, we report an extensive numerical study that demonstrates the robustness of our main results and illustrates several additional insights.

[20] `https://www.lacsd.org/services/solid-waste/tipping-fees-for-solid-waste-and-recyclables` accessed May 2, 2026.

B.5.1 Other Parameters That Influence Waste Changes

In the main paper, we show that monitoring is more likely to increase waste when p is smaller or ξ is lower. We report additional numerical results for other relevant model parameters.

We set the baseline parameters generally in line with the California strawberry example discussed in the main paper: $c_o = 2.01$, $c_1 = 0.02$, $c_2 = 0.38$, $c_d = 0.06$, $\beta_1 = 0.9$, and $\beta_2 = 0.1$. We assume $\xi \sim Tri(0.5, 0.7, 0.9)$, and D_1 and D_2 follow Gamma distributions with mean $\mu_1 = \mu_2 = 1000$, $cv_1 = cv/2$, and $cv_2 = cv$, where $cv = 1$ is the baseline.

Figure B.2 depicts how monitoring technology affects waste, measured by $W^I - W^N$, as cv and β_2 vary, with all other parameters fixed at their baseline values. Our results indicate that monitoring is more likely to increase waste when demand variability is higher, i.e., when cv is larger, or when the spoilage risk during transportation is higher, i.e., when β_2 is smaller.

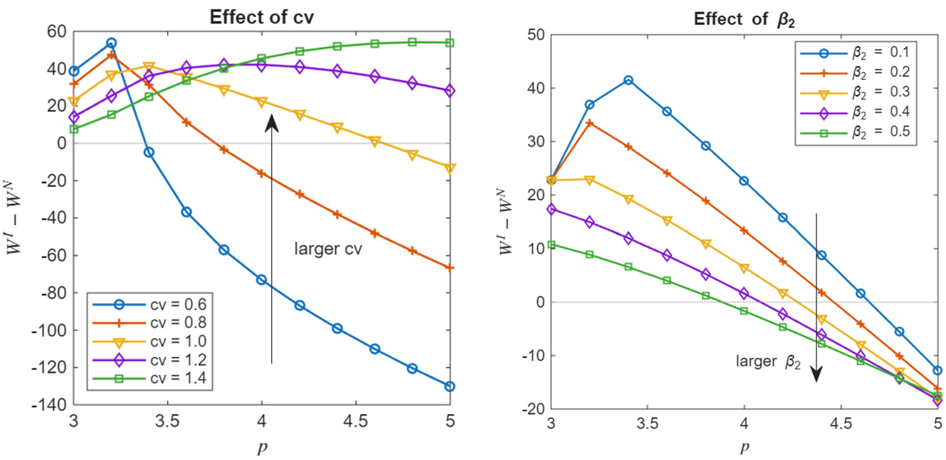


Figure B.2: Monitoring-induced changes in waste, $W^I - W^N$, for different demand coefficients of variation and transportation spoilage risks.

B.5.2 Waste-Conscious Solution

In §4.3, we show that waste-conscious firms can capture most of the profit gains from shipment monitoring while maintaining their waste reduction goals by adjusting the optimal order quantity Q^I as $Q^A = \eta Q^I + (1-\eta)\min(Q^I, Q^N)$ where $\eta \in [0,1]$. We now consider a wider range of parameter values to demonstrate the robustness of our findings. Keeping all the costs (i.e., c_1, c_2, c_d, c_o) and the mean demand μ as in the calibrated setting, we vary p from 3.0 to 4.0 in increments of 0.2, and cv_2 from $\{0.8, 1.0, 1.2\}$ with $cv_1 = cv_2/2$. Additionally, we consider different combinations of β_i's, with $\beta_1 = 0.6, 0.7, ..., 1.0$ and $\beta_2 = 0.1, 0.2, ..., \beta_1 - 0.1$ for any given β_1. Moreover, we allow ξ to be uncertain in this numerical study by assuming that ξ takes three different triangular distributions, Triangular$(0.4, 0.6, 0.8)$, Triangular$(0.5, 0.7, 0.9)$, and Triangular$(0.6, 0.8, 1.0)$. In total, there are 1,890 instances. For each of these instances,

Table B.2: Optimality gap of the waste-conscious solution ($\frac{\Pi^I - \Pi^A}{\Pi^I}$) and its impact on waste reduction

	Distribution of ξ					
	Tri$(0.4, 0.6, 0.8)$		Tri$(0.5, 0.7, 0.9)$		Tri$(0.6, 0.8, 1.0)$	
η	Opt Gap (%)	$W^A - W^N$	Opt Gap (%)	$W^A - W^N$	Opt Gap (%)	$W^A - W^N$
0	2.82	-27.30	1.57	-25.64	0.55	-19.96
0.25	1.58	-16.77	0.88	-17.64	0.31	-14.86
0.50	0.70	-5.41	0.39	-9.21	0.14	-9.60
0.75	0.18	6.78	0.10	-0.34	0.03	-4.20
1	0.00	19.80	0.00	8.95	0.00	1.36

Note: Each point is averaged over all other parameters for the given ξ and η

we compute the waste-conscious solution with $\eta = 0, 0.25, 0.5, 0.75$ and 1 as well as the optimal solution for the no-monitoring benchmark. Table B.2 reports the percent changes in profit and waste for each ξ and η, averaged over all other parameters.

The results are consistent with the main findings discussed in the paper. With the profit-maximizing solution ($\eta = 1$), the adoption of monitoring technology can increase total waste. However, if the firm does not increase its order quantity and uses monitoring only for shipment allocation ($\eta = 0$), it can still achieve near-optimal profit while reducing waste in all scenarios. Additionally, the optimality gaps are generally smaller when ξ is stochastically larger. Moreover, by moderating the increase in order quantity through an appropriate choice of η, firms can trade off profit gains against waste reduction, thereby better aligning their operational decisions with their waste-reduction goals.

C Proofs

C.1 The Karush–Kuhn–Tucker (KKT) Conditions for the Allocation Problem

Using the relation $min(x, y) = y - (y - x)^+$, we can rewrite the objective function as

$$\sum_{i \in \mathcal{M}} \left\{ (p - c_i)x_i + [p - c_i - (p + c_d)(1 - E[\epsilon_i])]y_i - (p + c_d)E\left[(x_i + y_i\epsilon_i - D_i)^+\right] \right\}.$$

Let λ_x, λ_y denote the Lagrangian dual variables associated with constraints (2) and (3), respectively. Let $\omega_i^x \geq 0$ and $\omega_i^y \geq 0$ denote the Lagrangian dual variables associated with nonnegativity constraints $x_i \geq 0$ and $y_i \geq 0$, respectively. By the complementary slackness, $\omega_i^x x_i = 0$ and $\omega_i^y y_i = 0$ for all $i \in \mathcal{M}$.

Let L denote the Lagrangian function. The Karush–Kuhn–Tucker (KKT) conditions for the allocation problem can be written as follows. For all $i \in \mathcal{M}$,

$$\frac{\partial L}{\partial x_i} = (p - c_i) - (p + c_d)E_{\epsilon_i}[F_i(x_i + y_i\epsilon_i)] - \lambda_x + \omega_i^x = 0, \tag{C.1}$$

$$\frac{\partial L}{\partial y_i} = (p - c_i) - (p + c_d)\left(1 - E_{\epsilon_i}[\epsilon_i \bar{F}_i(x_i + y_i\epsilon_i)]\right) - \lambda_y + \omega_i^y = 0. \tag{C.2}$$

Let $\bar{F}_i(x) = 1 - F_i(x)$ for all i. Using (C.1), (C.2), we can obtain the following relations which must hold in the optimal solution for any $i, j \in \mathcal{M}$.

$$\frac{\partial L}{\partial x_i} - \frac{\partial L}{\partial x_j} = c_j - c_i - (p + c_d)E[F_i(x_i + y_i\epsilon_i) - F_j(x_j + y_j\epsilon_j)] + \omega_i^x - \omega_j^x = 0, \tag{C.3}$$

$$\frac{\partial L}{\partial y_i} - \frac{\partial L}{\partial y_j} = c_j - c_i - (p + c_d)E[\epsilon_j \bar{F}_j(x_j + y_j\epsilon_j) - \epsilon_i \bar{F}_i(x_i + y_i\epsilon_i)] + \omega_i^y - \omega_j^y = 0, \tag{C.4}$$

$$\frac{\partial L}{\partial x_i} - \frac{\partial L}{\partial y_i} = (p + c_d)E[(1 - \epsilon_i)\bar{F}_i(x_i + y_i\epsilon_i)] - \lambda_x + \lambda_y + \omega_i^x - \omega_i^y = 0. \tag{C.5}$$

C.2 Proofs for the Base Model

Proof of Proposition 1. Given the Bernoulli distributed ϵ_i's, we can write the KKT conditions as

$$\frac{\partial L}{\partial x_1} - \frac{\partial L}{\partial x_2} = c_2 - c_1 - (p + c_d)\left[\beta_2\bar{F}_2(x_2 + y_2) + (1 - \beta_2)\bar{F}_2(x_2) - \beta_1\bar{F}_1(x_1 + y_1) - (1 - \beta_1)\bar{F}_1(x_1)\right] + \omega_1^x - \omega_2^x = 0 \tag{C.6}$$

and

$$\frac{\partial L}{\partial y_1} - \frac{\partial L}{\partial y_2} = c_2 - c_1 - (p + c_d)\left[\beta_2\bar{F}_2(x_2 + y_2) - \beta_1\bar{F}_1(x_1 + y_1)\right] + \omega_1^y - \omega_2^y = 0. \tag{C.7}$$

together with the complementary slackness $\omega_i^x x_i = 0$ and $\omega_i^y y_i = 0$.

We prove the proposition by verifying that the proposed solution satisfies the KKT conditions under each case.

Let us first prove cases (i), (iii)(iv) and (v), and then move to the more involved case (ii).

Case (i) By the complementary slackness, $\omega_1^x = \omega_1^y = 0$ and $\omega_2^x, \omega_2^y \geq 0$, and so (C.6) and (C.7) reduce to

$$c_2 - c_1 - (p + c_d)\left[\beta_2\bar{F}_2(x_2 + y_2) + (1 - \beta_2)\bar{F}_2(x_2) - \beta_1\bar{F}_1(x_1 + y_1) - (1 - \beta_1)\bar{F}_1(x_1)\right] \geq 0$$

and

$$c_2 - c_1 - (p + c_d)\left[\beta_2 \bar{F}_2(x_2 + y_2) - \beta_1 \bar{F}_1(x_1 + y_1)\right] \geq 0.$$

The solution $x_1^* = Q\xi, y_1^* = Q(1-\xi), x_2^* = y_2^* = 0$ satisfies the above two conditions iff $1 - \beta_1 \bar{F}_1(Q) - (1-\beta_1)\bar{F}_1(Q\xi) \leq \Phi$ and $\beta_2 - \beta_1 \bar{F}_1(Q) \leq \Phi$. The former inequality actually implies the latter, and so the latter is redundant. To see this, we show that $1 - \beta_1 \bar{F}_1(Q) - (1-\beta_1)\bar{F}_1(Q\xi) > \beta_2 - \beta_1 \bar{F}_1(Q)$. Notice that

$$\begin{aligned}
&(1 - \beta_1 \bar{F}_1(Q) - (1-\beta_1)\bar{F}_1(Q\xi)) - (\beta_2 - \beta_1 \bar{F}_1(Q)) \\
=&1 - (1-\beta_1)\bar{F}_1(Q\xi) - \beta_2 \geq \beta_1 - \beta_2 > 0
\end{aligned}$$

where the first inequality follows as $\bar{F}_1(Q\xi) \leq 1$ and the second is by the assumption of $\beta_1 > \beta_2$. Hence, the solution $x_1^* = Q\xi, y_1^* = Q(1-\xi), x_2^* = y_2^* = 0$ is optimal iff

$$C_1(Q,\xi) := 1 - \beta_1 \bar{F}_1(Q) - (1-\beta_1)\bar{F}_1(Q\xi) \leq \Phi. \tag{C1}$$

Case (iii) We consider the solution $x_1^* = 0, y_1^* = Q(1-\xi), x_2^* = Q\xi, y_2^* = 0$. With the complementary slackness we can express (C.6) and (C.7) as

$$c_2 - c_1 - (p + c_d)\left[\beta_2 \bar{F}_2(x_2 + y_2) + (1-\beta_2)\bar{F}_2(x_2) - \beta_1 \bar{F}_1(x_1 + y_1) - (1-\beta_1)\bar{F}_1(x_1)\right] \leq 0$$

and

$$c_2 - c_1 - (p + c_d)\left[\beta_2 \bar{F}_2(x_2 + y_2) - \beta_1 \bar{F}_1(x_1 + y_1)\right] \geq 0.$$

The solution $x_1^* = 0, y_1^* = Q(1-\xi), x_2^* = Q\xi, y_2^* = 0$ satisfies the above inequalities iff

$$C_2(Q,\xi) := \beta_1 F_1(Q(1-\xi)) - F_2(Q\xi) \geq \Phi \tag{C2}$$

$$C_3(Q,\xi) := \beta_2 \bar{F}_2(Q\xi) - \beta_1 \bar{F}_1(Q(1-\xi)) \leq \Phi \tag{C3}$$

Case (iv) Consider the solution such that $x_1^* = 0, x_2^* = Q\xi, y_1^* > 0, y_2^* > 0$. By complementary slackness, we have $\omega_1^x \geq 0, \omega_2^x = \omega_1^y = \omega_2^y = 0$. Then, together with $y_2 = Q(1-\xi) - y_1$, (C.6) reduces to

$$\beta_2 \bar{F}_2(Q - y_1^*) + (1-\beta_2)\bar{F}_2(Q\xi) - \beta_1 \bar{F}_1(y_1^*) - (1-\beta_1) \geq \Phi, \tag{C.8}$$

and (C.7) reduces to

$$\beta_2 \bar{F}_2(Q - y_1^*) - \beta_1 \bar{F}_1(y_1^*) = \Phi. \tag{C.9}$$

Since $\beta_2\bar{F}_2(Q-y_1)-\beta_1\bar{F}_1(y_1)$ is increasing in y_1, there exists a $y_1=y^\dagger\in(0,Q(1-\xi))$ such that (C.9) holds iff $\beta_2\bar{F}_2(Q)-\beta_1<\Phi$ and

$$C_3(Q,\xi)=\beta_2\bar{F}_2(Q\xi)-\beta_1\bar{F}_1(Q(1-\xi))>\Phi. \tag{C3'}$$

The former inequality always holds as $\beta_2<\beta_1$ and $\Phi>0$. The latter (C3') is all we need for the existence of an $y_1^*=y^\dagger\in(0,Q(1-\xi))$ such that (C.9) holds.[21] Moreover, given (C.9), (C.8) is equivalent to

$$C_4(Q,\xi):=(1-\beta_2)\bar{F}_2(Q\xi)-(1-\beta_1)\geq 0. \tag{C4}$$

Thus, conditions (C3') and (C4) guarantee that the proposed solution is optimal.

Case (v) In this case, $\omega_1^x=\omega_1^y=\omega_2^x=\omega_2^y=0$. Along with $Q\xi=x_1+x_2$ and $Q(1-\xi)=y_1+y_2$, conditions (C.6) and (C.7) reduce

$$\beta_2\bar{F}_2(Q-x_1-y_1)+(1-\beta_2)\bar{F}_2(Q\xi-x_1)-\beta_1\bar{F}_1(x_1+y_1)-(1-\beta_1)\bar{F}_1(x_1)=\Phi \tag{C.10}$$

$$\beta_2\bar{F}_2(Q-x_1^*-y_1^*)-\beta_1\bar{F}_1(x_1^*+y_1^*)=\Phi. \tag{C.11}$$

Moreover, (C.10) and (C.11) imply

$$(1-\beta_1)\bar{F}_1(x_1^*)-(1-\beta_2)\bar{F}_2(Q\xi-x_1^*)=0. \tag{C.12}$$

First, note that the left-hand side of (C.12) is decreasing in x_1 over $(0,Q\xi)$. Thus, there exists an $x_1^*=x^o\in(0,Q\xi)$ such that (C.12) holds iff $(1-\beta_1)\bar{F}_1(Q\xi)-(1-\beta_2)<0$ and

$$(1-\beta_1)-(1-\beta_2)\bar{F}_2(Q\xi)>0,\ \text{i.e., } C_4(Q,\xi)<0. \tag{C4'}$$

The former inequality is always true since $\beta_1>\beta_2$. The latter (C4') is all we need for the existence of x^o. Provided that (C4') holds, we consider when (C.11) holds for $x_1^*=x^o$ and an $y_1^*\in(0,Q(1-\xi))$. Let $q=x_1+y_1$. Note that $\beta_2\bar{F}_2(Q-q)-\beta_1\bar{F}_1(q)$ is increasing in $q\in(0,Q)$. Thus, there exists a $q^o\in(x^o,x^o+Q(1-\xi))$ such that $\beta_2\bar{F}_2(Q-q^o)-\beta_1\bar{F}_1(q^o)=\Phi$ iff $\beta_2\bar{F}_2(Q-x^o)-\beta_1\bar{F}_1(x^o)<\Phi$ and

$$\beta_2\bar{F}_2(Q\xi-x^o)-\beta_1\bar{F}_1(x^o+Q(1-\xi))>\Phi. \tag{C5}$$

We show that the former inequality is automatically satisfied because $\beta_2\bar{F}_2(Q-x^o)-\beta_1\bar{F}_1(x^o)\leq$

[21] In equation tags, we use $'$ to denote the complement of a condition. For instance, $(C3')$ is the complement of $(C3)$.

0 (and $\Phi > 0$). Note that

$$\begin{aligned}\beta_2\bar{F}_2(Q-x^o)-\beta_1\bar{F}_1(x^o) &\underset{(1)}{=} \beta_2\bar{F}_2(Q-x^o)-\beta_1\frac{1-\beta_2}{1-\beta_1}\bar{F}_2(Q\xi-x^o)\\ &\underset{(2)}{\leq} \beta_2\bar{F}_2(Q\xi-x^o)-\beta_1\frac{1-\beta_2}{1-\beta_1}\bar{F}_2(Q\xi-x^o)\\ &= \left(\beta_2-\beta_1\frac{1-\beta_2}{1-\beta_1}\right)\bar{F}_2(Q\xi-x^o) \underset{(3)}{\leq} 0\end{aligned}$$

where (1) follows by substituting $(1-\beta_1)\bar{F}_1(x^o) = (1-\beta_2)\bar{F}_2(Q\xi-x^o)$, (2) follows as $\xi \leq 1$ and $\bar{F}_2$ is decreasing, and (3) holds since $\beta_1 > \beta_2$. Therefore, under conditions (C4') and (C5), interior points x^o and y^o exist and the solution described in Case (iv) is optimal. By definition of C_5, given (C4'), condition (C5) can be written as $C_5(Q,\xi) > \Phi$.

Case (ii) We are ready to prove case (ii), which is more involved.[22] Putting together the cases already explored above on a β_1-β_2 coordinate plane, we can see that the entire parameter range is partitioned as shown in Figure 3. The key is to notice that Lines 2,3,4, 5 in Figure 3, i.e., $\beta_1 F_1(Q(1-\xi)) - F_2(Q\xi) = \Phi$, $(1-\beta_2)\bar{F}_2(Q\xi) = (1-\beta_1)$, $\beta_2\bar{F}_2(Q\xi) - \beta_1\bar{F}_1(Q(1-\xi)) = \Phi$ and $\beta_2\bar{F}_2(Q\xi-x^o) - \beta_1\bar{F}_1(x^o+Q(1-\xi)) \leq \Phi$ where $(1-\beta_2)\bar{F}_2(Q\xi-x^o) = (1-\beta_1)\bar{F}_1(x^o)$ intersect at the *same* point. As a result, the remaining parameter range is characterized by three conditions:

$$C_1(Q,\xi) = \beta_1[F_1(Q) - F_1(Q\xi)] + F_1(Q\xi) > \Phi \tag{C1'}$$

$$C_2(Q,\xi) = \beta_1 F_1(Q(1-\xi)) - F_2(Q\xi) < \Phi, \tag{C2'}$$

and, in case of $C_4(Q,\xi) = (1-\beta_2)\bar{F}_2(Q\xi) - (1-\beta_1) < 0$,

$$C_5(Q,\xi) = \beta_2\bar{F}_2(Q\xi-x^o) - \beta_1\bar{F}_1(x^o+Q(1-\xi)) \leq \Phi. \tag{C5'}$$

From Figure 3, it can be seen that if $C_4(Q,\xi) = (1-\beta_2)\bar{F}_2(Q\xi) - (1-\beta_1) \geq 0$, condition (C5') is no longer needed. We define $C_5(Q,\xi) = 0$ if $C_4(Q,\xi) \geq 0$, and as in (C5') otherwise. So, (C5') is automatically if $C_4(Q,\xi) \geq 0$ satisfied.

We now show that under conditions (C1'), (C2') and (C5'), the solution of Case (ii) is indeed optimal.

By complementary slackness, we have $\omega_1^x = \omega_2^x = \omega_1^y = 0$ and $\omega_2^y \geq 0$. Together with

[22] An alternative and indirect approach to prove Case (ii) is to show by contradiction that any solution form other than $\{x_1^* > 0, x_2^* > 0, y_1^* > 0, y_2^* = 0\}$ and those established in Cases (i), (iii),(iv) and (v) must violate the optimality conditions. Here, we present a direct approach to verify the proposed solution meets the optimality conditions.

$y_1^* = Q(1-\xi)$,$y_2^* = 0$ and $x_2^* = Q\xi - x_1^*$, the optimality condition (C.6) reduces to

$$\bar{F}_2(Q\xi - x_1^*) - \beta_1 \bar{F}_1(x_1^* + Q(1-\xi)) - (1-\beta_1)\bar{F}_1(x_1^*) = \Phi \tag{C.13}$$

while the optimality condition (C.7) reduces to

$$\beta_2 \bar{F}_2(Q\xi - x_1^*) - \beta_1 \bar{F}_1(x_1^* + Q(1-\xi)) \leq \Phi. \tag{C.14}$$

As the left-hand side of (C.13) is increasing in x_1^*, there exists an $x_1^* = x^\dagger \in (0, Q\xi)$ such that

$$\bar{F}_2(Q\xi - x^\dagger) - \beta_1 \bar{F}_1(x^\dagger + Q(1-\xi)) - (1-\beta_1)\bar{F}_1(x^\dagger) = \Phi \tag{C.15}$$

iff $\bar{F}_2(Q\xi - x_1) - \beta_1 \bar{F}_1(x_1 + Q(1-\xi)) - (1-\beta_1)\bar{F}_1(x_1) < \Phi$ for $x_1 = 0$ and $\bar{F}_2(Q\xi - x_1) - \beta_1 \bar{F}_1(x_1 + Q(1-\xi)) - (1-\beta_1)\bar{F}_1(x_1) > \Phi$ for $x_1 = Q\xi$, respectively, which are equivalent to conditions (C2') and (C1').

It remains to prove that $x_1^* = x^\dagger$ also satisfies (C.14). This needs to be done by analyzing three subcases:

- **Subcase 1:** $\beta_2 - \beta_1 \bar{F}_1(Q) \leq \Phi$. Note that with $x_1^* = x^\dagger$, (C.14) can be written as $\beta_2 \bar{F}_2(Q\xi - x^\dagger) - \beta_1 \bar{F}_1(x^\dagger + Q(1-\xi)) \leq \Phi$. The left-hand side is increasing in $x_1^\dagger$, and so $\beta_2 \bar{F}_2(Q\xi - x_1^\dagger) - \beta_1 \bar{F}_1(x_1^\dagger + Q(1-\xi))$ is upper bounded by $\beta_2 - \beta_1 \bar{F}_1(Q)$ (i.e., its value when $x_1^\dagger = Q\xi$). So, in the case of $\beta_2 - \beta_1 \bar{F}_1(Q) \leq \Phi$, (C.14) holds for any possible value of $x_1^\dagger$.

- **Subcase 2:** $\beta_2 - \beta_1 \bar{F}_1(Q) > \Phi$ and $(1-\beta_2)\bar{F}_2(Q\xi) \geq 1 - \beta_1$. Note that for any $x \in [0, Q\xi]$, we have

  $$(1-\beta_1)\bar{F}_1(x) - (1-\beta_2)\bar{F}_2(Q\xi - x) \leq (1-\beta_1) - (1-\beta_2)\bar{F}_2(Q\xi) \leq 0$$

  where the first inequality follows since $(1-\beta_1)\bar{F}_1(x) - (1-\beta_2)\bar{F}_2(Q\xi - x)$ is decreasing in x and the second inequality holds in the case of $(1-\beta_2)\bar{F}_2(Q\xi) \geq 1 - \beta_1$. Therefore, the above inequality holds for $x^\dagger$, i.e., $(1-\beta_1)\bar{F}_1(x^\dagger) - (1-\beta_2)\bar{F}_2(Q\xi - x^\dagger) \leq 0$. Plugging this into (C.15), we have $\beta_2 \bar{F}_2(Q\xi - x^\dagger) - \beta_1 \bar{F}_1(x^\dagger + Q(1-\xi)) \leq \Phi$, which proves (C.14) satisfied by $x_1^* = x^\dagger$ and $x_2^* = Q\xi - x^\dagger$.

- **Subcase 3:** $\beta_2 - \beta_1 \bar{F}_1(Q) > \Phi$ and $(1-\beta_2)\bar{F}_2(Q\xi) < 1 - \beta_1$. Recall that $(1-\beta_2)\bar{F}_2(Q\xi) < 1 - \beta_1$, i.e., (C4'), guarantees that an $x^o \in (0, Q\xi)$ exists such that $(1-\beta_2)\bar{F}_2(Q\xi - x^o) = (1-\beta_1)\bar{F}_1(x^o)$.

First, suppose $x^o \geq x^\dagger$. Then, it follows that $\beta_2 \bar{F}_2(Q\xi - x^o) - \beta_1 \bar{F}_1(x^o + Q(1-\xi)) \geq \beta_2 \bar{F}_2(Q\xi - x^\dagger) - \beta_1 \bar{F}_1(x^\dagger + Q(1-\xi))$ by the monotonicity of $\bar{F}_i$. Moreover, using condition (C5'), we obtain $\Phi \geq \beta_2 \bar{F}_2(Q\xi - x^\dagger) - \beta_1 \bar{F}_1(x^\dagger + Q(1-\xi))$, i.e., (C.14) holds for $x_1^* = x^\dagger$.

Alternatively, suppose $x^o < x^\dagger$. By the definition of x^o and the monotone property of $\bar{F}_i$, it follows that $(1-\beta_2)\bar{F}_2(Q\xi - x^\dagger) > (1-\beta_1)\bar{F}_1(x^\dagger)$. Substituting this inequality into (C.15) yields $\Phi = \bar{F}_2(Q\xi - x^\dagger) - \beta_1 \bar{F}_1(x^\dagger + Q(1-\xi)) - (1-\beta_1)\bar{F}_1(x^\dagger) > \beta_2 \bar{F}_2(Q\xi - x^\dagger) - \beta_1 \bar{F}_1(x^\dagger + Q(1-\xi))$, that is, (C.14) holds for $x_1^* = x^\dagger$.

In summary, under conditions (C1') (C2') and (C5'), optimality conditions (C.13) (C.14) hold for the solution described Case (ii). This completes the proof. □

Proof of Corollary 1. Consider any solution in Case (v). Substituting $(1-\beta_1)\bar{F}_1(x^o) = (1-\beta_2)\bar{F}_2(Q\xi - x^o)$ into the condition $\beta_2 \bar{F}_2(Q\xi - x^o) - \beta_1 \bar{F}_1(x^o + Q(1-\xi)) > \Phi$, we have $\frac{\beta_2(1-\beta_1)}{1-\beta_2}\bar{F}_1(x^o) - \beta_1 \bar{F}_1(x^o + Q(1-\xi)) > \Phi$. Clearly, this cannot hold if $\beta_2(1-\beta_1)/(1-\beta_2) \leq \Phi$. □

Proof of Proposition 2. With Proposition 1 and (C.1)-(C.2) in the KKT conditions, it is straightforward to find the closed-form expressions for dual variables λ_x^* and λ_y^* and thereby derive the expressions of $\frac{\partial R(Q,\xi)}{\partial Q}$. Note that there can be alternative expressions for the derivative when more than two decision variables among x_1^*, x_2^*, y_1^* and y_2^* are strictly positive in the optimal solution.

To show that $r(Q)$ is decreasing, we apply the preservation of concavity under maximization. Note that $R(Q,\xi)$ is concave in Q because concavity is preserved under maximization; see, e.g., section 3.2.5 and example 3.17 of Boyd et al. (2004) for a proof of preservation of convexity under minimization. As a result, $E_\xi[R(Q,\xi)]$ is concave in Q since concavity is preserved under expectation. Therefore, $\frac{\partial E_\xi[R(Q,\xi)]}{\partial Q}$ is decreasing in Q, and so is $r(Q)$ by the Leibniz integral rule. □

Proof of Proposition 3. Because $\beta_1 = 1$ in the special case, in the proof we will write β_2 simply as β.

Without Monitoring. In this special case, $z_1 = 1$ with probability one, and

$$z_2 = \begin{cases} 1 \text{ with probability } \beta, \\ \xi \text{ with probability } 1-\beta. \end{cases}$$

With Proposition B.1 and the uniform demand distribution, it is easy to show that $q_1^N = 2\mu\phi_1$

and

$$q_2^N = \begin{cases} 0 & \text{if } \phi_2 \le (1-\xi)(1-\beta) \\ \frac{2\mu(\phi_2-(1-\xi)(1-\beta))}{\beta+\xi^2(1-\beta)} & \text{if } (1-\xi)(1-\beta) < \phi_2 \le 1-\xi(1-\xi)(1-\beta) \\ \frac{2\mu(\phi_2+\xi(1-\beta)-1)}{\xi^2(1-\beta)} & \text{otherwise.} \end{cases}$$

With Monitoring.

We derive the first-order derivative $r(Q) = \frac{\partial R(Q,\xi)}{\partial Q}$ based on Proposition 2. To do so, we first examine the optimal second-stage allocation given any Q and each realization of ξ.

Given any ξ, the thresholds $C_i(Q,\xi)$ reduce to the following: $C_1(Q,\xi) = F(Q)$, $C_2(Q,\xi) = F(Q(1-\xi)) - F(Q\xi)$, $C_3(Q,\xi) = \beta\bar{F}(Q\xi) - \bar{F}(Q(1-\xi))$, $C_4(Q,\xi) = (1-\beta)\bar{F}(Q\xi)$, and $C_5(Q,\xi) = 0$.

Case of $\xi \ge \frac{1}{2}$. For $\xi \ge 1/2$, $C_2(Q,u) \le 0$ and $C_3(Q,u) \le 0$ and $C_5(Q,\xi) = 0$, so only cases (i) and (ii) can occur. If $F(Q) \le \Phi \iff Q \le 2\mu\Phi$, case (i) occurs in which, by Proposition 2, $\frac{\partial R(Q,\xi)}{\partial Q} = p - c_1 - (p+c_d)F(Q)$. If $F(Q) > \Phi \iff Q > 2\mu\Phi$, case (ii) occurs in which, by Proposition 1, $x^\dagger$ is determined by $F(x^\dagger + Q(1-\xi)) - F(Q\xi - x^\dagger) = \Phi$. Substituting the uniform distribution function F, we have

$$x^\dagger = \begin{cases} Q(\xi - \frac{1}{2}) + \mu\Phi \text{ if } Q \le 2\mu(2-\Phi), \\ Q\xi - 2\mu(1-\Phi) \text{ if } Q > 2\mu(2-\Phi). \end{cases}$$

Substituting $x^\dagger$ into $\frac{\partial R(Q,\xi)}{\partial Q}$ derived in Proposition 2 and combining cases (i) and (ii), we obtain

$$\frac{\partial R(Q,\xi)}{\partial Q} = \begin{cases} p - c_1 - (p+c_d)F(Q) & \text{if } Q \le 2\mu\Phi \\ p - c_1 - (p+c_d)F(\frac{Q}{2} + \mu\Phi) & \text{if } 2\mu\Phi < Q \le 2\mu(2-\Phi) \\ p - c_1 - (p+c_d)F(Q - 2\mu(1-\Phi)) & \text{otherwise.} \end{cases} \tag{C.16}$$

Define $\phi_i = \frac{p-c_i-c_o}{p+c_d}$. Because $\frac{\partial R(Q,\xi)}{\partial Q}$ is piecewise decreasing in Q, the optimal Q^I can be determined by solving $\frac{\partial R(Q,\xi)}{\partial Q} = c_o$. For the first case of (C.16), we have the interior solution as $Q^I = \frac{2\mu(p-c_1-c_o)}{p+c_d} = 2\mu\phi_1$, which is attained if $Q^I \le 2\mu\Phi$, or equivalently, $\phi_2 \le 0$ since $\Phi = \phi_1 - \phi_2$. For the second case of (C.16), we have $Q^I = 2\mu\left(\frac{2p-c_1-c_2-2c_o}{p+c_d}\right) = 2\mu(\phi_1+\phi_2)$, which satisfies $2\mu\Phi < Q^I \le 4\mu - 2\mu\Phi$. For the third case, the condition implies $\frac{\partial R(Q,\xi)}{\partial Q} = -c_1 - c_d < 0$ and thus will not permit an optimal Q. The optimal order quantity can be summarized as follows.

$$Q^I = \begin{cases} 2\mu\phi_1 & \text{if } \phi_2 \le 0, \\ 2\mu(\phi_1+\phi_2) & \text{if } \phi_2 > 0. \end{cases} \tag{C.17}$$

The resulting allocation quantities follow immediately by Proposition 1. If $\phi_2 \le 0$, allocate all the units to market 1; if $\phi_2 > 0$, allocate the units according to Case (ii) of Proposition 1, i.e., $x_1^* = x^\dagger = 2\mu(\phi_1+\phi_2)\xi - 2\mu\phi_2$, $x_2^* = 2\mu\phi_2$, $y_1^* = 2\mu(\phi_1+\phi_2)(1-\xi)$ and $y_2^* = 0$. As a result, the allocated quantity to market 1 is $x_1^* + y_1^* = 2\mu\phi_1$, and that to market 2 is $x_2^* + y_2^* = 2\mu\phi_2$.

Case of $\xi < \frac{1}{2}$. We now consider the case of $\xi < \frac{1}{2}$, which is more involved. With thresholds $C_1(Q,\xi) = F(Q)$, $C_2(Q,\xi) = F(Q(1-\xi)) - F(Q\xi)$, $C_3(Q,\xi) = \beta\bar{F}(Q\xi) - \bar{F}(Q(1-\xi))$, $C_4(Q,\xi) = (1-\beta)\bar{F}(Q\xi)$, and $C_5(Q,\xi) = 0$, cases (iii) and (iv) of the optimal allocation policy become possible.

In the following, we derive the optimal Q^I according to which allocation case it results in. By Proposition 2, since the first-order derivative $r(Q)$ is monotone, it suffices to find a solution Q^I solving $r(Q) = c_o$ and identify the conditions such that this solution Q^I is achievable within each case.

Case (i) Suppose that the optimal Q^I is attained under case (i). By Proposition 2, it must solve the first-order condition: $F(Q^I) = \phi_1$, while satisfying $C_1(Q^I,\xi) = F(Q^I) \le \Phi$. Thus, $Q^I = 2\mu\phi_1$ and allocate all units to market 1 if $\phi_1 - \Phi = \phi_2 \le 0$.

Case (ii) Analogous to the earlier analysis for the case of $\xi \ge \frac{1}{2}$, we have $Q^I = 2\mu(\phi_1+\phi_2)$, and $x_1^* = 2\mu(\phi_1+\phi_2)\xi - 2\mu\phi_2$, $x_2^* = 2\mu\phi_2$, $y_1^* = 2\mu(\phi_1+\phi_2)(1-\xi)$, and $y_2^* = 0$. The Q^I is achievable in Case (ii) if $C_2(Q^I,\xi) < \Phi < C_1(Q^I,\xi)$ and $C_5(Q^I,\xi) \le \Phi$. The latter condition trivially holds under the special case of $\beta_1 = 1$. Invoking $Q^I = 2\mu(\phi_1+\phi_2)$, we have $C_1(Q^I,\xi) = \min\{1, \phi_1+\phi_2\}$ and

$$C_2(Q^I,\xi) = \begin{cases} (1-2\xi)(\phi_1+\phi_2) & \text{if } (\phi_1+\phi_2)(1-\xi) \le 1, \\ 1-(\phi_1+\phi_2)\xi & \text{if } (\phi_1+\phi_2)\xi \le 1 < (\phi_1+\phi_2)(1-\xi), \\ 0 & \text{if } 1 < (\phi_1+\phi_2)\xi. \end{cases}$$

Thus, $C_1(Q^I,\xi) > \Phi = \phi_1 - \phi_2$ always holds, and $C_2(Q^I,\xi) < \Phi \iff$

$$\phi_1 + \phi_2 > \tfrac{1}{\xi} \tag{C.18}$$

$$\text{or} \quad \tfrac{1}{1-\xi} < \phi_1+\phi_2 \le \tfrac{1}{\xi} \text{ and } (1+\xi)\phi_1 - (1-\xi)\phi_2 > 1 \tag{C.19}$$

$$\text{or} \quad \phi_1+\phi_2 \le \tfrac{1}{1-\xi} \text{ and } \xi\phi_1 > (1-\xi)\phi_2. \tag{C.20}$$

Note that (C.18) never holds for $\xi < \frac{1}{2}$. Moreover, since $\phi_1 < 1$, $\frac{1}{1-\xi} < \phi_1+\phi_2$ implies $\frac{\xi}{1-\xi} < \phi_2$ whereas $(1+\xi)\phi_1 - (1-\xi)\phi_2 > 1$ implies $\frac{\xi}{1-\xi} > \phi_2$. So, (C.19) amounts to an empty set. Therefore, the condition for case (ii) to be optimal reduces to

$$\phi_1 + \phi_2 \le \frac{1}{1-\xi} \text{ and } \xi\phi_1 > (1-\xi)\phi_2. \tag{C.21}$$

Case (iii) Suppose that the optimal Q^I leads to case (iii). Then, solving the first-order condition $r(Q) = c_o$, we obtain

$$Q^I = \begin{cases} 2\mu \cdot \frac{(1-\xi)\phi_1 + \xi\phi_2}{\xi^2 + (1-\xi)^2} & \text{if } (1-\xi)^2(1-\phi_1) + \xi^2 \ge (1-\xi)\xi\phi_2 \\ 2\mu \cdot \frac{\xi\phi_2 - (1-\xi)(1-\phi_1)}{\xi^2} & \text{if } (1-\xi)^2(1-\phi_1) + \xi^2 < (1-\xi)\xi\phi_2 \end{cases} \tag{C.22}$$

where $Q^I\xi < Q^I(1-\xi) \le 2\mu$ in the first case and $Q^I\xi < 2\mu < Q^I(1-\xi)$ in the second case. For the Q^I to be achievable, we need $C_3(Q^I, \xi) \le \Phi \le C_2(Q^I, \xi)$ and recall that $\Phi = \phi_1 - \phi_2$.

Case (iii-a). For the case of $(1-\xi)^2(1-\phi_1)+\xi^2 \ge (1-\xi)\xi\phi_2$, we have $\Phi \le C_2(Q^I, \xi) \iff \xi\phi_1 \le (1-\xi)\phi_2$, and

$$C_3(Q^I, \xi) \le \Phi \iff -(1-\beta)(1-2\xi+2\xi^2) - (\beta(1-\xi)+\xi)\xi\phi_1 + (1-\xi+(1-\beta)\xi^2)\phi_2 \le 0.$$

Case (iii-b). For the case of $(1-\xi)^2(1-\phi_1) + \xi^2 < (1-\xi)\xi\phi_2$, we have

$$\Phi \le C_2(Q^I, \xi) \iff (1-2\xi+2\xi^2) - (1-\xi+\xi^2)\phi_1 + (1-\xi)^2\phi_2 \ge 0 \tag{C.23}$$

and

$$C_3(Q^I, \xi) \le \Phi \iff -(\beta - \beta\xi + \xi)\phi_1 + (1-\beta)\xi\phi_2 + \beta \le 0. \tag{C.24}$$

Note that since $\phi_1 < 1$, $(1-\xi)^2(1-\phi_1) + \xi^2 < (1-\xi)\xi\phi_2 \iff 1 - 2\xi + 2\xi^2 < (1-\xi)^2\phi_1 + (1-\xi)\xi\phi_2 \Rightarrow \frac{\xi}{1-\xi} < \phi_2$. Substituting $\frac{\xi}{1-\xi} < \phi_2$, one can show that (C.23) is implied by $(1-\xi)^2(1-\phi_1) + \xi^2 < (1-\xi)\xi\phi_2$.

Altogether, case (iii) is optimal with two possible order quantities: (iii-a) $Q^I = 2\mu \cdot \frac{(1-\xi)\phi_1 + \xi\phi_2}{\xi^2 + (1-\xi)^2}$ if

$$1 - 2\xi + 2\xi^2 \ge (1-\xi)^2\phi_1 + (1-\xi)\xi\phi_2 \text{ and } \xi\phi_1 \le (1-\xi)\phi_2 \tag{C.25}$$

and (iii-b) $Q^I = 2\mu \cdot \frac{\xi\phi_2 - (1-\xi)(1-\phi_1)}{\xi^2}$ if

$$1 - 2\xi + 2\xi^2 > (1-\xi)^2\phi_1 + (1-\xi)\xi\phi_2 \text{ and } -(\beta - \beta\xi + \xi)\phi_1 + (1-\beta)\xi\phi_2 + \beta \le 0. \tag{C.26}$$

Case (iv) We first solve $\beta\bar{F}(Q-y^\dagger)-\bar{F}(y^\dagger)=\Phi$ to obtain $y^\dagger$. There are two possible cases, depending on whether $y^\dagger \le 2\mu$.

$$y^\dagger = \begin{cases} \frac{\beta Q+2\mu(1+\Phi-\beta)}{1+\beta} & \text{if } Q \le 2\mu(2-\frac{\Phi}{\beta}), \\ \frac{\beta Q+2\mu(\Phi-\beta)}{\beta} & \text{if } Q > 2\mu(2-\frac{\Phi}{\beta}). \end{cases}$$

where $y^\dagger \le 2\mu$ in the first case and $y^\dagger > 2\mu$ in the second case.

Moreover, case (iv) is achievable if $C_3(Q,\xi) > \Phi$ and $C_4(Q,\xi) \ge 0$. The latter inequality is automatically satisfied given $\beta_1 = 1$. It suffices to consider the former condition $C_3(Q,\xi) > \Phi$.

Note that $C_3(Q,\xi) > \Phi$ implies $y^\dagger < Q(1-\xi)$. Thus, $Q - y^\dagger > Q\xi$. Consequently, when analyzing the first-order condition, $\frac{\partial R(Q,\xi)}{\partial Q} = p - c_2 - (p+c_d)[1-\beta\bar{F}(Q-y^\dagger) - \xi(1-\beta)\bar{F}(Q\xi)] = c_o$, we have two possible solutions: $Q\xi < Q - y^\dagger \le 2\mu$ and $Q\xi < 2\mu < Q - y^\dagger$. On the other hand, since $y^\dagger$ solves $\beta\bar{F}(Q-y^\dagger) - \bar{F}(y^\dagger) = \Phi$. It must be the case that $Q - y^\dagger < 2\mu$; otherwise, there is no solution. Therefore, only the case of $Q\xi < Q - y^\dagger \le 2\mu$ is possible.

Then, we can substitute $\bar{F}(x) = 1 - \frac{x}{2\mu}$ to simplify the first-order condition.

- **Case (iv-a).** For the case of $y^\dagger \le 2\mu$, solving the first-order condition yields $Q^I = \frac{2\mu(\beta\phi_1+\phi_2-1+\xi+\beta-\beta^2\xi)}{\beta+\xi^2-\beta^2\xi^2}$. For this solution to be achieved, we need

$$\begin{aligned} y^\dagger \le 2\mu &\iff Q^I \le 2\mu(2-\frac{\Phi}{\beta}) \\ &\iff (\beta+(1-\beta)\xi^2)\phi_1 \le (1-\beta)\xi^2\phi_2 + \beta(1-\xi+\beta\xi+2(1-\beta)\xi^2), \end{aligned}$$

and $C_3(Q^I,\xi) > \Phi$, which simplifies to

$$\begin{aligned} &\text{if } Q^I(1-\xi) \le 2\mu \iff (1-\xi)(\beta\phi_1+\phi_2) \le 1-(2-\beta-\beta^2)\xi + 2(1-\beta^2)\xi^2, \\ &C_3(Q^I,\xi) > \Phi \iff (1-\xi+(1-\beta)\xi^2)\phi_2 > (\beta\xi+(1-\beta)\xi^2)\phi_1 + (1-\beta)(1-2\xi+2\xi^2); \\ &\text{if } Q^I(1-\xi) > 2\mu \iff (1-\xi)(\beta\phi_1+\phi_2) > 1-(2-\beta-\beta^2)\xi + 2(1-\beta^2)\xi^2, \\ &C_3(Q^I,\xi) > \Phi \iff \phi_1\left(\beta^2(1-\xi)\xi+\beta+\xi^2\right) + \phi_2\left(\beta^2\xi^2+\beta(\xi-1)-\xi^2\right) + \beta(\beta(\xi-1)-\xi) < 0 \end{aligned}$$

Putting together the above inequalities on the ϕ_1-ϕ_2 coordinate plane, we find that all the inequalities intersect at one point

$$(\phi_1,\phi_2) = \left(\frac{1-2\xi+2\xi^2+\beta\xi(1-2\xi)}{1-\xi}, \frac{1-2\xi+2\xi^2-\beta(1-\xi)(1-2\xi)}{1-\xi}\right).$$

With some algebra, we can simplify the set of inequalities to

$$\begin{aligned}&(\beta+(1-\beta)\xi^2)\phi_1 \le (1-\beta)\xi^2\phi_2+\beta(1-\xi+\beta\xi+2(1-\beta)\xi^2) \text{ and}\\ &(1-\xi+(1-\beta)\xi^2)\phi_2 > (\beta\xi+(1-\beta)\xi^2)\phi_1+(1-\beta)(1-2\xi+2\xi^2)\end{aligned} \tag{C.27}$$

– **Case (iv-b).** For the case of $y^\dagger > 2\mu$, the first-order condition leads to $Q^I = \frac{2\mu((1-\beta)\xi+\phi_1-1)}{(1-\beta)\xi^2}$. For this solution to be achievable, we have

$$\begin{aligned}y^\dagger > 2\mu \iff & Q^I > 2\mu(2-\frac{\Phi}{\beta})\\ \iff & (\beta+(1-\beta)\xi^2)\phi_1 > (1-\beta)\xi^2\phi_2+\beta(1-\xi+\beta\xi+2(1-\beta)\xi^2),\end{aligned}$$

and $C_3(Q^I,\xi) > \Phi$. To simplify $C_3(Q^I,\xi)$, first note that $C_3(Q^I,\xi) > \Phi$ implies $y^\dagger \le Q(1-\xi)$ and given $y^\dagger > 2\mu$, it further implies $Q(1-\xi) > 2\mu$. Hence,

$$C_3(Q^I,\xi) > \Phi \iff -(\beta-\beta\xi+\xi)\phi_1+(1-\beta)\xi\phi_2+\beta > 0.$$

To simplify the notation, define $A_1(\phi_1) = \frac{\xi}{1-\xi}\phi_1$, $A_2(\phi_1) = -\frac{1-\xi}{\xi}\phi_1+\frac{1-2\xi+2\xi^2}{\xi-\xi^2}$, $A_3(\phi_1) = \frac{(\beta\xi+(1-\beta)\xi^2)\phi_1+(1-\beta)(1-2\xi+2\xi^2)}{1-\xi+(1-\beta)\xi^2}$, $A_4(\phi_1) = \frac{(\beta+(1-\beta)\xi^2)\phi_1-\beta(1-(1-\beta)\xi+2(1-\beta)\xi^2)}{(1-\beta)\xi^2}$ and $A_5(\phi_1) = \frac{(\beta+(1-\beta)\xi)\phi_1-\beta}{(1-\beta)\xi}$. With the above notation, we can summarize the conditions for each case as presented in Proposition 3 and illustrated in Figure 4. □

Proof of Proposition 4. Part (i) We examine the sign of $Q^N - Q^I$ for different values of ϕ_2.

For $\phi_2 \le 0$, $Q^N = Q^I$. For $0 < \phi_2 \le (1-\beta)(1-\xi)$, $Q^N - Q^I = -2\mu\phi_2 < 0$. For $(1-\beta)(1-\xi) < \phi_2 \le 1-\xi(1-\xi)\beta$, we have $\frac{\partial(Q^N-Q^I)}{\partial\phi_2} = 2\mu\left(\frac{1}{\beta(1-\xi^2)+\xi^2}-1\right) > 0$ and so $Q^N - Q^I$ is increasing in ϕ_2. Moreover, $Q^N < Q^I$ iff $\phi_2 < \frac{1}{1+\xi}$. For $\phi_2 > 1-\xi(1-\xi)\beta$, we have $\frac{\partial(Q^N-Q^I)}{\partial\phi_2} = 2\mu\left(\frac{1}{\xi^2(1-\beta)}-1\right) > 0$, and so $Q^N - Q^I$ is increasing in ϕ_2. Since $Q^N - Q^I$ is continuous in ϕ_2, its monotone property implies that over the entire range of ϕ_2, $Q^N < Q^I$ iff $\phi_2 < \frac{1}{1+\xi}$, or equivalently, $p < \frac{(c_2+c_o)(1+\xi)+c_d}{\xi}$.

Part (ii) Given any salable quantity x, the expected leftover in each market can be written as $E[(x-D)^+] = \int_0^x F(u)du$. Hence, for the special case of $\beta_1 = 1$ and $\beta_2 = \beta$, we have

$$W^I = \int_0^{x_1^*+y_1^*} F(u)du + \beta\int_0^{x_2^*+y_2^*} F(u)du + (1-\beta)\left(\int_0^{x_2^*} F(u)du + y_2^*\right)$$

and

$$W^N = \int_0^{q_1^N} F(u)du + \beta\int_0^{q_2^N} F(u)du + (1-\beta)\left(\int_0^{\xi q_2^N} F(u)du + (1-\xi)q_2^N\right)$$

where $F(u) = \frac{u}{2\mu}$ if $u \le 2\mu$ and $F(u) = 1$ if $u > 2\mu$.

For $\phi_2 \le 0$, it is easy to see that $W^I = W^N$ since the same quantity is sent to market 1 and nothing is sent to market 2 for both cases. For $\phi_2 > 0$, we have $x_1^* + y_1^* = 2\mu\phi_1$, $x_2^* = 2\mu\phi_2$, and $y_2^* = 0$. Hence, $x_1^* + y_1^* = q_1^N$, and so the first terms of W^I and W^N cancel out when comparing W^I and W^N. It suffices to compare the expected waste in market 2, which depends on the value of q_2^N. Now consider the three cases of q_2^N as given in (8).

Case 1. For $0 < \phi_2 \le (1-\beta)(1-\xi)$, without monitoring, market 1 receives $q_1^N = 2\mu\phi_1$ and market 2 receives $q_2^N = 0$. With monitoring, market 1 receives the same quantity $x_1^* + y_1^* = 2\mu\phi_1$, whereas market 2 receives $x_2^* = 2\mu\phi_2$ units of good items and $y_2^* = 0$ units of risky items. It follows that $W^I - W^N = \mu\phi_2^2 > 0$.

Case 2. For $(1-\beta)(1-\xi) < \phi_2 \le 1 - \xi(1-\xi)(1-\beta)$, substituting the corresponding expression of q_2^N, we have

$$W^I - W^N = \frac{\mu(1-\beta)(1-\xi)((1-\xi)(1-\beta) - (1+\xi)\phi_2^2)}{\xi^2 + \beta(1-\xi^2)}.$$

It follows that $W^I - W^N$ is decreasing in ϕ_2, and $W^I > W^N$ iff $\phi_2 < \sqrt{\frac{(1-\xi)(1-\beta)}{1+\xi}}$.

Case 3. For $\phi_2 > 1 - \xi(1-\xi)(1-\beta)$, notice that $q_2^N \ge 2\mu > x_2^*$. For this case, we prove that $q_2^N \ge x_2^*$ implies $W^I \le W^N$ by establishing the following lemma.

Lemma C.1. *For any nonnegative real numbers x, y, z, we have $(x-y)^+ + z \ge (x-y+z)^+$.*

Proof of Lemma C.1. Note that

$$(x-y)^+ + z = \begin{cases} z & \text{if } x \le y \\ x - y + z & \text{if } x > y. \end{cases}$$

For $x \le y$, $z \ge (x-y+z)^+$. For $x > y$, $x - y + z = (x-y+z)^+$. Hence, the desired inequality holds. $\square$

By Lemma C.1, it follows that $(z_2 q_2^N - D_2)^+ + (1-z_2)q_2^N \ge (q_2^N - D_2)^+$ for any $0 \le z_2 \le 1$. Taking expectation on both sides, we obtain a lower bound of W^N. That is, $W^N \ge E[(q_1^N - D_1)^+] + E[(q_2^N - D_2)^+]$. Since $q_1^N = x_1^* + y_1^*$, $x_2^* < q_2^N$ and $y_2^* = 0$, the lower bound $E[(q_1^N - D_1)^+] + E[(q_2^N - D_2)^+] \ge W^I$, and so $W^N \ge W^I$.

Since W^I and W^N are continuous in ϕ_2, putting together the above cases, we conclude that $W^I > W^N$ iff $\phi_2 < \hat{\phi} := \sqrt{\frac{(1-\xi)(1-\beta)}{1+\xi}}$. Using $\phi_2 = \frac{p - c_2 - c_o}{p + c_d}$, the condition is equivalent to $p < \hat{p} = \frac{c_2 + c_o + c_d\hat{\phi}}{1-\hat{\phi}}$.

At $\phi_2 = \tilde{\phi}$, we have $Q^N = Q^I$ and $q_2^N = x_2^*$. By Lemma C.1, it follows that in this case, $W^N \geq E[(q_1^N - D_1)^+] + E[(q_2^N - D_2)^+] = W^I$ since $q_1^N = x_1^* + y_1^*$ and $q_2^N = x_2^*$. In other words, $W^I \leq W^N$ at $\phi_2 = \tilde{\phi}$. Moreover, $\tilde{\phi}$ must fall in Case 2 of ϕ_2, because q_2^N exceeds 2μ in Case 3 while at $\phi_2 = \tilde{\phi}$, $q_2^N = 2\mu\phi_2 < 2\mu$. Within Case 2, $W^I - W^N$ is decreasing in ϕ_2 and changes its sign from positive to negative at $\phi_2 = \hat{\phi}_2$. Therefore, we must have $\hat{\phi} \leq \tilde{\phi}$.

Part (iii) As shown above, we have $x_1^* + y_1^* = q_1^N = 2\mu\phi_1$, that is, market 1 receives the same quantity whether or not monitoring technology is adopted. Because of $\beta_1 = 1$, it follows that the waste and sales are unchanged in market 1. Then, the proof of Part (ii) implies $S_2^I \geq S_2^N$, and under the same condition when total waste is increased, $W_2^I > W_2^N$.

Part (iv) We now prove that $\frac{W^I}{Q^I} \leq \frac{W^N}{Q^N}$. Consider the following cases.

(a) For $\phi_2 \leq 0$, clearly, $\frac{W^I}{Q^I} = \frac{W^N}{Q^N}$.

(b) For $0 < \phi_2 \leq (1-\xi)(1-\beta)$, we have $\frac{W^I}{Q^I} = \frac{\phi_1^2+\phi_2^2}{2(\phi_1+\phi_2)}$ and $\frac{W^N}{Q^N} = \frac{\phi_1}{2}$. It follows that $(\frac{W^I}{Q^I})/(\frac{W^N}{Q^N}) = \frac{\phi_1^2+\phi_2^2}{\phi_1^2+\phi_1\phi_2} < 1$ since $\phi_1 > \phi_2$. Thus, $\frac{W^I}{Q^I} < \frac{W^N}{Q^N}$.

(c) For $(1-\xi)(1-\beta) < \phi_2 \leq 1 - \xi(1-\xi)(1-\beta)$, we can write

$$\frac{W^I}{Q^I} - \frac{W^N}{Q^N} = -\frac{(1-\beta)(1-\xi)N}{2(\phi_1+\phi_2)(\phi_2 - (1-\beta)(1-\xi) + \phi_1(1-(1-\beta)(1-\xi^2)))}$$

where $N = -(1-\beta)(1-\xi)(\phi_1+\phi_2) + \phi_1^2(1-\phi_2-\xi\phi_2) + \phi_2^2(1+\phi_1+\xi\phi_1)$. Thus, $\frac{W^I}{Q^I} < \frac{W^N}{Q^N}$ iff $N > 0$. Note that N is decreasing $1-\beta$, and within the focal case, $1-\beta < \frac{\phi_2}{1-\xi}$ and $1-\beta \leq \frac{1-\phi_2}{\xi(1-\xi)}$.

- If $\frac{\phi_2}{1-\xi} \leq \frac{1-\phi_2}{\xi(1-\xi)}$ ($\iff$ $\phi_2 \leq \frac{1}{1+\xi}$), then $1-\beta$ is bounded above by $\frac{\phi_2}{1-\xi}$. As a result, $N > \phi_1(\phi_1-\phi_2)(1-(1+\xi)\phi_2) \geq 0$.
- Otherwise, if $\frac{\phi_2}{1-\xi} > \frac{1-\phi_2}{\xi(1-\xi)}$ ($\iff$ $\phi_2 > \frac{1}{1+\xi}$), then $1-\beta$ is bounded above by $\frac{1-\phi_2}{\xi(1-\xi)}$. As a result, $N \geq \frac{1}{\xi}((1+\xi)\phi_2 - 1)(\phi_1(1-\xi\phi_1) + \phi_2(1+\xi\phi_1)) > 0$.

In both cases, N is positive and hence $\frac{W^I}{Q^I} < \frac{W^N}{Q^N}$.

(d) For $\phi_2 > 1 - \xi(1-\xi)(1-\beta)$, $q_2^N \geq 2\mu$. As in the proof of Part (ii), by Lemma C.1, it follows that

$$\frac{W^N}{Q^N} \geq B = \frac{E[(q_1^N - D_1)^+] + E[(q_2^N - D_2)^+]}{q_1^N + q_2^N} = \frac{E[(q_1^N - D_1)^+] + q_2^N - \mu}{q_1^N + q_2^N}$$

where the last inequality follows because q_2^N exceeds the support of D_2. Note that B is increasing in q_2^N while q_2^N is decreasing in β. Thus, the smallest value of B, denoted by $\underline{B}$, occurs at the largest β within the focal case, that is, at $\beta = 1 - \frac{1-\phi_2}{\xi(1-\xi)}$. Hence, $\frac{W^I}{Q^I} - \frac{W^N}{Q^N} \leq \frac{W^I}{Q^I} - \underline{B}$. Because $\frac{W^I}{Q^I} = \frac{\phi_1^2+\phi_2^2}{2(\phi_1+\phi_2)}$ for both cases (c) and (d), by continuity, $\frac{W^I}{Q^I} - \underline{B} < 0$ at $\beta = 1 - \frac{1-\phi_2}{\xi(1-\xi)}$ has been established in case (c). Thus, $\frac{W^I}{Q^I} < \frac{W^N}{Q^N}$. □

Proof of Proposition 5. Proposition 3 does not apply to this special case since demands D_1 and D_2 are not identically distributed. In what follows, we first characterize the optimal shipping quantities (without and with monitoring) as stated in the proposition, and then prove statements (i) - (iv).

Optimal Shipping Quantities.

Without monitoring, given a deterministic demand $D_1 = \mu$, the expected profit from market 1 is piece-wise linear and concave in q_1. It is easy to show that the optimal shipping quantity is given by

$$q_1^N = \begin{cases} 0 \text{ if } \phi_1 \leq (1-\xi)(1-\beta_1), \\ \mu \text{ if } (1-\xi)(1-\beta_1) < \phi_1 \leq (1-\xi)(1-\beta_1)+\beta_1, \\ \frac{\mu}{\xi} \text{ if } \phi_1 > (1-\xi)(1-\beta_1)+\beta_1. \end{cases} \tag{C.28}$$

For market 2, the optimal shipping quantity q_2^N is as derived in Proposition 3. Given $(1-\xi)(1-\beta_i) < \phi_i < (1-\xi)(1-\beta_i)+\beta_i$, the optimal shipping quantities without monitoring are given by $q_1^N = \mu$ and $q_2^N = \frac{2\mu(\phi_2-(1-\xi)(1-\beta_2))}{\beta_2+\xi^2(1-\beta_2)}$.

Next, we consider the case with monitoring. Rather than characterizing all possible cases, we establish the conditions under which the optimal solution follows the same pattern as in allocation policy (ii) of Proposition 1 and the quantity sent to market 1 equals its demand μ. That is, we consider the optimal allocation quantities as

$$\mathbf{x}^* = [\mu - Q^I(1-\xi), Q^I - \mu], \ \mathbf{y}^* = [Q^I(1-\xi), 0]. \tag{C.29}$$

Substituting (C.29) into the objective function, one can reduce the problem to a single-variable maximization and it follows that the optimal purchase quantity Q^I is determined by $F(Q^I - \mu) = \phi_2 - (1-\xi)(1-\beta_1)$. In what follows, we show that the above solution is indeed optimal when ξ is greater than a threshold $\underline{\xi}$.

Because D_1 is deterministic, the objective function may not be differentiable in x_1 or y_1. Thus, we leverage the subdifferentials of the Lagrangian, denoted by ∂L, to verify if the solution satisfies the generalized KKT condition. For any optimal solution with $x_1 + y_1 = \mu$ and $Q > 0$, there must exist $(\mathbf{x}, \mathbf{y}, Q)$ and the corresponding dual variables $\lambda_x, \lambda_y, \omega_i^x, \omega_i^y$ for $i = 1, 2$ that satisfy the KKT conditions below.

$$0 \in \partial L_{x_1} = [p - c_1 - \beta_1(p + c_d) - \lambda_x + \omega_1^x, p - c_1 - \lambda_x + \omega_1^x], \tag{C.30}$$

$$0 \in \partial L_{y_1} = [-c_1 - c_d - \lambda_y + \omega_1^y, p - c_1 - (p + c_d)(1-\beta_1) - \lambda_y + \omega_1^y], \tag{C.31}$$

$$0 = \frac{\partial L}{\partial x_2} = p - c_2 - (p + c_d)[\beta_2 F(x_2 + y_2) + (1 - \beta_2)F(x_2)] - \lambda_x + \omega_2^x, \tag{C.32}$$

$$0 = \frac{\partial L}{\partial y_2} = p - c_2 - (p + c_d)[1 - \beta_2 + \beta_2 F(x_2 + y_2)] - \lambda_y + \omega_2^y, \tag{C.33}$$

$$0 = \frac{\partial L}{\partial Q} = \xi\lambda_x + (1 - \xi)\lambda_y - c_o \tag{C.34}$$

and complementary slackness $x_i\omega_i^x = y_i\omega_i^y = 0$, $\omega_i^x, \omega_i^y \geq 0$ for all $i = 1, 2$.

We verify when the solution given in (C.29) satisfies the above KKT conditions. By complementary slackness, we have $\omega_1^x = \omega_1^y = \omega_2^x = 0$ and $\omega_2^y \geq 0$. Then, (C.30) and (C.31) reduce to

$$p - c_1 - \beta_1(p + c_d) \leq \lambda_x^* \leq p - c_1$$

and

$$-c_1 - c_d \leq \lambda_y^* \leq -c_1 - c_d + \beta_1(p + c_d).$$

And (C.32) and (C.33) simplifies to

$$p - c_2 - (p + c_d)F(x_2^*) = \lambda_x^*$$

and

$$-c_2 - c_d + (p + c_d)\beta_2(1 - F(x_2^*)) \leq \lambda_y^*.$$

Note that some λ_y^* always exists to satisfy the above conditions since $c_2 > c_1$ and $\beta_2 < \beta_1$. To have some λ_x^* that simultaneously satisfies the above conditions, we need $\frac{c_2 - c_1}{p + c_d} \leq \beta_1 - F(x_2^*)$.

Finally, for λ_x^* and λ_y^* to satisfy (C.34), it is required that $\phi_2 \leq (\xi + (1 - \xi)\beta_2)F(x_2^*) + (1 - \xi)(1 - \beta_2)$.

In summary, given $x_2^* = Q^* - \mu$, the following condition needs to be satisfied:

$$\frac{\phi_2 - (1 - \xi)(1 - \beta_2)}{\xi + (1 - \xi)\beta_2} \leq F(Q^* - \mu) \leq \beta_1 - \phi_1 + \phi_2.$$

Substituting $F(Q^I - \mu) = \phi_2 - (1 - \xi)(1 - \beta_1)$, the first inequality holds iff $\phi_2 \leq 1 - \frac{1 - \beta_1}{1 - \beta_2}(\xi + (1 - \xi)\beta_2)$, whereas the second inequality automatically holds under the assumption of $\phi_1 \leq (1 - \xi)(1 - \beta_1) + \beta_1$. Moreover, $x_1^* > 0$ requires $Q^I < \frac{\mu}{1 - \xi}$, or equivalently, $F(Q^I - \mu) < F(\frac{\xi\mu}{1 - \xi}) \iff \phi_2 < (1 - \xi)(1 - \beta_1) + \frac{\xi}{2(1 - \xi)}$.

In summary, the desired solution is indeed optimal if

$$(1 - \xi)(1 - \beta_1) < \phi_1 \leq (1 - \xi)(1 - \beta_1) + \beta_1 \tag{C.35}$$

and

$$(1-\xi)(1-\beta_1) < \phi_2 < \min\left\{1 - \frac{(1-\beta_1)(\xi+(1-\xi)\beta_2)}{1-\beta_2}, (1-\xi)(1-\beta_1) + \frac{\xi}{2(1-\xi)}\right\}. \quad \text{(C.36)}$$

Note that (C.35) and the left inequality of (C.36) are satisfied within the assumed ranges of ϕ_1 and ϕ_2. Rearranging the terms, we have the right inequality of (C.36) holds for any $\phi_2 < (1-\xi)(1-\beta_2) + \beta_2$ iff $\xi \geq \frac{(1-\beta_1)\beta_2}{(1-\beta_2)(\beta_1-\beta_2)}$ and $\xi \geq \frac{1+4\beta_1-2\beta_2-\sqrt{8\beta_1+(1-2\beta_2)^2}}{4(\beta_1-\beta_2)}$. Thus, we can define $\underline{\xi} = \max\left(\frac{(1-\beta_1)\beta_2}{(1-\beta_2)(\beta_1-\beta_2)}, \frac{1+4\beta_1-2\beta_2-\sqrt{8\beta_1+(1-2\beta_2)^2}}{4(\beta_1-\beta_2)}\right)$ and conclude that if $\xi \geq \underline{\xi}$, the desired solution in (C.29) is optimal.

Evaluating the impact of monitoring.

Because the total quantity sent to market 1 remains μ, $Q^I > Q^N$ iff $Q^I - \mu > q_2^N \iff \phi_2 < (1-\beta_1)(1-\xi) + \frac{\beta_1-\beta_2}{(1-\beta_2)(1+\xi)}$. Writing the condition in terms of p proves statement (i).

In the following, we first consider the waste amount and sales in each market to prove statement (iii), then examine the total waste to prove statement (ii), and finally prove statement (iv).

For market 1, it is easy to see that the changes in expected sales and waste are given by $S_1^I - S_1^N = -(1-\xi)(1-\beta_1)x_2^* < 0$ and $W_1^I - W_1^N = (1-\xi)(1-\beta_1)x_2^* > 0$.

For market 2, we define $z = \phi_2 - (1-\xi)(1-\beta_1)$. Under the assumption of $(1-\xi)(1-\beta_2) < \phi_2 < (1-\xi)(1-\beta_2) + \beta_2$, it suffices to focus on the values of z in the range $(1-\xi)(\beta_1-\beta_2) < z < (1-\xi)(\beta_1-\beta_2)+\beta_2$. Furthermore, provided that $\xi \geq \underline{\xi}$, or equivalently, the right inequality of (C.36) holds, z is also upper bounded by $\min(\frac{\beta_1-\beta_2}{1-\beta_2}, \frac{\xi}{2(1-\xi)})$. Within the relevant range of z, absent monitoring, we have $q_2^N = \frac{2\mu(z-(1-\xi)(\beta_1-\beta_2))}{\beta_2+\xi^2(1-\beta_2)}$. With monitoring, we have $x_2^* = 2\mu z$, and $y_2^* = 0$. Invoking $D_2 \sim U[0, 2\mu]$, we obtain $W_2^I = \frac{x_2^{*2}}{4\mu} = \mu z^2$ and $S_2^I = \mu(2z - z^2)$. Furthermore, we can derive $S_2^I - S_2^N = \frac{\mu(1-\xi)}{\xi^2+\beta_2(1-\xi^2)}H(z)$ where $H_s(z) = (1+\xi)(1-\beta_2)z^2 - 2(\beta_1-\beta_2+\xi(1-\beta_2))z + (\beta_1-\beta_2)((1-\xi)(\beta_1+\beta_2)+2\xi)$. It can be verified that $H_s'(z) < 0$ for all $0 < z \leq \frac{\beta_1-\beta_2}{1-\beta_2}$. So, $H_s(z)$ is decreasing in z within the focal parameter range, and consequently, $H_s(z) \geq H_s(\frac{\beta_1-\beta_2}{1-\beta_2}) = \frac{(1-\xi)\beta_2(\beta_1-\beta_2)(2-\beta_1-\beta_2)}{1-\beta_2} > 0$. Therefore, $S_2^I > S_2^N$.

Then, consider the change in expected waste $W_2^I - W_2^N = \frac{\mu(1-\xi)}{\xi^2+\beta_2(1-\xi^2)}H_w(z)$ where $H_w(z) = -(1-\beta_2)(1+\xi)z^2 - 2(1-\beta_1)z + (1-\xi)(\beta_1-\beta_2)(2-\beta_1-\beta_2)$. Note that $H_w'(z) \leq 0$ for all $z > 0$. Thus, $H_w(z)$ and $W_2^I - W_2^N$ are decreasing in z. At the smallest $z = (1-\xi)(\beta_1-\beta_2)$, $H_w(z) = (\beta_1-\beta_2)^2(1-\xi)(\xi^2+\beta_2(1-\xi^2)) > 0$ and thus $W_2^I > W_2^N$. As $W_2^I - W_2^N$ decreases with z, there exists a ζ such that $W_2^I > W_2^N$ iff $z < \zeta$, or equivalently, $\phi_2 < \check{\phi} := \zeta + (1-\xi)(1-\beta_1)$.

Now consider the change in total waste. Notice that $\Delta W(z) = (W_1^I + W_2^I) - (W_1^N + W_2^N)$ is quadratic and concave in z, and moreover, $\Delta W'(z) = -\frac{2\mu(1-\beta_2)(1-\xi^2)(z+(1-\beta_1)(1-\xi))}{\xi^2+\beta_2(1-\xi^2)} \leq 0$ for

$z > 0$. Hence, $\Delta W(z) > 0$ iff $z < \hat{z}$ where $\hat{z}$ is the larger root of $\Delta W(z) = 0$, that is,

$$\hat{z} = -(1-\xi)(1-\beta_1) + \frac{\sqrt{(1-\beta_2)(1-\xi^2)\left((1-\beta_2)^2 - (1-\beta_1)^2\left(\beta_2 + (1-\beta_2)\xi^2\right)\right)}}{(1-\beta_2)(1+\xi)}. \quad \text{(C.37)}$$

Equivalently, $\Delta W > 0$ iff $\phi_2 < \hat{\phi} = \frac{\sqrt{(1-\beta_2)(1-\xi^2)((1-\beta_2)^2-(1-\beta_1)^2(\beta_2+(1-\beta_2)\xi^2))}}{(1-\beta_2)(1+\xi)}$. Expressing this condition in terms of p leads to the condition in statement (ii).

To prove statement (iv), we identify a sufficient condition under which $\frac{W^I}{Q^I} > \frac{W^N}{Q^N}$. Observe that, as $\phi_2 \to (1-\xi)(1-\beta_2)$, $\frac{W^I}{Q^I} - \frac{W^N}{Q^N} \to \frac{(\beta_1-\beta_2)^2(1-\xi)^2}{1+2(\beta_1-\beta_2)(1-\xi)} > 0$. By continuity, there exists $\delta > 0$ such that $\frac{W^I}{Q^I} > \frac{W^N}{Q^N}$ for all $\phi_2 \in [(1-\xi)(1-\beta_2), (1-\xi)(1-\beta_2)+\delta]$. □

Proof of Corollary 2. For both Propositions 4 and 5, $W^I > W^N$ iff $\phi_2 < \hat{\phi}$ where $\hat{\phi}$ is decreasing in ξ. Thus, the condition for $W^I > W^N$ translates to that ξ is lower than a threshold. Moreover, under the setting of Proposition 5, as shown in the proof of Proposition 5(iv), there exists some $\delta > 0$ such that $\frac{W^I}{Q^I} > \frac{W^N}{Q^N}$ iff $\phi_2 \leq (1-\xi)(1-\beta_2) + \delta$. Thus, the condition for $\frac{W^I}{Q^I} > \frac{W^N}{Q^N}$ translates to $\xi \leq \check{\xi} := 1 - \frac{\phi_2 - \delta}{1-\beta_2}$. □

D Model Extensions

In this appendix, we present the details of several model extensions that are summarized in §5 of the main paper. Mathematical proofs for these extensions can be found in §D.5 at the end of this appendix.

D.1 More Than Two Markets

In general, the firm can sell to more than two markets. Let $\mathcal{M} = \{1, 2, ..., n\}$ denote the set of n markets. Without loss of generality, the markets are ordered according to their distances from the origin; that is, market i is closer to the origin than market $i+1$ for all $i = 1, 2, ..., n-1$. As before, a nearer market has a lower transportation cost and spoilage risk, i.e., $c_1 < c_2 < ... < c_n$ and $\beta_1 > \beta_2 > ... > \beta_n$. Proposition D.1 generalizes the structure of optimal allocation policies for the n-market problem.

Proposition D.1. *Given any $Q \geq 0$ and $\xi \in [0,1]$, the optimal allocation follows a market-partition rule: There exist integers $0 \leq z_1 \leq z_2 \leq z_3 \leq n$ such that $\mathcal{M}$ is partitioned into four subsets $\mathcal{M}_1 = \{i : i \leq z_1\}$, $\mathcal{M}_2 = \{i : z_1 + 1 \leq i \leq z_2\}$, $\mathcal{M}_3 = \{i : z_2 + 1 \leq i \leq z_3\}$, and $\mathcal{M}_4 = \{i : i \geq z_3 + 1\}$. In the optimal allocation, $x_i^* = 0$ and $y_i^* > 0$ for all $i \in \mathcal{M}_1$; $x_i^* > 0$ and $y_i^* > 0$ for all $i \in \mathcal{M}_2$; $x_i^* > 0$ and $y_i^* = 0$ for all $i \in \mathcal{M}_3$, and $x_i^* = y_i^* = 0$ for all $i \in \mathcal{M}_4$.*

Market	1	…	z_1	z_1+1	z_1+2	…	z_2	z_2+1	z_2+2	…	z_3	z_3+1	z_3+2	…	n
x_i^*	0	0	0	+	+	…	+	+	+	…	+	0	0	…	0
y_i^*	+	+	+	+	+	…	+	0	0	…	0	0	0	…	0

$\mathcal{M}_1$ Risky units only

$\mathcal{M}_2$ Both risky and good units

$\mathcal{M}_3$ Good units only

$\mathcal{M}_4$ Shutdown

Figure D.1: Illustration of the market-partition rule when there are $n > 2$ markets.

As illustrated by Figure D.1, the optimal solution partitions the n markets into four groups. The first group $\mathcal{M}_1$, consisting of the nearest z_1 markets, will receive only risky units. The second group $\mathcal{M}_2$, consisting of the markets farther away than those in group $\mathcal{M}_1$ but closer than those in the third group $\mathcal{M}_3$, will receive both risky and good units. The third group comprises even farther markets which should receive only good units. Finally, there is a cutoff point z_3 such that any market more distant than market z_3 will not be served. (Some of the groups may degenerate as we allow $z_i = z_{i+1}$.) To the best of our knowledge, no existing research has documented this market-partition rule for allocating two types of resources that differ in yield risk.[23]

We numerically show that our main findings established based on the two-market model continue to hold for the n-market setting. We consider five markets ($n = 5$) with $\mathbf{c} = (0.1, 0.2, 0.3, 0.4, 0.5)$ and $\boldsymbol{\beta} = (0.8, 0.6, 0.4, 0.2, 0)$. Demand in each market follows a uniform distribution $U[0, 100]$. Let $c_o = 5$ and $c_d = 1$. We vary the selling price p from 6 to 32 in increments of 2 and set ξ as a deterministic value taking 0.25, 0.5 or 0.75. For each instance, we computed the expected profit, waste and sales under the profit-maximizing solution (denoted by superscript I), the waste-conscious solution (denoted by superscript A) with $\eta = 0$ (that is, $Q^A = \min(Q^I, Q^N)$), and the benchmark scenario without monitoring (denoted by superscript N). Figure D.2 plots the changes in the expected profits, waste and sales due to monitoring technology as the selling price p increases and ξ varies. The panels in the second column show that under the profit-maximizing solution, monitoring technology increases waste (i.e., $W^I - W^N > 0$) when p is small and this negative consequence is more likely with a lower ξ. Moreover, if the firm follows the waste-conscious solution as discussed in §4.3.2, it can capture most of the profit gains from monitoring, reduce waste, and increase sales. For illustrative purposes, we set $\eta = 0$ in the example, while firms can adjust $\eta \in [0, 1]$ to balance their profit gain and environmental goal.

[23]Zipkin (1980) proves a ranking property when allocating one resource. To some extent, Proposition D.1 generalizes this earlier result by considering two types of resources differing in yield risk. An interesting question for future research is if one can further generalize the solution structure with more than two product types.

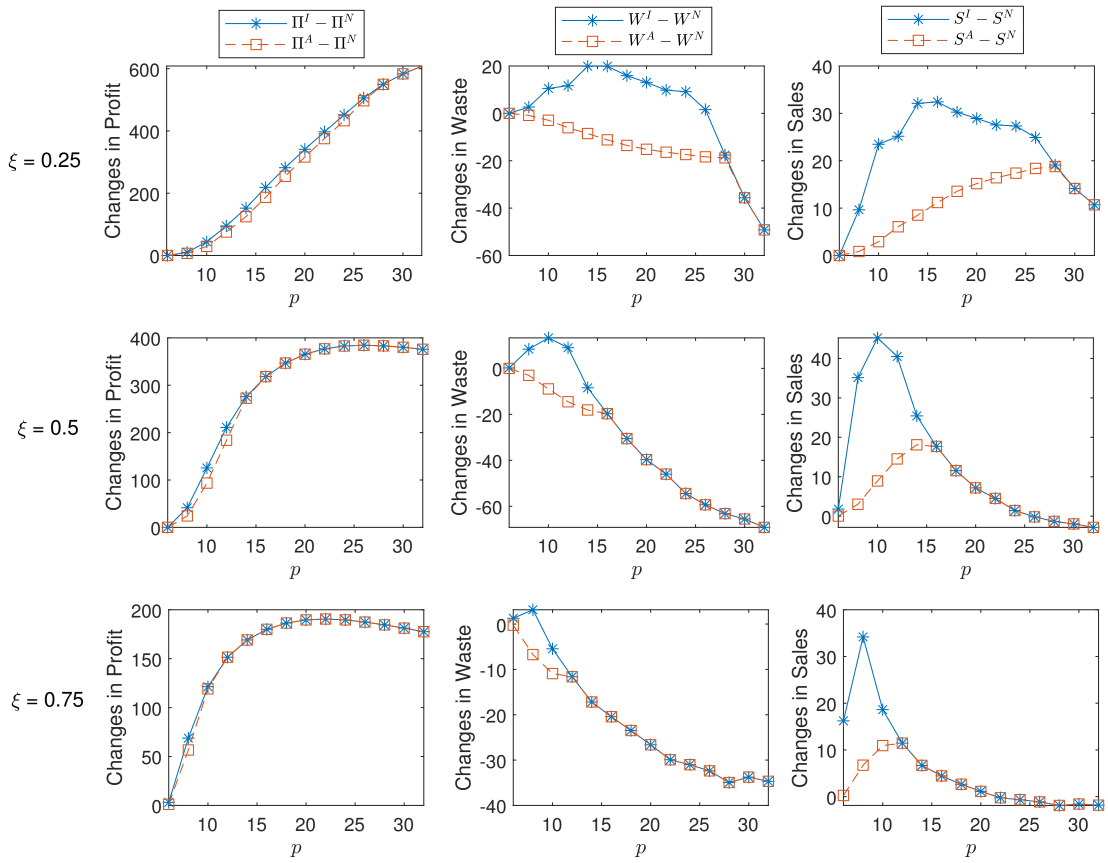


Figure D.2: Changes in expected profits, waste and sales as a function of p due to shipment monitoring under the profit-maximizing and waste-conscious solutions: The case of $n = 5$ markets.

D.2 Disposal Before Allocation

Our analysis has so far assumed that all units must be sent to the destination markets. In some situations, the firm might discard some risky units before allocating them to different markets from a central warehouse. We refer to those discarded units as in-transit disposal. Let c_t denote the cost incurred for each unit disposed of in transit. To incorporate in-transit disposal, we modify our base model by replacing the equality constraint (3) with inequality $\sum_{i \in \mathcal{M}} y_i \leq Q(1-\xi)$, and subtracting the in-transit disposal cost $c_t(Q - \sum_{i \in \mathcal{M}}(x_i + y_i))$ from the objective function (1).

Proposition D.2. *Suppose that the firm can dispose of some risky units at a per-unit cost c_t instead of sending them to any market. Let λ_y^B denote the optimal Lagrangian dual variable associated with constraint $\sum_{i \in \mathcal{M}} y_i = Q(1-\xi)$ in the original allocation problem without in-transit disposal. If $c_t \geq -\lambda_y^B$, the optimal allocation remains as characterized in Proposition 1, resulting in no in-transit disposal. Otherwise, the optimal allocation is given by one of the following cases.*

(i) If $F_1(Q\xi) \leq \Phi$ and $\beta_1 \bar{F}_1(Q\xi) \leq 1 - \psi_1$ and $\beta_2 \leq 1 - \psi_2$, then $\mathbf{x}^* = [Q\xi, 0]$ and $\mathbf{y}^* = [0, 0]$;

(ii) If $F_1(Q\xi) > \Phi$ and $\beta_1 \bar{F}_1(x^\dagger) \leq 1 - \psi_1$ and $\beta_2 \bar{F}_2(Q\xi - x^\dagger) \leq 1 - \psi_2$ where $x^\dagger$ is the unique solution to $F_1(x^\dagger) - F_2(Q\xi - x^\dagger) = \Phi$, then $\mathbf{x}^* = [x^\dagger, Q\xi - x^\dagger]$ and $\mathbf{y}^* = [0, 0]$;

(iii) if $\beta_1 \bar{F}_1(Q) < 1 - \psi_1 < \beta_1 \bar{F}_1(Q\xi)$ and $(1 - \beta_1)\bar{F}_1(Q\xi) \geq \psi_2$, then $\mathbf{x}^* = [Q\xi, 0]$ and $\mathbf{y}^* = [y^\dagger, 0]$ where $y^\dagger$ is the unique solution to $\beta_1 \bar{F}_1(Q\xi + y^\dagger) = 1 - \psi_1$;

(iv) if $\beta_1 \bar{F}_1(Q(1 - \xi)) < 1 - \psi_1 < \beta_1$ and $\beta_1 - F_2(Q\xi) \geq 1 - \psi_1$, then $\mathbf{x}^* = [0, Q\xi]$ and $\mathbf{y}^* = [y^\dagger, 0]$ where $y^\dagger$ is the unique solution to $\beta_1 \bar{F}_1(y^\dagger) = 1 - \psi_1$;

(v) if $\beta_1 - F_2(Q\xi) < 1 - \psi_2$ and $(1 - \beta_1)\bar{F}_1(Q\xi) < \psi_2$ and $\beta_2 \bar{F}_2(Q\xi - x^\dagger) \leq 1 - \psi_2$ where $x^\dagger$ is the unique solution to $\bar{F}_2(Q\xi - x^\dagger) - (1 - \beta_1)\bar{F}_1(x^\dagger) = 1 - \psi_2$, then $\mathbf{x}^* = [x^\dagger, Q\xi - x^\dagger]$ and $\mathbf{y}^* = [y^\dagger, 0]$ where $y^\dagger$ is the unique solution to $\beta_1 \bar{F}_1(x^\dagger + y^\dagger) = 1 - \psi_1$;

(vi) if $\beta_1 \bar{F}_1(Q(1 - \xi) - y_2^\dagger) < 1 - \psi_1 < \beta_1$ and $\beta_2 \bar{F}_2(Q) < 1 - \psi_2 < \beta_2 \bar{F}_2(Q\xi)$ and $(1 - \beta_2)\bar{F}_2(Q\xi) \geq 1 - \beta_1$ where $y_2^\dagger$ is the unique solution solving $\beta_2 \bar{F}(Q\xi + y_2^\dagger) = 1 - \psi_2$, then $\mathbf{x}^* = [0, Q\xi]$ and $\mathbf{y}^* = [y_1^\dagger, y_2^\dagger]$ where $y_1^\dagger$ solves $\beta_1 \bar{F}_1(y_1^\dagger) = 1 - \psi_1$;

(vii) otherwise, $\mathbf{x}^* = [x^\dagger, Q\xi - x^\dagger]$ and $\mathbf{y}^* = [y_1^\dagger, y_2^\dagger]$ are the solution to the system of equations: $\beta_1 - \beta_2 = (1 - \beta_2)F_2(Q\xi - x^\dagger) - (1 - \beta_1)F_1(x^\dagger)$, $\beta_1 \bar{F}_1(x^\dagger + y_1^\dagger) = 1 - \psi_1$, $\beta_2 \bar{F}_2(Q\xi - x^\dagger + y_2^\dagger) = 1 - \psi_2$.

In Proposition D.2, we establish that if c_t is greater than a threshold, the firm will not dispose of any risky unit and so the optimal solution remains the same as in our base model. The threshold can be computed based on a Lagrangian dual variable from the base model. If c_t is smaller than the threshold, the optimal allocation quantities fall into seven different cases. Compared to the base model, more cases emerge because the firm can now control how many risky units should move forward: shipping no risky units to either market (cases (i)(ii)), sending some of them only to market 1 (cases (iii)-(v)), or shipping to both markets (cases (vi)(vii)). Despite this complexity, Proposition D.2 explicitly characterizes the condition for each case to be optimal. As in the base model, this enables us to easily evaluate the first-order derivative for the first-stage purchase problem, thereby determining the optimal order quantity.

Allowing in-transit disposal can affect some of our findings on waste. Without monitoring, the total waste is the same as in the base model, since the firm is unable to identify risky units. However, with monitoring, the total waste now includes not only the leftover and spoilage as discussed earlier but also the units discarded in transit. The possibility of in-transit disposal gives the firm an additional incentive to over-order when the profit margin is exceedingly high. The firm may deliberately purchase an excessive quantity, knowing that some units will be disposed

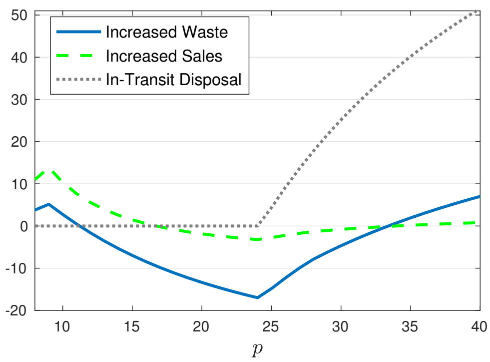


Figure D.3: Changes in expected waste and sales when in-transit disposal is possible. [parameter values: $c_t = 0.2$, $D_1, D_2 \sim U[0, 100]$, $\xi = 0.6$, $\beta_1 = 0.5$, $\beta_2 = 0.1$, $c_o = 5$, $(c_1, c_2) = (0.1, 0.2)$, $c_d = 1$]

of in transit. Figure D.3 presents a numerical example in which the firm disposes of units when selling price $p \geq 24$. As a result, monitoring technology can induce more waste either when the profit margin is low (as explained by our base-model analysis) or when the profit margin is very high (resulting in extensive in-transit disposal). Furthermore, with in-transit disposal, the increase in waste can exceed the increase in sales (for $p \geq 35$). A caveat to the above analysis is that in-transit disposal may have less environmental impact than unsold units at the retail level. In practice, units disposed of in transit are easier to collect and recycle because they can be processed in bulk, unlike unsold units at the retail level. Our findings suggest that if monitoring technology induces in-transit disposal, greater efforts should be made by supply chain firms or nonprofit organizations to handle the discarded units in transit.

D.3 General Yield Distributions

We now consider the case where ϵ_1 and ϵ_2 follow general probability distributions on support $[0, 1]$. Recall that ϵ_1 is first-order stochastically larger than ϵ_2. The structural properties characterized in Proposition 1 can be extended to this general case but with more implicit boundary conditions. Define $H_i(x, y) = E_{\epsilon_i}[F_i(x + y\epsilon_i)]$ and $M_i(x, y) = E_{\epsilon_i}[\epsilon_i \bar{F}_i(x + y\epsilon_i)]$. Let $C_1(Q, \xi) = H_1(Q\xi, Q(1-\xi))$, $C_2(Q, \xi) = H_1(0, Q(1-\xi)) - H_2(Q\xi, 0)$, and $C_3(Q, \xi) = M_2(Q\xi, 0) - M_1(0, Q(1-\xi))$. The following proposition generalizes Proposition 1 and fully characterizes the optimal allocation when the ϵ_i's follow general distributions.

Proposition D.3. *Given any $Q \geq 0$ and $\xi \in [0, 1]$, the optimal allocation $\mathbf{x}^*$ and $\mathbf{y}^*$ is characterized as follows. (i) If $C_1(Q, \xi) \leq \Phi$, then $\mathbf{x}^* = [Q\xi, 0]$, $\mathbf{y}^* = [Q(1-\xi), 0]$.*

(ii) If $C_2(Q, \xi) < \Phi < C_1(Q, \xi)$ and $M_2(Q\xi - x^\dagger, 0) - M_1(x^\dagger, Q(1-\xi)) \leq \Phi$ where $x^\dagger$ is the solution to $H_1(x^\dagger, Q(1-\xi)) - H_2(Q\xi - x^\dagger, 0) = \Phi$, then $\mathbf{x}^* = [x^\dagger, Q\xi - x^\dagger]$, $\mathbf{y}^* = [Q(1-\xi), 0]$.

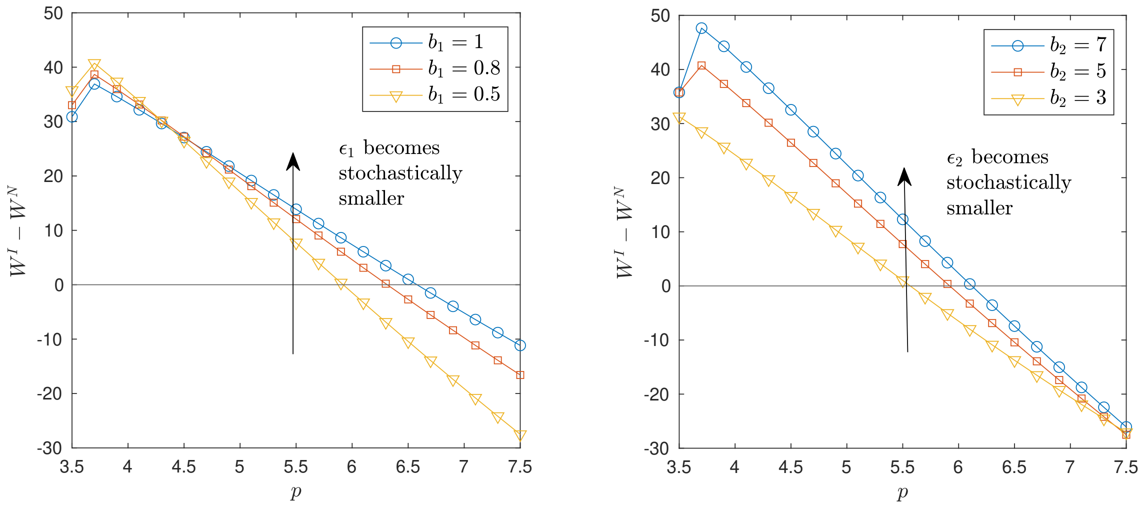
(iii) If $C_3(Q, \xi) \leq \Phi \leq C_2(Q, \xi)$, then $\mathbf{x}^* = [0, Q\xi]$, $\mathbf{y}^* = [Q(1-\xi), 0]$.

(iv) If $C_3(Q,\xi) > \Phi$ and $H_1(0,y^\dagger) - H_2(Q\xi, Q(1-\xi)-y^\dagger) \geq \Phi$ where $y^\dagger$ is the solution to $M_2(Q\xi, Q(1-\xi)-y^\dagger) - M_1(0,y^\dagger) = \Phi$, then $\mathbf{x}^* = [0, Q\xi]$, $\mathbf{y}^* = [y^\dagger, Q(1-\xi)-y^\dagger]$.

(v) If $M_2(0,Q(1-\xi)) - M_1(Q\xi,0) < \Phi < M_2(0,0) - M_1(Q\xi, Q(1-\xi))$ and $H_1(Q\xi, y^\ddagger) - H_2(0, Q(1-\xi)-y^\ddagger) \leq \Phi$ where $y^\ddagger$ is the solution to $M_2(0, Q(1-\xi)-y^\ddagger) - M_1(Q\xi, y^\ddagger) = \Phi$, then $\mathbf{x}^* = [Q\xi, 0]$ and $\mathbf{y}^* = [y^\ddagger, Q(1-\xi)-y^\ddagger]$.

(vi) Otherwise, $\mathbf{x}^* = [x^o, Q\xi - x^o]$ and $\mathbf{y}^* = [y^o, Q(1-\xi)-y^o]$ where x^o and y^o are determined by a system of equations: $H_1(x^o, y^o) - H_2(Q\xi - x^o, Q(1-\xi)-y^o) = \Phi$; $M_2(Q\xi - x^o, Q(1-\xi) - y^o) - M_1(x^o, y^o) = \Phi$.

Proposition D.3 is established by ruling out the other forms of allocation. For instance, we can show that it must not be optimal to assign all good units to market 1 while all risky units to market 2, or to assign all risky units to market 2 while splitting good units. [24]



(a) Effect of b_1 with $b_2 = 5$

(b) Effect of b_2 with $b_1 = 0.5$

Figure D.4: Monitoring-induced change in waste as a function of p: The case of general random yield [$\epsilon_i \sim G_i$ where $G_i(x) = 1-(1-x)^{b_i}$ for $i = 1, 2$; $\xi \sim Tri(0, 0.8, 1)$, $c_o = 2.01$, $c_1 = 0.13, c_2 = 0.38, c_d = 0.05$; the D_i's follow a Gamma distribution with mean 600 and $cv = 1$.]

Given the full characterization of the optimal allocation, we can follow the same approach as discussed in §4.1.2 to calculate the gradient of the first-stage problem and thereby determine the optimal purchase quantity. To check the robustness of our main findings, assume each ϵ_i to follow a probability distribution with cdf $G_i(x) = 1-(1-x)^{b_i}$ over support $[0, 1]$ and $0 < b_1 < b_2$. Note that G_i is first-order stochastically larger as b_i becomes smaller, and it reduces to a uniform distribution on $[0, 1]$ when $b_i = 1$. Figure D.4 presents a numerical study,

[24] Compared to the base model, we have one additional case that cannot be ruled out, i.e., case (v) of Proposition D.3 where risky units are split and all good units are directed to market 1. When computing the gradient of the first-stage problem, we simply need to include this additional case, which does not impose much computational burden.

showing that monitoring tends to increase waste for low-margin products, and moreover, this adverse outcome is more likely when ϵ_i is stochastically smaller.

Table D.1: Percentage changes in profit, waste, and sales due to shipment monitoring with $Q^A = \eta Q^I + (1-\eta)\min(Q^I, Q^N)$ and $\epsilon_i \sim G_i(x) = 1-(1-x)^{b_i}$.

η	$\frac{\Pi^A-\Pi^N}{\Pi^N}$ (%)	$\frac{W^A-W^N}{W^N}$ (%)	$\frac{S^A-S^N}{S^N}$ (%)
0.0	20.5	-12.8	4.5
0.1	21.0	-11.2	5.6
0.2	21.6	-9.6	6.6
0.3	22.1	-8.1	7.7
0.4	22.7	-6.5	8.8
0.5	23.3	-5.0	9.9
0.6	23.9	-3.5	11.0
0.7	24.6	-2.0	12.1
0.8	25.3	-0.6	13.3
0.9	26.2	0.8	14.4
Profit-Maximizing Solution:			
1.0	27.3	2.1	15.6

Note: $b_1 = 1/4$, $b_2 = 5$, $\xi = 0.8$, $c_o = 2.01$, $c_1 = 0.13, c_2 = 0.38, c_d = 0.05$; the D_i's follow a Gamma distribution with mean 600 and $cv = 1$.

Furthermore, based on the calibrated numerical setting, Table D.1 showcases that the waste-conscious solution proposed in §4.3.2 remains a valid approach. By adjusting the purchase quantity via the parameter η, firms can capture much of the value from shipment monitoring while achieving their waste reduction goals. For instance, using the monitoring technology without increasing the order quantity ($\eta = 0$), the firm can improve the profit by 20.5% while reducing waste by 12.8%.

D.4 Supplier-Managed Shipping

In this appendix, we present a detailed mathematical formulation for the case of supplier-managed shipping.

As in the base model, in each market $i \in \mathcal{M}$, the retailer sells the product at a given retail price p to fulfill a random variable D_i. The retailer determines an order quantity, denoted by o_i, for each market i.[25] The supplier in turn chooses a production quantity Q at a unit cost c_o and then distributes the product, either with or without monitoring technology, to each market i at a unit transportation cost c_i. In line with the supply uncertainty literature (Tomlin and

[25] We assume that the retailer (e.g., a grocery chain) serves two markets and makes the purchase decision to maximize the total profit. If two independent retailers serve the two markets, respectively, they will engage in a noncooperative game when the supplier allocates the product with monitoring technology, because each retailer's order quantity will influence the allocation received by the other retailer. We leave this more complex situation for future research.

Wang 2005, Wang et al. 2010), the supply chain contract depends generally on both the amount ordered and the amount effectively delivered. We assume that the retailer pays τw per unit ordered and $(1-\tau)w$ per unit delivered under salable condition (but up to its order quantity in case the supplier over-delivers) where parameter $\tau \in [0,1]$. Alternatively, one can interpret the contract agreement as the retailer paying w per unit ordered in advance and receiving a refund of $(1-\tau)w$ per unit undelivered or delivered but with unsalable condition.

The sequence of events is as follows. (i) First, the retailer determines the order quantities o_i's for the two markets; (ii) the supplier chooses how much to produce and allocate to fulfill these orders; (iii) shipments are received and the payment is made according to the realized spoilage and the contract described above; (iv) finally, the retailer earns revenue from the salable units received in each market and incurs the disposal cost for the spoiled and leftover units.[26]

D.4.1 Without Monitoring

We first analyze the problem without monitoring. Given any order quantities $\mathbf{o} = (o_1, o_2)$, the supplier determines the shipping quantities $\mathbf{q} = (q_1, q_2)$ for each market. There will be $q_i z_i$ units effectively delivered where the random yield factor is equal to $z_i = \xi + (1-\xi)\epsilon_i$ as in our base model. Accordingly, the supply chain payment is equal to $\tau w o_i + \bar{\tau} w \min(o_i, q_i z_i)$ where we define $\bar{\tau} = 1 - \tau$. Note that the supplier may have an incentive to over-deliver to mitigate spoilage risk, and the term $\min(o_i, q_i z_i)$ ensures that the retailer will not pay for any extra units beyond what is ordered.

The supplier maximizes the following expected profit by choosing the shipping quantities $\mathbf{q}$.

$$\Pi_S^N(\mathbf{q}|\mathbf{o}) = \sum_{i \in \mathcal{M}} \left\{ E\left[\tau w o_i + \bar{\tau} w \min(o_i, q_i z_i)\right] - (c_i + c_o) q_i \right\} \tag{D.1}$$

Without monitoring, condition-based allocation is impossible and hence the supplier will determine each q_i independently. Consequently, the optimal shipping quantity to market i depends only on order quantity o_i, denoted by $q_i^*(o_i)$. Anticipating the supplier's optimal shipping quantities, the retailer chooses o_i to maximize the profit in each market i given below:

$$U_i^N(o_i) = E[p \min(D_i, q_i^*(o_i) z_i) - \tau w o_i - \bar{\tau} w \min(o_i, q_i^*(o_i) z_i) - c_d((q_i^*(o_i) z_i - D_i)^+ + q_i^*(o_i)(1 - z_i))]. \tag{D.2}$$

For the problem without monitoring, if ξ is deterministic, we can characterize the equilibrium

[26] In the model extension, we assume that the retailer is responsible for the disposal cost of unsalable units to reflect the fact that those "spoiled" units are found at the retail level. One may alternatively assume that the supplier shares some of the disposal cost. We expect that our main findings will qualitatively hold under this alternative setting.

in closed-form as follows. For any given o_i and ξ, the supplier's objective Π_S^N is piece-wise linear and concave in q_i. It is easy to show that the supplier's optimal shipping quantity $q_i^*(o_i)$ is given below. Define $\phi_i = \frac{\bar{\tau}w - c_i - c_o}{\bar{\tau}w}$.

$$q_i^*(o_i) = \begin{cases} 0 & \text{if } \phi_i \leq (1-\beta_i)(1-\xi), \\ o_i & \text{if } (1-\beta_i)(1-\xi) < \phi_i \leq 1-(1-\beta_i)\xi, \\ o_i/\xi & \text{if } 1-(1-\beta_i)\xi < \phi_i. \end{cases} \tag{D.3}$$

Substituting $q_i^*(o_i)$ into the retailer's problem, we have the following three cases.

(i) If $\phi_i \leq (1-\beta_i)(1-\xi)$, the supplier always produces nothing, and so no trade will occur in equilibrium.

(ii) If $(1-\beta_i)(1-\xi) < \phi_i \leq 1-(1-\beta_i)\xi$, then $U_i^N(o_i)$ can be written as

$$\begin{aligned} U_i^N(o_i) =& po_i(\beta_i + (1-\beta_i)\xi) - (p+c_d)(\beta_i E[o_i - D_i]^+ + (1-\beta_i)E[o_i\xi - D_i]^+) \\ & - c_d(1-\beta_i)(1-\xi)o_i - (\tau w + (\beta_i + (1-\beta_i)\xi)\bar{\tau}w)o_i. \end{aligned}$$

(iii) If $\phi_i > 1-(1-\beta_i)\xi$, then

$$\begin{aligned} U_i^N(o_i) =& p(\beta_i\frac{o_i}{\xi} + (1-\beta_i)o_i) - (p+c_d)(\beta_i E[\frac{o_i}{\xi} - D_i]^+ + (1-\beta_i)E[o_i - D_i]^+) \\ & - c_d(1-\beta_i)(1-\xi)\frac{o_i}{\xi} - wo_i. \end{aligned}$$

By analyzing the first-order derivative of U_i^N for cases (ii)(iii), we can derive the retailer's optimal order quantity o_i^N for each market i as follows:

(i) If $\phi_i \leq (1-\beta_i)(1-\xi)$, then $o_i^N = 0$;

(ii) if $(1-\beta_i)(1-\xi) < \phi_i \leq 1-(1-\beta_i)\xi$, then o_i^N is determined by equation

$$\beta_i\bar{F}_i(o_i^N) + (1-\beta_i)\xi\bar{F}_i(o_i^N\xi) = 1 - \frac{p - \tau w - (\beta_i + (1-\beta_i)\xi)\bar{\tau}w}{p+c_d};$$

(iii) if $\phi_i > 1-(1-\beta_i)\xi$, then o_i^N is determined by equation

$$\beta_i\bar{F}_i(\frac{o_i^N}{\xi}) + (1-\beta_i)\xi\bar{F}_i(o_i^N) = 1 - \frac{p - \xi w}{p+c_d}.$$

D.4.2 With Monitoring

With monitoring technology, receiving the retailer's order quantities $\mathbf{o}$, the supplier will determine a total production quantity Q, and then decide on the allocation quantities $\mathbf{x}$ and $\mathbf{y}$. Thus, there will be $x_i + y_i\epsilon_i$ units delivered to each market i. Accordingly, the supply chain payment is equal to $\tau w o_i + (1-\tau)w\min\{o_i, x_i + y\epsilon_i\}$.

Given any order quantities $\mathbf{o} = (o_1, o_2)$, the supplier solves a two-stage optimization similar to that in our base model. It determines a production quantity Q to maximize

$$\Pi_S^I(Q|\mathbf{o}) = E[R(Q,\xi|\mathbf{o})] - c_o Q. \tag{D.4}$$

where $R(Q,\xi|\mathbf{o})$ is the optimal objective value of the following allocation problem:

$$R(Q,\xi|\mathbf{o}) = \max_{\mathbf{x},\mathbf{y}} \sum_{i\in\mathcal{M}} \tau w o_i + \bar{\tau} w E\left[\min(o_i, x_i + y_i\epsilon_i)\right] - c_i(x_i + y_i) \tag{D.5}$$

$$\sum_{i\in\mathcal{M}} x_i = Q\xi \tag{D.6}$$

$$\sum_{i\in\mathcal{M}} y_i = Q(1-\xi) \tag{D.7}$$

$$x_i \geq 0,\ y_i \geq 0 \text{ for all } i \in \mathcal{M} \tag{D.8}$$

Let $\mathbf{x}^*(\mathbf{o},\xi)$ and $\mathbf{y}^*(\mathbf{o},\xi)$ denote the supplier's optimal allocation given order quantities $\mathbf{o} = (o_1, o_2)$ and realized ξ. Anticipating the supplier's optimal allocation policy, the retailer maximizes its expected profit by choosing order quantities (o_1, o_2). The retailer's expected profit is given by

$$\begin{aligned} U^I(o_1,o_2) =& E[p\min(D_i, x_i^*(\mathbf{o},\xi) + y_i^*(\mathbf{o},\xi)\epsilon_i) - \tau w o_i - \bar{\tau} w \min(o_i, x_i^*(\mathbf{o},\xi) + y_i^*(\mathbf{o},\xi)\epsilon_i) \\ & - c_d((x_i^*(\mathbf{o},\xi) + y_i^*(\mathbf{o},\xi)\epsilon_i - D_i)^+ + (1-\epsilon_i)y_i^*(\mathbf{o},\xi))] \end{aligned} \tag{D.9}$$

We have a Stackelberg game in which the retailer first determines the order quantities (o_1, o_2) and then the supplier chooses the production and allocation quantities. By backward induction, we need to solve the supplier's problem given any (o_1, o_2). Since the o_i's are deterministic, the supplier's allocation problem is a piece-wise concave maximization, which can be solved efficiently. However, depending on the values of o_1, o_2, and ξ, the optimal allocation $\mathbf{x}^*(\mathbf{o},\xi)$ and $\mathbf{y}^*(\mathbf{o},\xi)$ can fall into numerous different cases. This makes the retailer's problem analytically intractable, because $\mathbf{x}^*(\mathbf{o},\xi)$ and $\mathbf{y}^*(\mathbf{o},\xi)$, as the supplier's best response, must be incorporated into the retailer's problem.

Therefore, we resort to a numerical approach to explore the impact of supply chain decentralization. Specifically, we conduct an exhaustive search to find the retailer's optimal order quantities (o_1^I, o_2^I), where for every possible (o_1, o_2), the optimal allocation quantities $\mathbf{x}^*(\mathbf{o}, \xi)$ and $\mathbf{y}^*(\mathbf{o}, \xi)$ are determined by solving the supplier's optimization problem (D.4)-(D.8). The numerical results are as discussed in the main paper.

D.5 Proofs for Model Extensions

Proof of Proposition D.1. We prove the market-partition rule by establishing a series of lemmas. Moreover, we will prove the proposition for the general model where in-transit disposal is allowed. Let λ_x and λ_y denote the Lagrangian dual variables associated with constraints (2) and (3), respectively. Let $\omega_i^x, \omega_i^y \geq 0$ denote the Lagrangian dual variables associated with nonnegativity constraints $x_i \geq 0$ and $y_i \geq 0$, respectively. By the complementary slackness, $\omega_i^x x_i = 0$ and $\omega_i^y y_i = 0$ for all $i \in \mathcal{M}$. Let L denote the Lagrangian function. By the KKT conditions, for all $i \in \mathcal{M}$, we have

$$\frac{\partial L}{\partial x_i} = (p - c_i) - (p + c_d)E_{\epsilon_i}[F_i(x_i + y_i\epsilon_i)] + c_t - \lambda_x + \omega_i^x = 0, \tag{D.10}$$

$$\frac{\partial L}{\partial y_i} = (p - c_i) - (p + c_d)(1 - E[\epsilon_i \bar{F}_i(x_i + y_i\epsilon_i)]) + c_t - \lambda_y + \omega_i^y = 0. \tag{D.11}$$

Step 1. The following lemma establishes that once it is optimal to shut down market i, it is also optimal to do so for all $j \geq i$.

Lemma D.1. *In the optimal solution, if $x_i^* = y_i^* = 0$ for some i, then $x_j^* = y_j^* = 0$ for any $j > i$.*

Proof of Lemma D.1. If $x_i^* = y_i^* = 0$, then $\omega_i^x \geq 0$ and $\omega_i^y \geq 0$. By (D.10) and (D.11), we have

$$\frac{\partial L}{\partial x_i} = 0 \iff p - c_i + c_t - \lambda_x \leq 0 \tag{D.12}$$

and

$$\frac{\partial L}{\partial y_i} = 0 \iff p - c_i - (p + c_d)(1 - \beta_i) + c_t - \lambda_y \leq 0. \tag{D.13}$$

Suppose to the contrary that $x_j^* > 0$ for some $j > i$. Then, the KKT conditions $\frac{\partial L}{\partial x_j} = 0$ and $\omega_j^x = 0$ imply $0 = p - c_j - (p + c_d)E_{\epsilon_j}[F_j(x_j^* + y_j^*\epsilon_j)] + c_p - \lambda_x < p - c_i + c_t - \lambda_x$ (because $c_j > c_i$) which contradicts (D.12). Similarly, suppose $y_j^* > 0$ for some $j > i$. Then, the KKT conditions $\frac{\partial L}{\partial y_j} = 0$ and $\omega_j^y = 0$ imply $0 = (p - c_j) - (p + c_d)(1 - \beta_j + E_{\epsilon_j}[\epsilon_j F_j(x_j^* + y_j^*\epsilon_j)]) + c_p - \lambda_y <$ $p - c_i - (p + c_d)(1 - \beta_i) + c_p - \lambda_y$ because $c_i < c_j$ and $\beta_i > \beta_j$, which contradicts (D.13). $\square$

Step 2. Consider any pair of markets $i < j$ with allocation $\begin{pmatrix} x_i & x_j \\ y_i & y_j \end{pmatrix}$. Let us use sign() to represent an element-wise sign function that returns $+$, 0 or $-$ based on the sign of each element in a given matrix. In Lemma D.2, we rule out any solution such that sign $\begin{pmatrix} x_i & x_j \\ y_i & y_j \end{pmatrix} = \begin{pmatrix} + & 0 \\ + & + \end{pmatrix}$ or $\begin{pmatrix} + & 0 \\ 0 & + \end{pmatrix}$.

Lemma D.2. *Any solution such that $x_i > 0, y_i \geq 0$ and $x_j = 0, y_j > 0$, where $i < j$, must not be optimal.*

Proof of Lemma D.2. We prove the lemma by contradiction. Suppose that the optimal solution is as described, the Lagrangian multipliers satisfy $\omega_i^x = 0, \omega_i^y \geq 0, \omega_j^x \leq 0$, and $\omega_j^y = 0$. Note that given these Lagrangian multipliers,

$$\frac{\partial L}{\partial x_i} - \frac{\partial L}{\partial y_i} = 0 \iff (p + c_d)E[(1 - \epsilon_i)\bar{F}_i(x_i + y_i\epsilon_i)] - \lambda_x + \lambda_y \geq 0$$

and

$$\frac{\partial L}{\partial x_j} - \frac{\partial L}{\partial y_j} = 0 \iff (p + c_d)E[(1 - \epsilon_j)\bar{F}_j(y_j\epsilon_j)] - \lambda_x + \lambda_y \leq 0.$$

It follows that

$$E[(1 - \epsilon_i)\bar{F}_i(x_i + y_i\epsilon_i)] \geq E[(1 - \epsilon_j)\bar{F}_j(y_j\epsilon_j)]. \tag{D.14}$$

Under the Bernoulli yield, the above reduces to $(1-\beta_i)\bar{F}_i(x_i) \geq (1-\beta_j)\bar{F}_j(0) = 1-\beta_j$. Because $\beta_i > \beta_j$, we have $1 - \beta_j > 1 - \beta_i \geq (1 - \beta_i)\bar{F}_i(x_i)$, leading to a contradiction.

□

Next, in Lemma D.3, we rule out any solution such that sign $\begin{pmatrix} x_i & x_j \\ y_i & y_j \end{pmatrix} = \begin{pmatrix} + & + \\ 0 & + \end{pmatrix}$.

Lemma D.3. *Any solution such that $x_i > 0, y_i = 0$ and $x_j > 0, y_j > 0$, where $i < j$, must not be optimal.*

Proof of Lemma D.3. We prove the lemma by contradiction. Suppose that the optimal solution is as described. Then, the Lagrangian multipliers satisfy $\omega_i^x = 0, \omega_i^y \geq 0$, and $\omega_j^x = 0, \omega_j^y = 0$. Then, it follows that

$$\frac{\partial L}{\partial x_i} - \frac{\partial L}{\partial x_i} = 0 \iff c_j - c_i - (p + c_d)[F_i(x_i) - E[F_j(x_j + y_j\epsilon_j)]] = 0 \tag{D.15}$$

which holds only if $F_i(x_i) > E[F_j(x_j + y_j\epsilon_j)]$, or equivalently, $E[\bar{F}_j(x_j + y_j\epsilon_j)] > \bar{F}_i(x_i)$, since $c_j > c_i$.

On the other hand,

$$\frac{\partial L}{\partial y_i} - \frac{\partial L}{\partial y_i} = 0 \iff c_j - c_i - (p + c_d)E[\epsilon_j \bar{F}_j(x_j + y_j\epsilon_j) - \epsilon_i \bar{F}_i(x_i)] \leq 0. \tag{D.16}$$

Note that (D.15) and (D.16) together imply that

$$E[\epsilon_j \bar{F}_j(x_j + y_j\epsilon_j)] - E[\epsilon_i]\bar{F}_i(x_i) \geq E[\bar{F}_j(x_j + y_j\epsilon_j)] - \bar{F}_i(x_i),$$

or equivalently, $E[(1-\epsilon_i)]\bar{F}_i(x_i) \geq E[(1-\epsilon_j)\bar{F}_j(x_j + y_j\epsilon_j)]$. Under Bernoulli yield, it becomes $(1-\beta_i)\bar{F}_i(x_i) \geq (1-\beta_j)\bar{F}_j(x_j)$. Since $\beta_i > \beta_j$, it implies $\bar{F}_i(x_i) > \bar{F}_j(x_j)$. It follows that $F_i(x_i) < F_j(x_j) \leq E[F_j(x_j + y_j\epsilon_j)]$. However, because $c_j - c_i > 0$, for (D.15) to hold, we must have $F_i(x_i) \geq E[F_j(x_j + y_j\epsilon_j)]$, thus leading to a contradiction. $\square$

Step 3. To characterize the optimal allocation rule, we start with any market i and show that the structure of the optimal allocation for all $j > i$ can be implied by the preceding lemmas. Depending on the sign of x_i^* and y_i^*, there are four possible cases: sign $\begin{pmatrix} x_i^* \\ y_i^* \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$, $\begin{pmatrix} + \\ 0 \end{pmatrix}$, $\begin{pmatrix} + \\ + \end{pmatrix}$ or $\begin{pmatrix} 0 \\ + \end{pmatrix}$. We examine the four cases one by one for any given i.

If sign $\begin{pmatrix} x_i^* \\ y_i^* \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$, then by Lemma D.1, it follows that for all $j > i$, we must have sign $\begin{pmatrix} x_j^* \\ y_j^* \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ as well. Consequently, there exists an integer $0 \leq z_3 \leq n$ such that $x_i^* = y_i^* = 0$ for all $i \in \mathcal{M}_4 = \{i : i \geq z_3 + 1\}$. Note that $\mathcal{M}_4$ can be empty in which case we set $z_3 = n$.

Moreover, since Lemmas D.2 and D.3 together rule out the possibility of sign $\begin{pmatrix} x_i & x_j \\ y_i & y_j \end{pmatrix} = \begin{pmatrix} + & + \\ 0 & + \end{pmatrix}$ or $\begin{pmatrix} + & 0 \\ 0 & + \end{pmatrix}$, the following holds true. If sign $\begin{pmatrix} x_i^* \\ y_i^* \end{pmatrix} = \begin{pmatrix} + \\ 0 \end{pmatrix}$, then for any $j > i$, we must have sign $\begin{pmatrix} x_j^* \\ y_j^* \end{pmatrix} = \begin{pmatrix} + \\ 0 \end{pmatrix}$ or sign $\begin{pmatrix} x_j^* \\ y_j^* \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$. We also know that once sign $\begin{pmatrix} x_j^* \\ y_j^* \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$, then j and all the other $j' > j$ will consist of set $\mathcal{M}_4$ as derived earlier. Thus, there exists an integer $z_2 \leq z_3$ such that $x_i^* > 0$ and $y_i^* = 0$ for all

$i \in \mathcal{M}_3 = \{i : z_2 + 1 \le i \le z_3\}$. Note that $\mathcal{M}_3$ can be empty in which case we can set $z_2 = z_3$.

Following an analogous argument, Lemma D.2 rules out any solution such that sign $\begin{pmatrix} x_i & x_j \\ y_i & y_j \end{pmatrix} = \begin{pmatrix} + & 0 \\ + & + \end{pmatrix}$. Thus, if sign $\begin{pmatrix} x_i^* \\ y_i^* \end{pmatrix} = \begin{pmatrix} + \\ + \end{pmatrix}$, then for any $j > i$, we can have three possibilities: sign $\begin{pmatrix} x_j \\ y_j \end{pmatrix} = \begin{pmatrix} + \\ + \end{pmatrix}$, $\begin{pmatrix} + \\ 0 \end{pmatrix}$, or $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$. For the second and third possibilities, as already discussed above, j and following indexes will consist of sets $\mathcal{M}_3$ and $\mathcal{M}_4$. Hence, there exists an integer $z_1 \le z_2$ such that $x_i^* > 0$ and $y_i^* > 0$ for all $i \in \mathcal{M}_2 = \{i : z_1 + 1 \le i \le z_2\}$. Let $z_1 = z_2$ if $\mathcal{M}_2$ is empty.

Lastly, if sign $\begin{pmatrix} x_i^* \\ y_i^* \end{pmatrix} = \begin{pmatrix} 0 \\ + \end{pmatrix}$, then for any $j > i$, we can have all the four combinations: sign $\begin{pmatrix} x_j^* \\ y_j^* \end{pmatrix} = \begin{pmatrix} 0 \\ + \end{pmatrix}$, $\begin{pmatrix} + \\ + \end{pmatrix}$, $\begin{pmatrix} + \\ 0 \end{pmatrix}$, or $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$. Yet, the last three possibilities have been considered in the above discussions. There exists an integer $z_1 \le z_2$ such that $x_i^* = 0$ and $y_i^* > 0$ for $i \in \mathcal{M}_1 = \{i : 1 \le i \le z_1\}$ where we set $z_1 = 0$ if $\mathcal{M}_1$ is empty. Any $i > z_1$ will fall into set $\mathcal{M}_2$, $\mathcal{M}_3$ or $\mathcal{M}_4$ as discussed earlier. □

Proof of Proposition D.2. With in-transit disposal cost, the objective function of the allocation problem is given by

$$\max_{\mathbf{x},\mathbf{y}\ge 0} \sum_{i\in\mathcal{M}} \left\{ E\left[p\min(D_i, x_i + y_i\epsilon_i) - c_d((1-\epsilon_i)y_i + (x_i + y_i\epsilon_i - D_i)^+)\right] - c_i(x_i + y_i)\right\} - c_t(Q - \sum_{i\in\mathcal{M}}(x_i + y_i)) \tag{D.17}$$

subject to

$$\sum_{i\in\mathcal{M}} x_i = Q\xi, \tag{D.18}$$

$$\sum_{i\in\mathcal{M}} y_i \le Q(1-\xi). \tag{D.19}$$

Let λ_x and λ_y denote the Lagrangian dual variables associated with constraints (D.18) and (D.19), respectively, where λ_x is free and $\lambda_y \ge 0$.[27] Let $\omega_i^x, \omega_i^y \ge 0$ denote the Lagrangian dual variables associated with nonnegativity constraints $x_i \ge 0$ and $y_i \ge 0$, respectively. By the complementary slackness, $\omega_i^x x_i = 0$ and $\omega_i^y y_i = 0$ for all $i \in \mathcal{M}$, $\lambda_x(Q\xi - \sum_{i\in\mathcal{M}} x_i) \ge 0$ and $\lambda_y(Q(1-\xi) - \sum_{i\in\mathcal{M}} y_i) \ge 0$.

[27] In fact, we can consider a more general case in which (D.18) is also replaced by inequality $\sum_{i\in\mathcal{M}} x_i \le Q\xi$ with a nonnegative dual variable $\lambda_x \ge 0$. It can be shown that whenever the constraint for y_i is binding, the constraint for x_i will be binding too. Intuitively, the firm always prefers to dispose of risky units first. Moreover, when ξ is deterministic, it is without loss of optimality to assume $\sum_{i\in\mathcal{M}} x_i = Q\xi$, because if any good unit is discarded in the second stage, then the firm should reduce the order quantity Q in the first stage.

Let L denote the Lagrangian function. By the KKT conditions, for all $i \in \mathcal{M}$, we have

$$\frac{\partial L}{\partial x_i} = (p - c_i) - (p + c_d)E_{\epsilon_i}[F_i(x_i + y_i\epsilon_i)] + c_t - \lambda_x + \omega_i^x = 0, \tag{D.20}$$

$$\frac{\partial L}{\partial y_i} = (p - c_i) - (p + c_d)(1 - E[\epsilon_i \bar{F}_i(x_i + y_i\epsilon_i)]) + c_t - \lambda_y + \omega_i^y = 0. \tag{D.21}$$

Let λ_x^B and λ_y^B denote respectively the Lagrangian dual variables associated with the equality constraints $Q^*\xi = \sum_{i\in\mathcal{M}} x_i^*$ and $Q^*(1-\xi) = \sum_{i\in\mathcal{M}} y_i^*$ in the optimal solution of the base model. Note that λ_y^B can be either positive or negative for the base model. For the optimal solution of the base model to remain optimal when in-transit disposal is allowed, we can construct a new solution with the Lagrangian variable λ_y^B replaced by $\lambda_y^* = c_t + \lambda_y^B$ and λ_x^B replaced by $\lambda_x^* = c_t + \lambda_x^B$ while all the other variables unchanged. The new solution satisfies the KKT condition (3) if $c_t \geq -\lambda_y^B$ such that the new Lagrangian dual λ_y^* is nonnegative. Thus, if $c_t \geq -\lambda_y^B$, the optimal solution to our base model is also optimal to the problem with in-transit disposal.

Now suppose that $c_t < -\lambda_y^B$. For a solution to satisfy the KKT condition, we will have $\sum_{i\in\mathcal{M}} y_i^* < Q(1-\xi)$ and $\lambda_y = 0$. We conducted a case-by-case analysis based on $\text{sign}(x_1, x_2, y_1, y_2)$. Define notation $\psi_i = \frac{p-c_i+c_t}{p+c_d}$. By assumption, $0 < \psi_2 < \psi_1 < 1$.

- Case $(+, 0, 0, 0)$. For $x_1^* = Q\xi$ and $x_2^* = y_1^* = y_2^* = 0$ to be optimal, we have $\omega_1^x = 0$, $\omega_2^x, \omega_1^y, \omega_2^y \geq 0$ and the KKT conditions require

$$\frac{\partial L}{\partial x_1} - \frac{\partial L}{\partial x_2} = 0 \iff F_1(Q\xi) \leq \Phi = \psi_1 - \psi_2,$$

$$\frac{\partial L}{\partial y_1} = 0 \iff \beta_1 \bar{F}_1(Q\xi) \leq 1 - \psi_1$$

and

$$\frac{\partial L}{\partial y_2} = 0 \iff \beta_2 \leq 1 - \psi_2.$$

- Case $(0, +, 0, 0)$. This case is never optimal because of the following. Given $x_1^* = 0, x_2^* = Q\xi$ and $y_1^* = y_2^* = 0$ and hence $\omega_1^x > 0$ and $\omega_2^x = 0$,

$$\frac{\partial L}{\partial x_1} - \frac{\partial L}{\partial x_2} = (c_2 - c_1) + (p + c_d)F_2(Q\xi) + \omega_1^x$$

which cannot equal zero since $c_2 > c_1$.

- Case $(+, +, 0, 0)$. We have $\omega_1^x = \omega_2^x = 0$ and $\omega_1^y, \omega_2^y \geq 0$. Let $x_1^* = x^\dagger$ and $x_2^* = Q\xi - x^\dagger$.

By the KKT conditions,

$$\frac{\partial L}{\partial x_1} - \frac{\partial L}{\partial x_2} = 0 \iff F_1(x^\dagger) - F_2(Q\xi - x^\dagger) = \Phi.$$

There exists $x^\dagger \in (0, Q\xi)$ that solves the above equation iff $F_1(Q\xi) > \Phi$. Moreover,

$$\frac{\partial L}{\partial y_1} = 0 \iff \beta_1 \bar{F}_1(x^\dagger) \leq 1 - \psi_1$$

and

$$\frac{\partial L}{\partial y_2} = 0 \iff \beta_2 \bar{F}_2(Q\xi - x^\dagger) \leq 1 - \psi_2.$$

- Case $(+, 0, +, 0)$. In this case, $\omega_1^x = \omega_1^y = 0$ and $\omega_2^x, \omega_2^y \geq 0$. It follows that

$$\frac{\partial L}{\partial y_1} = 0 \iff \beta_1 \bar{F}_1(Q\xi + y_1^*) = 1 - \psi_1$$

where there exists $y_1^* \in (0, Q(1-\xi))$ solving the above equation iff

$$\beta_1 \bar{F}_1(Q) < 1 - \psi_1 < \beta_1 \bar{F}_1(Q\xi).$$

Moreover,

$$\frac{\partial L}{\partial y_2} = 0 \iff \beta_2 \leq 1 - \psi_2$$

and

$$\frac{\partial L}{\partial x_1} - \frac{\partial L}{\partial x_2} = 0 \iff (1 - \beta_1)\bar{F}_1(Q\xi) \geq \psi_2.$$

- Case $(0, +, +, 0)$. In this case, $\omega_1^x \geq 0, \omega_2^x = 0$ and $\omega_1^y = 0, \omega_2^y \geq 0$. It follows that

$$\frac{\partial L}{\partial y_1} = 0 \iff \beta_1 \bar{F}_1(y_1^*) = 1 - \psi_1$$

where there exists $y_1^* \in (0, Q(1-\xi))$ solving the above equation iff

$$\beta_1 \bar{F}_1(Q(1-\xi)) < 1 - \psi_1 < \beta_1.$$

Moreover,

$$\frac{\partial L}{\partial y_2} = 0 \iff \beta_2 \bar{F}_2(Q\xi) \leq 1 - \psi_2$$

and

$$\frac{\partial L}{\partial x_1} - \frac{\partial L}{\partial x_2} = 0 \iff \bar{F}_2(Q\xi) \geq 2 - \beta_1 - \psi_2.$$

- Case $(+,+,+,0)$. In this case, $\omega_1^x = \omega_2^x = \omega_1^y = 0$ and $\omega_2^y \geq 0$. Consequently,

$$\frac{\partial L}{\partial y_1} = 0 \iff \beta_1 \bar{F}_1(x_1^* + y_1^*) = 1 - \psi_1$$

and $\frac{\partial L}{\partial x_1} - \frac{\partial L}{\partial x_2} = 0 \iff \bar{F}_2(x_2^*) - \beta_1 \bar{F}_1(x_1^* + y_1^*) - (1-\beta_1)\bar{F}_1(x_1^*) = \Phi$. Substituting $\beta_1 \bar{F}_1(x_1^* + y_1^*) = 1 - \psi_1$, $\Phi = \psi_1 - \psi_2$ and $x_2^* = Q\xi - x_1^*$ into the above, we have $x_1^* = x^\dagger$ where $x^\dagger$ solves $\bar{F}_2(Q\xi - x_1^*) - (1-\beta_1)\bar{F}_1(x_1^*) = 1 - \psi_2$. There exists a solution $x^\dagger \in (0, Q\xi)$ iff

$$\beta_1 - F_2(Q\xi) < 1 - \psi_2 \text{ and } (1-\beta_1)\bar{F}_1(Q\xi) < \psi_2.$$

and there exists $y^\dagger \in (0, Q(1-\xi))$ solving $\beta_1 \bar{F}_1(x^\dagger + y^\dagger) = 1 - \psi_1$ iff

$$\beta_1 \bar{F}_1(Q(1-\xi)) < 1 - \psi_1 < \beta_1.$$

Additionally,

$$\frac{\partial L}{\partial y_2} = 0 \iff \beta_2 \bar{F}_2(Q\xi - x^\dagger) \leq 1 - \psi_2.$$

- Cases $(\cdot,\cdot,0,+)$. These cases are not possible to be optimal. Note that the proof of Proposition D.1 is applicable to the cases with in-transit disposal. The lemmas established therein imply the following: $(0,+,0,+)$ is ruled out by Lemma D.1; $(+,0,0,+)$ is ruled out by Lemma D.2; $(+,+,0,+)$ is ruled out by Lemma D.3.

- Case $(+,0,+,+)$. This case is not possible to be optimal. See Lemma D.2 in the proof of Proposition D.1.

- Case $(0,+,+,+)$. Given $\omega_1^x \geq 0$ and $\omega_2^x = \omega_1^y = \omega_2^y = 0$, we have

$$\frac{\partial L}{\partial y_1} = 0 \iff \beta_1 \bar{F}_1(y_1^*) = 1 - \psi_1$$

and

$$\frac{\partial L}{\partial y_2} = 0 \iff \beta_2 \bar{F}_2(Q\xi + y_2^*) = 1 - \psi_2.$$

There exist $y_2^* = y_2^\dagger \in (0, Q(1-\xi))$ that solves the second equation iff

$$\beta_2 \bar{F}_2(Q) < 1 - \psi_2 < \beta_2 \bar{F}_2(Q\xi).$$

Then, there exist $y_1^* = y_1^\dagger \in (0, Q(1-\xi) - y_2^\dagger)$ solving the first equation iff

$$\beta_1 \bar{F}_1(Q(1-\xi) - y_2^\dagger) < 1 - \psi_1 < \beta_1.$$

Moreover,

$$\frac{\partial L}{\partial x_1} - \frac{\partial L}{\partial x_2} = 0 \iff \beta_2 \bar{F}_2(Q\xi + y_2^*) + (1-\beta_2)\bar{F}_2(y_2^*) - \beta_1 \bar{F}_1(y_1^*) - (1-\beta_1) \geq \Phi.$$

Substituting $\beta_1 \bar{F}_1(y_1^*) = 1 - \psi_1$ and $\beta_2 \bar{F}_2(Q\xi + y_2^*) = 1 - \psi_2$, the above condition reduces to

$$(1-\beta_2)\bar{F}_2(Q\xi) \geq 1 - \beta_1.$$

- Case $(+,+,+,+)$. Since all the other cases have been explored, if the conditions for all the above cases are not met, the optimal solution must belong to this case. The optimal allocation can be obtained by the KKT conditions given $\omega_1^x = \omega_2^x = \omega_1^y = \omega_2^y = 0$.

□

Proof of Proposition D.3. For cases (i)-(v), the proof follows a similar approach as in that of Proposition 1, i.e., verifying the KKT conditions given the corresponding solutions. In particular, for cases (ii), (iv) and (v), the first inequality condition guarantees that there exists a solution $x^\dagger \in (0, Q\xi)$, $y^\dagger \in (0, Q(1-\xi))$ and $y^\ddagger \in (0, Q(1-\xi))$, respectively, to the corresponding equation.

We thus omit the details of verifying KKT conditions for cases (i)-(v), but will elaborate on the proof to establish the "otherwise" part, case (vi). We prove case (vi) by showing that all the other forms of solution must not be optimal: (1) $x_1^* = y_1^* = 0$, x_2^* or $y_2^* > 0$; (2) $x_1^* > 0, y_1^* = 0, x_2^* = 0, y_2^* > 0$; (3) $x_1^* > 0, y_1^* = 0, x^2* > 0, y_2^* = 0$. The following result due to Horn (1979) will be useful.

Lemma D.4. (Horn 1979) *Let ϵ be a real random variable with non-degenerate distribution. Let h_1 and h_2 be real functions both either monotonically not increasing or monotonically not decreasing with $E[h_1(\epsilon)] < \infty$ and $E[h_2(\epsilon)] < \infty$. Then,*

$$E[h_1(\epsilon)h_2(\epsilon)] \geq E[h_1(\epsilon)]E[h_2(\epsilon)]$$

where the inequality holds strictly if the monotony of h_1 and h_2 is strict.

We prove a series of lemmas below.

Lemma D.5. (Ruling Out Form (1)) *In the optimal solution, if $x_1^* = y_1^* = 0$, then neither x_2^* nor y_2^* can be positive.*

Proof of Lemma D.5. If $x_1^* = y_1^* = 0$, then the multipliers $\omega_1^x \geq 0$ and $\omega_1^y \geq 0$. By (C.1) and (C.2), it follows that

$$p - c_1 - \lambda_x \leq 0 \tag{D.22}$$

and

$$p - c_1 - (p + c_d)(1 - \epsilon_1) - \lambda_y \leq 0. \tag{D.23}$$

Suppose $x_2^* > 0$. Then, the KKT conditions imply $0 = p - c_2 - (p + c_d)E_{\epsilon_2}[F_2(x_2^* + y_2^*\epsilon_2)] - \lambda_x < p - c_1 - \lambda_x$ (because $c_2 > c_1$) which contradicts (D.22). Similarly, suppose $y_2^* > 0$. Then, the KKT conditions imply $0 = (p - c_2) - (p + c_d)(1 - \bar{\epsilon}_2 + E_{\epsilon_2}[\epsilon_2 F_2(x_2^* + y_2^*\epsilon_2)]) - \lambda_y < p - c_1 - (p + c_d)(1 - \bar{\epsilon}_1) - \lambda_y$ because $c_1 < c_2$ and $\bar{\epsilon}_1 > \bar{\epsilon}_2$, which contradicts (D.23). $\square$

Lemma D.6. (Ruling Out Form (2)) *Any solution such that $x_1 > 0, y_1 = 0$ and $x_2 = 0, y_2 > 0$ must not be optimal.*

Proof of Lemma D.6. Suppose that the optimal solution is as described, the Lagrangian multipliers satisfy $\omega_1^x = 0, \omega_1^y \geq 0, \omega_2^x \leq 0$, and $\omega_2^y = 0$. By (C.5), it follows that $(p + c_d)E[(1 - \epsilon_1)\bar{F}_1(x_1)] - \lambda_x + \lambda_y \geq 0$ and $(p + c_d)E[(1 - \epsilon_2)\bar{F}_2(y_2\epsilon_2)] - \lambda_x + \lambda_y \leq 0$. Thus,

$$(1 - E[\epsilon_1])\bar{F}_1(x_1) \geq E[(1 - \epsilon_2)\bar{F}_2(y_2\epsilon_2)]. \tag{D.24}$$

On the other hand, by (C.4), we have $(c_2 - c_1) - (p + c_d)E[\epsilon_2\bar{F}_2(y_2\epsilon_2) - \epsilon_1\bar{F}_1(x_1)] \leq 0$. Since $c_2 > c_1$, it follows

$$E[\epsilon_1]\bar{F}_1(x_1) < E[\epsilon_2\bar{F}_2(y_2\epsilon_2)]. \tag{D.25}$$

In fact, (D.24) and (D.25) contradict each other. To see this, notice that from (D.24), we have $(1 - E[\epsilon_1])\bar{F}_1(x_1) \geq E[(1 - \epsilon_2)\bar{F}_2(y_2\epsilon_2)] \geq (1 - E[\epsilon_2])E[\bar{F}_2(y_2\epsilon_2)]$, where the last inequality follows by Lemma D.4 because $(1 - \epsilon_2)$ and $\bar{F}_2(y_2\epsilon_2)$ are both decreasing in ϵ_2. Clearly, (D.24) and (D.25) cannot simultaneously hold if $E[\epsilon_1] = 1$. For $E[\epsilon_1] < 1$, the inequality $(1 - E[\epsilon_1])\bar{F}_1(x_1) \geq (1 - E[\epsilon_2])E[\bar{F}_2(y_2\epsilon_2)]$ implies that

$$\begin{aligned} E[\epsilon_1]\bar{F}_1(x_1) &\geq \tfrac{E[\epsilon_1](1 - E[\epsilon_2])}{1 - E[\epsilon_1]} E[\bar{F}_2(y_2\epsilon_2)] \\ &\underset{(a)}{>} E[\epsilon_2]E[\bar{F}_2(y_2\epsilon_2)] \\ &\underset{(b)}{\geq} E[\epsilon_2\bar{F}_2(y_2\epsilon_2)] \end{aligned}$$

Inequality (a) holds as $E[\epsilon_1] > E[\epsilon_2]$. Inequality (b) follows by invoking Lemma D.4 for an increasing function ϵ_2 multiplied by a decreasing function $\bar{F}_2(y_2\epsilon_2)$. Therefore, we have a contradiction to (D.25).

□

Lemma D.7. (Ruling Out Form (3)) *Any solution such that* $x_1 > 0, y_1 = 0$ *and* $x_2 > 0, y_2 > 0$ *must not be optimal.*

Proof of Lemma D.7. Suppose that the optimal solution is as described. Then, the Lagrangian multipliers satisfy $\omega_1^x = 0, \omega_1^y \geq 0$, and $\omega_2^x = 0, \omega_2^y = 0$. By (C.3), it follows that

$$c_2 - c_1 - (p + c_d)[F_1(x_1) - E[F_2(x_2 + y_2\epsilon_2)]] = 0 \tag{D.26}$$

which holds only if $F_1(x_1) > E[F_2(x_2 + y_2\epsilon_2)]$, or equivalently, $E[\bar{F}_2(x_2 + y_2\epsilon_2)] > \bar{F}_1(x_1)$, since $c_2 > c_1$.

On the other hand, by (C.4), we have

$$c_2 - c_1 - (p + c_d)E[\epsilon_2\bar{F}_2(x_2 + y_2\epsilon_2) - \epsilon_1\bar{F}_1(x_1)] \leq 0. \tag{D.27}$$

Note that (D.26) and (D.27) together imply that

$$E[\epsilon_2\bar{F}_2(x_2 + y_2\epsilon_2) - E[\epsilon_1]\bar{F}_1(x_1)] \geq F_1(x_1) - E[F_2(x_2 + y_2\epsilon_2)],$$

or equivalently, $E[(1 - \epsilon_1)]\bar{F}_1(x_1) \geq E[(1 - \epsilon_2)\bar{F}_2(x_2 + y_2\epsilon_2)]$. Using Lemma D.4, we have $E[(1 - \epsilon_2)\bar{F}_2(x_2 + y_2\epsilon_2)] \geq E[(1 - \epsilon_2)]E[\bar{F}_2(x_2 + y_2\epsilon_2)]$. It follows that $E[(1 - \epsilon_1)]\bar{F}_1(x_1) \geq E[(1 - \epsilon_2)]E[\bar{F}_2(x_2 + y_2\epsilon_2)] \geq E[(1 - \epsilon_1)]E[\bar{F}_2(x_2 + y_2\epsilon_2)]$ where the second inequality is due to the fact that ϵ_1 is first-order stochastically larger than ϵ_2. Consequently, we have $\bar{F}_1(x_1) \geq E[\bar{F}_2(x_2 + y_2\epsilon_2)]$, and so (D.26) must not hold since $c_2 - c_1 > 0$. □

Except for the solution form $x_i^* > 0, y_i^* > 0$, all the other forms have been either included in cases (i)-(v) or eliminated by the above lemmas. Hence, case (vi) follows. □

References

BioCycle (2021) Landfill tipping fee analysis. Available at `www.biocycle.net/landfill-tipping-fee-analysis` accessed Aug 9, 2025.

Boyd S, Boyd SP, Vandenberghe L (2004) *Convex optimization* (Cambridge university press).

Buzby J, Bentley J, Padera B, Campuzano J, Ammon C (2016) Updated supermarket shrink estimates for fresh foods and their implications for ers loss-adjusted food availability data United States Department of Agriculture.

Erwin J (2024) We are developing a game-changing technique to save strawberries. Available at `https://blogs.ifas.ufl.edu/fshndept/2024/03/04/we-are-developing-a-game-changing-technique-to-save-strawberries` accessed May 23, 2025.

Hammam N, Abouelwafa S (2022) Technical, economic and marketing assessment to determine the losses of strawberry fruits during harvesting in the west nubaria region. *Middle East Journal of Agriculture Research* 11(03):733–746.

Holmes GJ (2024) The california strawberry industry: Current trends and future prospects. *International Journal of Fruit Science* 24(1):115–129.

Horn M (1979) Some inequalities for the expectation of a product of functions of a random variable and for the multivariate distribution function at a random point. *Biometrical Journal* 21(3):243–245.

Lee W, Slater G, C K (2013) How walmart distributes strawberries to 200 stores. Available at `https://projects.scpr.org/interactives/from-farm-to-you` accessed July 21, 2025.

Stewart H, Hyman J (2024) Farmers received about half of what consumers paid for fresh strawberries from 2020 – 23. USDA Economic Research Service.

Tomlin B, Wang Y (2005) On the value of mix flexibility and dual sourcing in unreliable newsvendor networks. *Manufacturing & Service Operations Management* 7(1):37–57.

Wang Y, Gilland W, Tomlin B (2010) Mitigating supply risk: Dual sourcing or process improvement? *Manufacturing & Service Operations Management* 12(3):489–510.

Zipkin PH (1980) Simple ranking methods for allocation of one resource. *Management Science* 26(1):34–43.