Sourcing from a Self-Reporting Supplier: Strategic Communication of Social Responsibility in a Supply Chain

Tao Lu[†] and Brian Tomlin[‡]

[†]School of Business, University of Connecticut, Storrs, CT 06269

[‡]Tuck School of Business at Dartmouth, Hanover, NH 03755

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Abstract

Problem Definition: To manage supplier social responsibility (SR), some firms have adopted a self-assessment strategy whereby they ask suppliers to self-report SR capabilities. Self-reported information is difficult to verify and this leads to an important credibility question: can a buyer expect truthful reporting? We examine whether a supplier's SR capability can be credibly communicated through free and unverifiable self-reporting.
Academic/Practical Relevance: SR is a strategic focus for firms because consumers care about ethical production. Some firms rely on supplier self-assessment as part of their SR strategy. It is important to understand the value and challenges of this approach.
Methodology: We develop a cheap talk model of a supplier and a buyer. The supplier is endowed with a given SR level (privately known to her) that represents the probability of no violation. The buyer sells in a market that is sensitive to publicized SR violations. The supplier first communicates her SR level to the buyer and then the buyer chooses between two audit stringency levels to conduct on the supplier and also chooses how much to order.
Results: Influential truthful communication may emerge in equilibrium if (i) the buyer orders a larger quantity from the high-type supplier but imposes a more stringent audit than he would for the low type, and (ii) the high-type supplier opts for this larger order whereas the low-type, fearing audit failure, does not. The buyer can benefit as the audit becomes more expensive.
Managerial Implications: Supplier SR self-assessments can be a valuable strategy for buyers, but only if the buyer has access to auditing capabilities of different levels and does not pre-commit to a particular level. It is valuable for firms to engage in an upfront auditing step to ensure a minimum SR capability of approved suppliers because very low performing suppliers will never truthfully report. Implementing supplier self-assessments may or may not help reduce the social damage due to potential SR violations; we identify situations when it helps and when it does not.

Key words: Social responsibility; sustainability; cheap talk; procurement; audits.

1 Introduction

Consumers are increasingly concerned about the ethical implications of how products are made. After the infamous Rana Plaza collapse which killed over a thousand people, some firms with supply chains traceable to the building encountered media scrutiny and an associated sales decline (Francis, 2014; Boudreau et al., 2015). Zara faced consumer boycott calls after unpaid workers at a supplier entered stores and secretly placed notes in clothes to publicize their plight. Nike, who fostered a reputation for ethical sourcing after much publicized issues in the 1990's, was subjected in recent years to protests by US "college students who claimed that Nike wasn't allowing independent investigators into its factories" (Segran, 2017). Social responsibility (SR) in sourcing has become a strategic focus of supply chain executives as consumer awareness of environmental, health, labor, and safety practices has grown (Chen and Lee, 2016). However, according to The Sustainability Consortium (2016), a majority of companies surveyed lacked visibility into their supply chains, and there was particularly-low visibility into worker health and safety, labor rights, and other SR practices. This presents a major challenge for firms seeking to engage in responsible sourcing. Not knowing a supplier's SR capability, i.e., its practices, preventative measures and operational controls, a firm might unwittingly expose itself to SR risk in its supply chain.

Supplier auditing is one technique that firms have adopted to address this challenge. Global retailers such as Aldi, Carrefour, Inditex, and Primark require a preliminary SR audit as a pre-requisite to becoming an approved supplier. Inditex calls these evaluations "pre-assessment audits", and in 2017, 79% of its 2252 pre-assessment audits resulted in the supplier being approved (Inditex, 2017). Passing a pre-assessment audit indicates that some minimum level of SR capability has been demonstrated; however, qualified suppliers may differ in their SR capabilities and the buying firm still has some uncertainty regarding a supplier's capability.[1] Auditing supplier SR risk does not stop with pre-assessment; firms are increasingly requiring that SR audits be conducted on approved suppliers. Over four thousand of Inditex's approved suppliers underwent a social audit in 2017 and many of these were also subjected to a more stringent special audit (as a complement to the regular social audit) that evaluated particular aspects around worker health and safety (Inditex, 2017).

Auditing a supplier's SR capability is costly and imprecise, and therefore to promote SR and reduce risk, some firms implement a self-assessment strategy in which suppliers self-report their SR capability. In the automotive industry, the Volvo Group requires that suppliers follow "ethical and sustainable standards [and Volvo] uses a self-assessment approach to evaluate supplier performance and compliance with [their] requirements" (Volvo, 2019).[2] At the start

[1]A preliminary SR audit might include review of third-party certification but even that would not fully determine a supplier's SR capability because certification is often not fully reliable. A certified factory in Pakistan caught on fire, killing at least 262 workers (Walsh and Greenhouse, 2012).

[2]An example of their self-assessment questionnaire can be found at `https://www.volvogroup.com/content/dam/volvo/volvo-group/markets/global/en-en/suppliers/our-supplier-requirements/sustainability-saq-original-english.pdf`.

of its purchasing cycle, Aldi asks qualified suppliers (i.e., those already having passed a pre-assessment audit) to fill out a self-assessment questionnaire regarding their SR capability (ALDI South Group, 2017). Aldi may follow up later with audits. Aldi and Volvo are not alone in their use of self assessment. The Home Depot leverages a combination of self-reporting and auditing to ensure ethical practices in its supply base, as does Carrefour (Home Depot, 2015; Carrefour Group, n.d.). The pharmaceutical company Sanofi assigns SR scores to suppliers based in part on their responses to self-assessment questionnaires (Sanofi, 2018).

Much of the information requested in SR self-assessment questionnaires can be difficult if not impossible to verify, and there is anecdotal evidence that suppliers have falsified SR-related documents in the past (Business Week, 2006; Wong, 2007). Thus, there may be a credibility concern with SR self-assessments if suppliers believe that better reported capabilities result in larger purchase volumes. This raises the question of whether a supplier's actual SR capability can be credibly communicated to a buyer through unverifiable self-reporting. (Although some portion of self-reported information might be verifiable at a cost, we consider unverifiable self-reporting to sharpen the focus on the credibility concern.) To answer this question, we develop a cheap talk game between a supplier (that has already been approved through a pre-assessment audit) and a buyer in a supply chain. The buyer (he) sells in a market where the selling price of the product is given and the market demand is sensitive to publicized SR violations. The supplier (she) is endowed with a given SR level (privately known to her) that represents the probability of no violation occurring. The buyer has a belief about the supplier's SR level, perhaps based on the pre-assessment evaluation, codified as a probability distribution. The supplier first communicates her SR level to the buyer through a free and unverifiable message. The buyer then updates his belief about the supplier's SR level (recall that Sanofi assigns scores after self assessment) and then the buyer chooses between a basic and a stringent audit level, e.g., Inditex conducting a regular social audit or one complemented by a special audit. Auditing may be imperfect. After the audit, the buyer chooses how much to order. To focus on one of our key research questions, we assume a simple wholesale price contract (as in many cheap talk papers); we note that it can be difficult to impose a contingent penalty on a noncomplying supplier in an emerging economy (Chen and Lee, 2016, p12).

We establish that credible and influential communication can occur in equilibrium even when the supplier's self-report message is free and unverifiable. To prove this, we first fully characterize the buyer's optimal auditing and ordering decisions if information is symmetric and show that the buyer's auditing and ordering decisions are complementary in the sense that a (weakly) larger order will be placed whenever a stringent audit is conducted and the supplier passes it.

We prove that (because of this complementarity) influential truthful communication may emerge in equilibrium under asymmetric information if (i) the buyer would conditionally order a larger quantity from the high-type supplier but impose a more stringent audit than what he would do for the low type, and (ii) the high-type supplier opts for this larger conditional order whereas the low-type, fearing a failure in the stringent audit, does not. Contrary to intuition, we establish that the buyer can be better off as the option of the stringent audit becomes more expensive because a lower auditing cost may entice the buyer to "over-audit" and therefore prevent credible communication. Additionally, we find that although it improves the buyer's payoff, supplier self-reporting may increase the expected social damage resulting from potential SR violations. We develop several model extensions and show that self-reporting can reduce social damage if a supplier can deceive auditors or if audits occur after production. We also examine the value of implementing pre-assessment audits before self-reporting. To the best of our knowledge, our work is the first to examine whether a supplier's SR level can be credibly self-reported to a buyer through free and unverifiable communication. A key novelty in our results lies in the mechanism leading to influential and truthful communication, which is driven by the buyer's joint ordering and auditing decisions; to our knowledge, such a mechanism has not been documented in the literature.

Our results provide several practical implications for the use of self assessments and audits to manage SR risk. We show that to induce truthful self-reporting by suppliers, a buyer must have multiple audit levels at its disposal and not precommit to a certain audit level. This can be done by developing some special audit methods which focus on specific compliance issues that are used as a complement to a regular SR audit; for instance, similar practices have been adopted by Carrefour (Carrefour, 2017, p65) and Inditex (Inditex, 2017, p30-33). Our findings further reveal that an appropriately expensive stringent option may benefit the buyer because it gives the buyer a certain commitment power to the desired differentiated auditing strategy which induces credible communication. Our results also indicate the importance of a pre-assessment audit to ensure that approved suppliers demonstrate a certain minimum SR capability: pre-approved suppliers with very low SR levels will always misreport their SR capability and so the buying firm can benefit by eliminating such firms through pre-assessment. In the desirable truth revealing equilibrium, a low-type supplier receives a positive order quantity if she honestly reports, a result in line with the practice of only approving suppliers that reach a certain minimum capability.

The rest of the paper is organized as follows. §2 reviews the related literature and §3 describes the model. §4 presents the main results. We discuss several model extensions and additional insights in §5 and conclude the paper in §6.

2 Literature Review

There is a growing operations management literature on SR risk in supply chains, e.g., Lee and Tang (2017), Lewis et al. (2017), Guo et al. (2017), Chen et al. (2018), Kraft et al. (2019), and many others. Within that stream of research, our work connects to papers that study sourcing strategies under supplier SR risk and to papers that study SR auditing and compliance.

Guo et al. (2015) study a buyer's optimal sourcing strategies in the presence of a responsible supplier and a less expensive supplier with SR risk, provided that some customers in the end market are socially conscious. Orsdemir et al. (2019) examine the situation in which a firm chooses whether to vertically integrate its supply chain to eliminate SR risk and, if integrated, it may possibly supply to a horizontal competitor. Agrawal and Lee (2019) study how a buyer's sourcing policies (i.e., whether or not to commit to offering a sustainable product only) influence the adoption of sustainable processes by suppliers. When sourcing from suppliers with SR risks, a firm may audit potential suppliers to mitigate SR risks. Chen et al. (2019) and Zhang et al. (2019) examine the impact of supply chain network structure on a firm's sourcing and auditing strategies.

Chen and Lee (2016) investigate the effects of process audits, contingency payment and certification when sourcing from a supplier with private information about its ethical level.[3] We also consider sourcing under asymmetric information but, whereas Chen and Lee (2016) employ a mechanism design approach, we study the credibility of costless and informal communication using a cheap talk framework under a given simple wholesale price contract without any contingent payments. As mentioned earlier, penalties contingent on suppliers' actual performance may be difficult to enforce on small suppliers in emerging economies (Chen and Lee, 2016). Generally speaking, unlike the mechanism design approach, our cheap talk model does *not* assume the buyer to have commitment power to any pre-announced contract menu. Hence, our cheap talk model reflects settings in which it is difficult for the buyer to pre-commit to auditing levels and/or purchase quantities.

Another stream of the SR sourcing literature examines the strategic interaction between supplier SR compliance and audits (or inspections). Kim (2015) studies a dynamic game between a regulator choosing inspection policies and a production firm controlling noncompliance disclosure. Plambeck and Taylor (2016) analyze a model in which the buyer determines the level of its auditing effort whereas the supplier may choose a level of effort to avoid violations of social responsibility as well as a level of hiding effort to evade the buyer's audit. They find that

[3] Chen and Lee (2016) model a supplier's ethical level as a disutility experienced by the supplier when committing a violation and the amount of this disutility is privately known to the supplier, whereas in our paper the supplier's private information is modeled as its capability of safeguarding against violations.

under certain conditions the increased auditing effort can backfire by reducing the supplier's responsibility effort. Caro et al. (2018) consider a supply chain consisting of one supplier choosing the compliance level and two buyers conducting costly audits. They examine the buyers' different auditing and penalty mechanisms: independently auditing and penalizing the supplier, independently auditing but collectively penalizing the supplier, and jointly auditing and imposing penalties. Fang and Cho (2015) study this joint audit problem faced by multiple competing buyers and find that a cooperative approach may not be effective due to the externalities of violations on buyers' payoffs. Cho et al. (2019) analyze the effects of the buyer's wholesale price decision and the audit level in combating child labor, and show that these two decisions are strategic substitutes. This compliance-auditing literature often adopts a simultaneous-move game framework in which the buyer(s) and the supplier determine their auditing and compliance efforts respectively without observing each other's choice of the effort level (but knowing each other's cost function); see Plambeck and Taylor (2016) and Caro et al. (2018). We complement this literature by examining whether or not a supplier's unverifiable compliance level can be truthfully communicated with a buyer if it is predetermined before the buyer's auditing decision.

Besides the aforementioned SR literature, our paper relates to the cheap talk literature pioneered by Crawford and Sobel (1982). Whether cheap talk can lead to truthful information transmission has been studied in various operations-relevant contexts, such as delay announcement in service systems (Allon et al., 2011) and sharing inventory availability information by a retailer (Allon and Bassamboo, 2011). In the context of supply chain management, Ren et al. (2010) demonstrate a repeated interaction with a proper customer review strategy can incentivize truthful demand forecast sharing between supply chain parties. Özer et al. (2011) discuss the role of trust and trustworthiness in sharing demand forecast via cheap talk. Shamir and Shin (2015) examines the possibility of truthful demand information sharing by an incumbent to influence the entry decision of an entrant. Chu et al. (2017) show that when the supplier, as the receiver, determines both the wholesale price and capacity, the downstream retailer may truthfully report its demand forecast. Berman et al. (2018) study a cheap talk model where a supplier has private forecast about the probability of releasing a new product, and a retailer decides how to allocate its limited capacity between an existing product and the potential new product. The authors find that postponement may facilitate truthful communication in a supply chain. In this paper, we uncover a novel rationale for truthful cheap talk regarding a supplier's social responsibility level, which rests on the buyer's joint decisions on auditing stringency and the order quantity.

Finally, our work is somewhat related to the literature on sourcing under supply quality/reliability risks. Some papers explore incentive mechanisms (e.g., deferred payments) as a means to ensure supply quality (e.g., Babich and Tang, 2012; Rui and Lai, 2015, and references therein), and some examine how a supplier's private information can be signaled or screened through contract terms (e.g., Yang et al., 2009; Gümüş et al., 2012; Bakshi et al., 2015, and references therein).

3 The Model

We begin by providing a high level overview of the model before presenting and formalizing the detailed description.

The supply chain consists of a pre-approved supplier (she) and a buyer (he). The market into which the buyer sells reacts negatively if an SR violation occurs and becomes publicly known. Only the supplier is prone to SR violations; the buyer is not. The supplier is endowed with a given SR level which measures the supplier's capability of safeguarding against SR violations; a higher level indicates a lower probability of an SR violation. A violation may be occurring unbeknownst to the supplier; this reflects the fact that a supplier's SR control capability is imperfect. For example, environmental, health, labor, and safety standards may be unwittingly violated at a supplier even if it faithfully follows its own (imperfect) SR procedures. The buyer has a belief about the supplier's SR level (e.g., from a pre-assessment audit) but the actual SR level is privately known to the supplier. The supplier reports her SR level to the buyer through a free but unverifiable message, e.g., via some self-reported SR assessment or other communication. The buyer chooses an audit level to impose on the supplier in an attempt to discover any SR violation but this audit may be imperfect. After the audit, the buyer chooses the quantity to order from the supplier and pays an exogenously-given, per-unit wholesale price. Our main results continue to hold if the audit is conducted during (or after) production as discussed in Appendix B. The assumption that the sender (i.e., the supplier in our model) sells its product at an exogenous price is quite standard in the cheap talk literature (e.g., Allon and Bassamboo (2011) and Berman et al. (2018)) because it enables a focus on the role of free communication in information transmission rather than a focus on costly signaling through the use of endogenous and more sophisticated contracts. Our model represents a setting in which the supply chain parties first agree on a wholesale price and then engage in nonbinding communication. We now describe the model in detail.

Supplier SR Types The supplier's SR level, denoted by θ, is the probability that no SR violation occurs, and so $1 - \theta$ is the probability that a violation occurs. We refer to θ as the supplier's type and we assume that θ can take on one of two possible values in $\Theta = \{\theta_L, \theta_H\}$ where $0 \leq \theta_L < \theta_H \leq 1$, that is, SR violations are less likely to occur at a type-H supplier. If an SR violation occurs then it becomes discovered by the public with probability δ. For expositional simplicity, we assume that the supplier's per-unit production cost c is independent of its type although type-dependent production costs would not impact our findings. The prior probability distribution of θ is common knowledge and denoted by $\lambda(\theta)$. The supplier privately observes her type θ and sends a message m from a message space M to the buyer claiming her type. The buyer updates his belief about the supplier's type based on this message m. We will use $\beta(\theta|m)$ to denote the buyer's posterior probability that the supplier is of type-θ. This updating will be formalized later in this section when we describe our equilibrium approach.

Auditing After receiving the message m and updating his belief, the buyer chooses between a basic and a stringent audit to conduct on the supplier. In our model, the basic audit can be interpreted as a regular inspection whereas the stringent audit can represent a regular inspection together with a special audit which checks the specific compliance issues such as worker health and safety.[4] The cost of conducting a stringent audit is $c_A > 0$, and the cost of a basic audit is normalized to zero. An SR violation, if occurring, is detected by the basic (stringent) audit with probability γ_N (γ_A); the values of γ_N and γ_A represent the accuracies of the basic and stringent audits, respectively, where $0 \leq \gamma_N < \gamma_A \leq 1$. That is, the stringent audit is more accurate but more costly than the basic one. For ease of exposition, we will set $\gamma_N = 0$ and refer to the basic and stringent audit options as "no audit" and "audit", respectively, in the remainder of the paper. That is, we will say that the buyer decides to audit the supplier if he imposes an audit accuracy γ_A at a cost c_A, and that the buyer decides not to audit otherwise. Our main results can be extended when γ_N takes any value between 0 and γ_A.[5] The probability of a type-θ supplier passing an audit with accuracy γ is given by $\rho(\theta, \gamma) = \theta + (1 - \theta)(1 - \gamma)$. Conditional on a supplier passing an audit with accuracy γ, it follows from Bayes' rule that the buyer's posterior belief of the supplier's type $\beta(\theta|m)$ is updated to be $\hat{\beta}(\theta|m, \gamma) = \frac{\rho(\theta,\gamma)\beta(\theta|m)}{\sum_{\theta' \in \Theta} \rho(\theta',\gamma)\beta(\theta'|m)}$. Note that when the buyer chooses not to audit (i.e., $\gamma = 0$), then $\rho(\theta, 0) = 1$ and $\hat{\beta}(\theta|m, 0) = \beta(\theta|m)$. As is common in the literature (e.g., Babich and Tang, 2012; Plambeck and Taylor, 2016; Caro et al., 2018; Cho et al., 2019), we assume the audit cost structure, i.e., the values of c_A, γ, to be common knowledge.[6]

[4]As mentioned in the introduction, special audits can be used as a complement to regular social audits (Inditex, 2017, p33). In a similar spirit, Cho et al. (2019) also assumes the buyer can choose between two audit levels, which represents the practice where firms may or may not have specific evaluation processes on the risks of human trafficking and slavery; see the examples provided in their Online Appendix B.

[5]The analysis and results with a general γ_N are available upon request from the authors.

[6]We note that similar to our model, Cho et al. (2019) considers a buyer choosing between two auditing accuracy levels, whereas Plambeck and Taylor (2016) and Caro et al. (2018) allow for a continuous auditing cost function that is increasing and convex in the auditing accuracy.

Order Quantities and Market Revenues After deciding whether or not to audit the supplier and after the audit result is known, the buyer determines an order quantity $Q \geq 0$ to place with the supplier at a given per-unit wholesale price w. An exogenous wholesale price assumption is not uncommon in the SR literature, (e.g., Plambeck and Taylor, 2016), and as noted above is common in the cheap talk literature. We use the random variable ξ to denote whether or not there is a publicly-revealed SR violation: $\xi = 0$ if a violation occurs and is revealed and $\xi = 1$ if there is no violation or a violation occurs but is not discovered by the public. The buyer's revenue depends on both the quantity it has to sell (which is the same as the order quantity Q) and whether or not there is publicly-revealed SR violation, which we denote generally by $R(Q|\xi)$. In the base model, we specifically assume that the buyer sells the product at a given price $p > w$ and that the market demand is equal to d_1 if no violation is exposed to the public (i.e., $\xi = 1$) and $d_0 < d_1$ otherwise. That is, the total potential number of customers in the market is given by d_1, each demanding one unit of product; among them, $d_1 - d_0$ customers are socially responsible and will refuse to buy the product if an SR violation in the supply chain is discovered. Normalizing the salvage value of excess inventory to zero, we have $R(Q|1) = p \min\{d_1, Q\}$ and $R(Q|0) = p \min\{d_0, Q\}$. In Online Appendix C, we will establish that our key findings apply to a very general revenue function $R(Q|\xi)$.

In addition to the reduced revenue, the buyer incurs a fixed goodwill loss g in the event of a publicly-revealed SR violation. The two different forms of losses capture the reality that a publicly-revealed SR violation in a firm's supply chain often leads to a decline in sales volume due to boycotting by some consumers as well as a negative shock in the stock market; see, for instance, Boudreau et al. (2015).[7]

To focus on interesting cases, we make the following mild assumption throughout the paper.

Assumption 1. If the supplier fails an audit, it is optimal for the buyer to order nothing from the suppler.

This assumption is in line with the SR literature (e.g., Plambeck and Taylor, 2016; Caro et al., 2018) and satisfied in our setting as long as the fixed goodwill loss is sufficiently large: $g \geq \max\{(p-w)d_0, (p-w)d_1 - \delta p(d_1 - d_0)\}/\delta$; see Lemma A.1 in Appendix. So, as in most existing papers, an audit failure in our model means that a material breach is found which is significant enough to result in a termination of the contract with the supplier. In practice, if a supplier fails an audit, the buyer may instead source from an backup supplier. We can regard this as an outside option and normalize the expected profit from using the backup supplier to

[7]Our model can readily be extended if the goodwill loss is a (weakly) convex function of the available-to-sell quantity: one can incorporate this cost into the revenue function $R(Q|0)$.

Table 1: Summary of the notation for useful probabilities

δ	probability that a violation will be publicly revealed if it exists
γ	probability that a violation will be detected by the audit if it exists
$\rho(\theta,\gamma)$	probability that a type-θ supplier will pass an audit with accuracy γ
$\psi(\theta,\gamma,\delta)$	probability that no violation will be publicly revealed if the supplier is of type θ and passes an audit accuracy γ
$\lambda(\theta)$	prior probability that the supplier is of type θ
$\beta(\theta\|m)$	buyer's posterior probability that the supplier is of type θ given message m
$\hat{\beta}(\theta\|m,\gamma)$	buyer's updated posterior probability that the supplier is of type θ given message m and no violation detected in an audit with accuracy γ

zero.

If the supplier passes an audit, the buyer will determine an order quantity to maximize his expected profit. There is still a chance that a publicly-revealed SR violation happens, i.e., $\xi = 0$, because the audit may be imperfect. We assume that a violation, if occurring, will be revealed by the public with an exogenous probability δ. By Bayes' rule, if a type-θ supplier passes an audit with accuracy γ, the probability of no publicly-revealed SR violation occurring is given by

$$\psi(\theta,\gamma,\delta) = \frac{\theta + (1-\delta)(1-\gamma)(1-\theta)}{\theta + (1-\gamma)(1-\theta)}.$$

Note that $\psi(\theta, 0, \delta) = 1 - \delta(1-\theta)$ for the case when no audit is conducted.

If the buyer chooses to audit the supplier, then he determines an order quantity Q, conditional on the supplier passing the audit, to maximize his expected profit denoted by $\Pi_b^A(Q)$. If the buyer chooses not to audit, then he chooses Q to maximize an expected profit $\Pi_b^N(Q)$. For $i = \{N, A\}$, the buyer's (conditional) expected profit can be written as

$$\Pi_b^i(Q) = \sum_{\theta\in\Theta} \hat{\beta}(\theta|m,\gamma_i)\left[\psi(\theta,\gamma_i,\delta)R(Q|1) + (1-\psi(\theta,\gamma_i,\delta))(R(Q|0) - g\mathbf{1}(Q>0))\right] - wQ, \quad (1)$$

where $\mathbf{1}(x)$ is an indicator function which equals one if the statement x is true and equals zero otherwise. For $i \in \{N, A\}$, let $Q^i(m)$ denote the maximizer of (1); then, $Q^A(m)$ represents the buyer's optimal order quantity conditional on the supplier passing an audit whereas $Q^N(m)$ represents the optimal order quantity when no audit is conducted.[8] Table 1 summarizes the notation of useful probabilities in our model.

Buyer and Supplier Profit Functions Let $a(m)$ represent the buyer's audit decision based on the supplier's message m, where $a(m) = 1$ if the buyer chooses to audit and $a(m) = 0$

[8]If multiple optimal solutions exist for the buyer's ordering decision, we break ties by setting the quantity as the smallest maximizer. This applies to all the profit-maximizing order quantities throughout the paper.

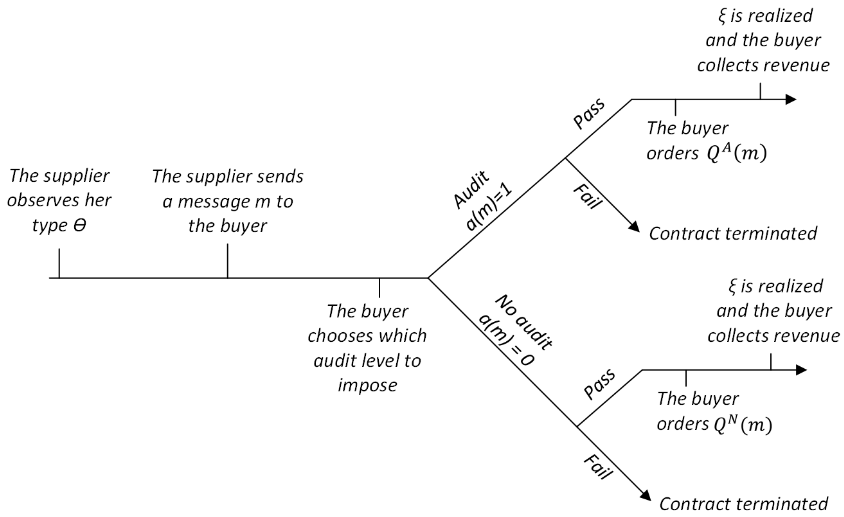


Figure 1: Sequence of events

otherwise. Given any message m, the buyer's ex ante expected profit can be written as

$$\Pi_b(a) = a\left[\Pi_b^A(Q^A(m))\sum_{\theta\in\Theta}\beta(\theta|m)\rho(\theta,\gamma_A) - c_A\right] + (1-a)\Pi_b^N(Q^N(m)). \tag{2}$$

The buyer maximizes (2) over $a \in \{0,1\}$. If the buyer is indifferent between auditing and not auditing, we assume that he will choose not to audit.

The supplier's ex-ante expected profit depends on the buyer's audit and order decisions as well as her own type θ. The supplier's expected profit is

$$\Pi_s(\theta, a(m), Q^N(m), Q^A(m)) = \begin{cases} (w-c)Q^A(m)\rho(\theta,\gamma_A) & \text{if } a(m) = 1 \\ (w-c)Q^N(m) & \text{if } a(m) = 0 \end{cases} \tag{3}$$

We intentionally omit any ex post penalty that might be imposed on the supplier. When the supplier fails in the audit, she may bear a loss of reputation if the buyer discloses the negative audit result to other buying firms. Our main results can readily be extended if the supplier incurs a fixed loss due to a failure in the audit; for ease of exposition, we choose not to include it in the model. Furthermore, a publicly-revealed violation may also have a negative consequence on the supplier, possibly imposed by the buyer through certain contingent contract terms. Our assumption of the simple wholesale price contract amplifies the conflict of interest between the buyer and supplier because the SR risk is completely transferred to the buyer once an order is placed. We are therefore able to focus on the key driver for truthful communication while relegating the effect of risk sharing mechanisms.

Timeline Summary The sequence of events is as follows: (i) first, the supplier realizes her SR level θ and sends a message $m \in M$ to the buyer; (ii) upon receiving the message m, the buyer forms a posterior belief $\beta(\theta|m)$ about the supplier's type, based on which he decides whether to conduct an audit by maximizing (2); (iii) if the buyer conducts an audit, then he orders $Q^A(m)$ conditional on the supplier passing to maximize (1) for the case of $i = A$ but orders nothing if the supplier fails; otherwise, the buyer orders $Q^N(m)$ to maximize (1) for the cases of $i = N$; (iv) the supplier's SR violation, if any, is potentially discovered by the public, i.e., ξ is realized, and the buyer receives the resulting revenue. See Figure 1 for an illustration of the timeline. Again, we emphasize that we analyze a setting in which the audit is conducted during (or after) production in Appendix B and our key results hold. We note that in practice the buyer might request a supplier to improve its SR level after a low self-assessment report. According to Inditex (2017, p31, p35), however, a supplier's improvement activities usually take many months during which production of the order continues. As such, we do not consider improvement requests or activities. Our model reflects the practice in which improvement take a long time and does not affect current production.

Equilibrium Approach We adopt the pure-strategy perfect Bayesian Nash equilibrium (PBNE) as our solution concept. The supplier's reporting strategy $\mu : \Theta \to M$, the buyer's auditing and ordering decisions $a(m)$, $Q^N(m)$ and $Q^A(m)$, and the buyer's posterior belief $\beta(\theta|m)$ constitute a PBNE if the following holds:

(i) Anticipating the buyer's optimal auditing and ordering decisions, the supplier chooses a message $m \in M$ to maximize her expected profit, i.e.,

$$\mu(\theta) = \arg\max_{m \in M} \Pi_s(\theta, a(m), Q^N(m), Q^A(m)) \text{ for all } \theta \in \Theta.$$

(ii) Given any message $m \in M$, $a(m) = \arg\max_{a \in \{0,1\}} \Pi_b(a)$, and $Q^i(m)$ maximize $\Pi_b^i(Q)$ defined in (1) for all $i \in \{N, A\}$.

(iii) The buyer's belief is consistent on the equilibrium path, i.e., $\beta(\theta|m) = \frac{\mathbf{1}(\mu(\theta)=m))\lambda(\theta)}{\sum_{\theta' \in \Theta} \mathbf{1}(\mu(\theta')=m)\lambda(\theta')}$ if $\sum_{\theta' \in \Theta} \mathbf{1}(\mu(\theta') = m)\lambda(\theta') > 0$ and $\beta(\theta|m) = 0$ otherwise.

Because the supplier's message is unverifiable and costless, the supply chain interaction is a cheap talk game. We refer to a PBNE as a *babbling equilibrium* if the communication is uninformative, that is, $\beta(\theta|m) = \lambda(\theta)$ for all $m \in M$ and $\theta \in \Theta$, i.e., in equilibrium the buyer's posterior belief remains the same as his prior. We use the term *truth revealing* equilibrium to refer to an informative PBNE in which each type of supplier honestly reports her SR level and the buyer's posterior belief is given by $\beta(\theta|m) = 1$ and $\beta(\theta'|m) = 0$ for all $m \in \mu(\theta)$, $\theta, \theta' \in \Theta$ and $\theta \neq \theta'$.

One important question of this work is whether or not a truth revealing equilibrium exists in our game.

4 Results

4.1 Symmetric Information

To better understand the rationale for truthful communication, it is instructive to first characterize the buyer's optimal auditing and ordering decisions under symmetric information. Suppose for now that the buyer knows the supplier's SR level θ. Then, the buyer's ordering decision is to determine a quantity Q to maximize

$$\Pi_b^i(Q|\theta) = \psi(\theta,\gamma_i,\delta)p\min(d_1,Q) + (1-\psi(\theta,\gamma_i,\delta))(p\min(d_0,Q) - g\mathbf{1}(Q>0)) - wQ$$

where $i = A$ or N for the case with or without an audit, respectively. For ease of notation, we will simply write $\Pi_b^i(Q|\theta)$ as $\Pi_b^i(Q)$ whenever appropriate.

Lemma 1. *Under symmetric information, for any SR level θ, the buyer's ordering decision is characterized as follows: (i) If an (no) audit is conducted, then conditional on the supplier passing, the buyer's optimal order quantity $Q^A(\theta)$ ($Q^N(\theta)$) is given by*

$$Q^i(\theta) = \begin{cases} d_1 & \text{if } \theta > \eta_1^i \text{ and } \theta > \eta_0^i \\ d_0 & \text{if } \theta \le \eta_1^i \text{ and } \theta > \eta_0^i \\ 0 & \text{if } \theta \le \eta_0^i; \end{cases} \tag{4}$$

where $i \in \{N, A\}$; the thresholds in the above expressions are defined as $\eta_1^i = H_1(\gamma_i)$, $\eta_0^i = H_0(\gamma_i)$ where $H_1(\gamma) = \left[1 - \frac{p-w}{(1-\gamma)\delta p + \gamma(p-w)}\right]^+$ and

$$H_0(\gamma) = \min\left(1 - \frac{(p-w)d_1}{(1-\gamma)\delta[g + p(d_1-d_0)] + \gamma(p-w)d_1}, 1 - \frac{(p-w)d_0}{(1-\gamma)\delta g + \gamma(p-w)d_0}\right).$$

(ii) These thresholds of θ satisfy $\eta_1^A \le \eta_1^N$ and $\eta_0^A \le \eta_0^N$, and consequently, $Q^N(\theta) \le Q^A(\theta)$ for any given θ. (iii) For all $i \in \{N, A\}$, $Q^i(\theta)$ is increasing in θ.

The proofs of all our analytical results are provided in the Supplementary Material of the paper, available from the authors upon request. Part (i) of Lemma 1 explicitly characterizes the buyer's order decisions with and without an audit. As shown in part (ii), the buyer orders a (weakly) larger quantity if he chooses to audit and the supplier passes it than if he conducts no audit at all. This order size increase reflects the role the audit plays in reducing the uncertainty of supplier compliance. Part (iii) presents an intuitive result that the buyer tends to order more given a more reliable supplier. This implies that if there was asymmetry in the supplier SR information then a supplier might exaggerate her SR level so as to attract a larger order. This could then be a barrier to the existence of a truth revealing equilibrium under asymmetric information. In general, Lemma 1 indicates that the buyer's optimal order quantity is tailored according to the supplier's SR risk. This is consistent with the practice of SR sourcing strategies. For instance, according to Inditex's annual report, the company allocates smaller (but positive) production volumes to those suppliers with lower SR performance (Inditex, 2017, p31).

To analyze the buyer's audit decision, we define the value of an audit on a supplier with SR level θ as

$$VoA(\theta) = \Pi_b^A(Q^A(\theta))\rho(\theta, \gamma_A) - \Pi_b^N(Q^N(\theta)),$$

where, as defined in §3, $\rho(\theta, \gamma_A) = 1 - \gamma_A(1-\theta)$, is the probability that supplier passes the audit. Clearly, it is optimal to conduct an audit if and only if the value of an audit $VoA(\theta)$ exceeds the audit cost c_A. Define $v(\theta, q^N, q^A) = \Pi_b^A(q^A)\rho(\theta, \gamma_A) - \Pi_b^N(q^N)$; it represents the value of audit on a supplier with SR level θ when the buyer's order quantities are q^N and q^A for the cases where no audit is conducted and where an audit is conducted (and the supplier passes it), respectively. Because the optimal order quantities $Q^A(\theta)$ and $Q^N(\theta)$ depend on the supplier's SR level θ, $VoA(\theta)$ is a piece-wise function consisting of $v(\theta, q^N, q^A)$ with different pairs of (q^N, q^A). The closed form expressions of $VoA(\theta)$ and $v(\theta, q^N, q^A)$ can be found in Lemma A.2 of Appendix A.

Is it more or less valuable for the buyer to conduct an audit if the supplier is less reliable? The answer is not straightforward because the supplier's SR level influences the value of audit in two opposing ways. On the one hand, a higher SR level reduces the necessity of audit because a violation is less likely to happen. On the other hand, as shown in part (ii) of Lemma 1, conducting an audit can potentially enable the buyer to order more and target a higher revenue (if the supplier passes) than what he would do without an audit. A higher SR level increases this potential, thereby giving the buyer a greater incentive to audit. This intuition is formalized in Lemma A.3 of Appendix A, which establishes that $VoA(\theta)$ is either unimodal or bimodal in θ. Consequently, there can be at most two disjoint intervals of θ in which the buyer should conduct an audit for a fixed c_A, as formally characterized in Proposition 1 below. For convenience, we define a threshold $\Delta = \frac{\gamma_A[\delta g - (p-w)d_0]}{(1-\gamma_A)\delta p + \gamma_A(p-w)}$.

Proposition 1. *Under symmetric information, for any given c_A, the buyer's optimal audit decision is characterized as follows*[9]*: (i) If $d_1 - d_0 > \Delta$ and $\eta_0^N \leq \eta_1^A$, it is optimal to conduct an audit, i.e., $a(\theta) = 1$, if and only if $\theta \in (\theta^{(4)}, \theta^{(3)}) \cup (\theta^{(2)}, \theta^{(1)})$ where $\theta^{(1)}$, $\theta^{(2)}$, $\theta^{(3)}$ and $\theta^{(4)}$ are*

[9] For ease of exposition, we allow the intervals $(\theta^{(1)}, \theta^{(2)})$, $(\theta^{(3)}, \theta^{(4)})$ and (θ^-, θ^+) to be empty.

the intersection between c_A and $v(\theta, d_1, d_1)$, $v(\theta, d_0, d_1)$, $v(\theta, d_0, d_0)$ and $v(\theta, 0, d_0)$, respectively; (ii) otherwise, there exist two thresholds θ^- and θ^+ such that it is optimal to conduct an audit, i.e., $a(\theta) = 1$, if and only if $\theta \in (\theta^-, \theta^+)$.

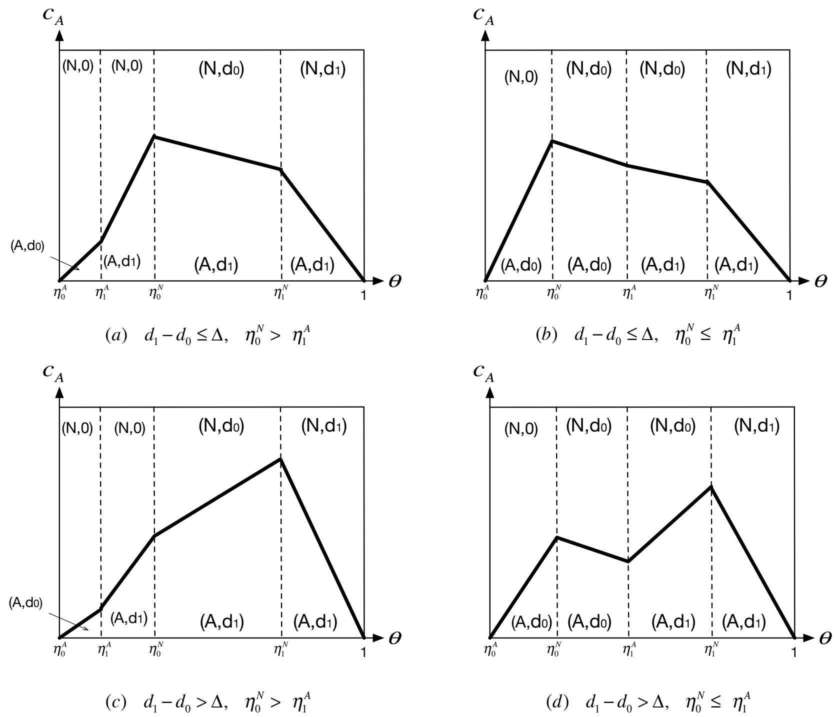


Figure 2: Illustration of the buyer's optimal solution under symmetric information; (N, q) =no audit and order q units; (A, q) =audit and conditionally order q units; for illustrative purpose, we assumed $\eta_0^N < \eta_1^N$ in the graphs.

The closed-form expressions of the $\theta^{(i)}$'s can be found in the proof (see Equation (A.1) in Appendix D) whereas the values of θ^- and θ^+ depend on where $VoA(\theta)$ intersects with c_A. Figure 2 depicts the buyer's optimal ordering and auditing decisions under symmetric information for any pair of c_A and θ; for illustrative purpose, we assume $\eta_0^N < \eta_1^N$ in the graphs.[10] The thick piecewise-linear curve in each panel represents $VoA(\theta)$. The buyer should conduct an audit if and only if c_A is below the curve. Panel (d) illustrates part (i) of Proposition 1, whereas the other panels correspond to part (ii) of Proposition 1. The space of θ is segmented by the thresholds defined in Lemma 1, by which one can pin down the buyer's optimal order quantity (with or without audit).[11] Finally, it is worthwhile noting that the buyer's auditing and ordering decisions are complementary: If an audit is imposed, the buyer's order quantity is (weakly)

[10]It follows from Lemma 1 that if $\theta \le \eta_0^A$, then $Q^A = Q^N = 0$ and $VoA(\theta) = 0$, and so the buyer will not audit or consider sourcing from such a low SR level supplier. Therefore, we focus on $\theta \in (\eta_0^A, 1]$ here.

[11]In the analysis, we generally allow $\eta_0^N \ge \eta_1^N$ and/or $\eta_0^A \ge \eta_1^A$. To illustrate all the possible decisions, $\eta_0^N < \eta_1^N$ and $\eta_0^A < \eta_1^A$ are assumed in Figure 2. When $\eta_0^N \ge \eta_1^N$ (reps. $\eta_0^A \ge \eta_1^A$), the interval $(\eta_0^N, \eta_1^N]$ (resp. $(\eta_0^A, \eta_1^A]$) will be degenerated.

higher (conditional on the supplier passing) than that if no audit is conducted. As will be shown later, this complementarity will play a crucial role in truthful communication.

4.2 Asymmetric Information

We now consider the asymmetric information setting described in §3 in which the supplier communicates an SR level to the buyer through a costless and unverifiable message m. We note that we can restrict the message space M to be $\{\theta_L, \theta_H\}$ without loss of generality, and therefore a message m can be interpreted as the type reported by the supplier. For expositional ease, we will use the following shorthand notation: $Q_t^N = Q^N(m_t)$, $Q_t^A = Q^A(m_t)$, and $a(m_t) = a_t$ for $t \in \{H, L\}$. We will refer to a supplier with an SR level θ_t as a type-t supplier. In the following, we will first provide conditions under which a truth revealing equilibrium exists and then explore how the audit cost c_A affects the equilibrium and the buyer's ex ante profit.

4.2.1 Characterization of Truth Revealing Equilibrium

In any truth revealing equilibrium (if one exists), a type-t supplier should not have any incentive to mimic the other type. This requirement can be formulated as the supplier's incentive compatible (IC) condition:

$$\Pi_s(\theta_t, a_t, Q_t^N, Q_t^A) \geq \Pi_s(\theta_t, a_{t'}, Q_{t'}^N, Q_{t'}^A), \quad \forall\, t, t' \in \{H, L\},\, t \neq t,' \tag{5}$$

where $\Pi_s(\cdot, \cdot, \cdot, \cdot)$ is given by (3). We will call a truth revealing equilibrium "noninfluential" if the buyer takes exactly the same action in equilibrium irrespective of the supplier's type; we will call it "influential" otherwise.[12] A truth revealing but noninfluential equilibrium may trivially exist if θ_L and θ_H are such that $a_H = a_L = 0$ and $Q_L^N = Q_H^N$, or $a_H = a_L = 1$ and $Q_L^A = Q_H^A$. In what follows, we will focus on the *influential* truth revealing equilibrium in which the buyer takes different actions towards different types of suppliers.

Proposition 2. *Under asymmetric information, an influential truth revealing equilibrium exists if and only if both following conditions hold: (i) if information was symmetric, the buyer would conduct an audit only on the high-type supplier, i.e., $VoA(\theta_H) > c_A \geq VoA(\theta_L)$; (ii) if information was symmetric, the low-type supplier would receive an order $Q_L^N = d_0$, whereas the high-type would receive an order $Q_H^A = d_1$ conditional on it passing the audit, and their SR levels θ_L and θ_H are such that $\rho(\theta_L, \gamma_A) \leq \frac{d_0}{d_1} \leq \rho(\theta_H, \gamma_A)$.*

To explain this result, we first note that an influential truth revealing equilibrium can exist

[12] See Berman et al. (2018) for a similar definition.

only if the buyer intends to follow a differentiated auditing strategy, i.e., to impose the audit on only one of the types. Otherwise, unless the buyer's order quantity is the same for both supplier types (in which case the equilibrium cannot be influential), the low-type supplier would always mimic the high type for a larger order because the buyer's audit decision is invariant to what type she reports. Furthermore, one can show that if the buyer intends to only audit the low-type supplier, then the supplier's IC condition (5) cannot be satisfied due to the fact that the low-type supplier is more vulnerable to an audit (i.e., a higher risk of failing) than a high type, i.e., $\rho(\theta_L, \gamma_A) < \rho(\theta_H, \gamma_A)$.[13] Thus, an influential truth revealing equilibrium can exist only when the buyer intends to only audit the high type, i.e., under condition (i) of Proposition 2. Given this auditing strategy, the supplier's IC condition (5) is reduced to $Q_L^N \geq Q_H^A \rho(\theta_L, \gamma_A)$ and $Q_H^A \rho(\theta_H, \gamma_A) \geq Q_L^N$. These inequalities will be satisfied if and only if condition (ii) of Proposition 2 holds, i.e., the buyer's order quantities are given by $Q_L^N = d_0$ and $Q_H^A = d_1$ and the range of d_0/d_1 is as specified. (Note that the low-type supplier will misreport for a positive expected payoff whenever receiving a zero order $Q_L^N = 0$.) Intuitively, the rationale is that the low-type supplier, who is more likely to fail in an audit, would not mimic the high type if she receives a positive order quantity even without being audited. On the other hand, the high-type supplier may prefer to reveal its true type and undergo an audit if she will be compensated with a larger conditional order.

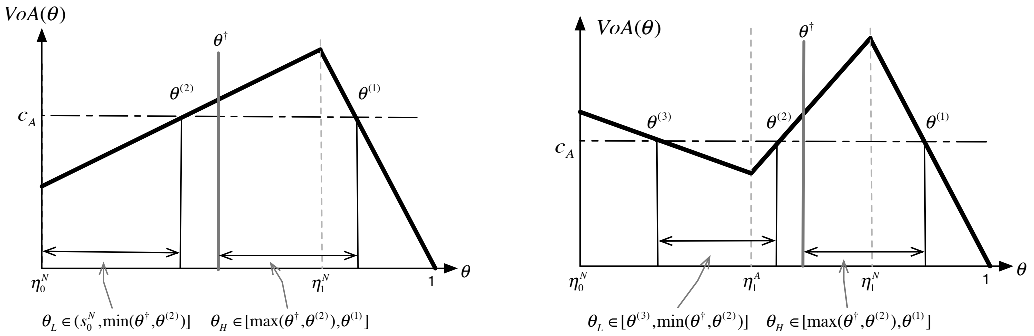
Proposition 2 can be recast in terms of certain truth revealing intervals such that an influential truth revealing equilibrium exists if and only if θ_L and θ_H belong to their corresponding intervals. Figure 3 depicts these intervals which are formally characterized in Proposition 3. Note that Figure 3 includes only the cases of $\eta_0^N < \eta_1^N$, because it is a necessary condition for the influential truth revealing equilibrium to exist.[14] If $\eta_0^N \geq \eta_1^N$, then Q_L^N cannot be d_0 (by Lemma 1), thus violating condition (ii) of Proposition 2.

Proposition 3. *Under asymmetric information, an influential truth revealing equilibrium can exist only if* $d_1 - d_0 > \Delta$ *and* $VoA(\max(\eta_1^A, \eta_0^N)) < c_A < VoA(\eta_1^N)$. *Suppose that these inequalities are satisfied, and let* $\theta^\dagger := \frac{d_0 - (1-\gamma_A) d_1}{\gamma_A d_1}$. *For* $\eta_0^N > \eta_1^A$, *an influential truth revealing equilibrium exists if and only if* $\theta_L \in (\eta_0^N, \min(\theta^\dagger, \theta^{(2)})]$ *and* $\theta_H \in (\max(\theta^\dagger, \theta^{(2)}), \theta^{(1)})$; *for* $\eta_0^N \leq \eta_1^A$, *an influential truth revealing equilibrium exists if and only if* $\theta_L \in (\theta^{(3)}, \min(\theta^\dagger, \theta^{(2)})]$ *and* $\theta_H \in (\max(\theta^\dagger, \theta^{(2)}), \theta^{(1)})$.

Two corollaries follow from Proposition 2 (or similarly Proposition 3) that have important

[13] If the audit is conducted only on the low-type supplier, then the supplier IC condition 5 will be reduced to $Q_L^A \rho(\theta_L, \gamma_A) \geq Q_H^N$ (for $t = L$) and $Q_H^N \geq Q_L^A \rho(\theta_H, \gamma_A)$ (for $t = H$). For any order quantities, these two inequalities cannot hold simultaneously as $\rho(\theta_L, \gamma_A) < \rho(\theta_H, \gamma_A)$.

[14] The inequality $\eta_0^N < \eta_1^N$ holds when d_0 is larger or g is smaller such that the buyer chooses d_0 as the order quantity to cope with the SR risk instead of ordering either d_1 or 0.



(a) For $\eta_0^N > \eta_1^A$

(b) For $\eta_0^N \leq \eta_1^A$

Figure 3: Truth revealing intervals of θ_L and θ_H; the relative position of $\theta^{\dagger}$ to other thresholds are indeterminate; $\theta^{\dagger} > \theta^{(2)}$ is assumed for illustrative purposes.

managerial implications.

Corollary 1. *If the buyer restricts himself to a single audit level, then the influential and truth revealing equilibrium will not exist.*

In other words, requesting SR self-reports from suppliers is useful (in the sense that it induces credible reports) only if the buyer has different audit options and does not pre-commit to a particular option. If the buyer never audits, or if he always audits regardless of the supplier type, then influential and truthful communication will not arise in equilibrium. As mentioned earlier, the audit and non-audit options in our model can be extended to two general audit levels: a basic level and a more stringent one. In general, supplier self-assessment as strategy can be a valuable SR tool for buyers, but only if the buyer has access to auditing capabilities of different levels. One way to achieve this is to develop some special audit procedures which focus on specific SR issues that can be used as a complement to regular SR audits. This is done by some firms; see page 65 of Carrefour (2017) and pages 30-33 of Inditex (2017). We also note that the influential truth revealing equilibrium arises only if the buyer imposes a higher auditing level on a self-reported better supplier. This is not without precedent in practice: Carrefour employs an independent auditing firm to visit the suppliers achieving the best results in the self-assessment exercise (Carrefour Group, n.d., p33). Our results thus corroborate Carrefour's selective use of independent audits in combination with supplier self-reporting.

As shown in the next corollary, a supplier will lie in its self report if its SR level is too low to receive a positive order if truthfully communicated.

Corollary 2. *If $\theta_L \leq \eta_0^N$ such that $Q_L^N = 0$, then the influential and truth revealing equilibrium will not exist.*

To limit the self-reporting challenge associated with such suppliers, the buyer should shape its approved supplier base in advance of self-reporting such that all approved suppliers demonstrate some minimum SR level. In other words, firms should engage in an upfront step to screen suppliers before they are approved for use. As noted in the introduction, Inditex and other firms use such pre-assessment audits and we model and explore pre-assessment in §5.1.

4.2.2 Effect of Audit Cost on Equilibria

We now explore how the audit cost c_A influences the possible equilibria and the buyer's ex ante profit. To focus on interesting cases, we assume that $\eta_0^N < \theta_L \leq \eta_1^N < \theta_H$ because these conditions on the supplier SR levels ensure that a low-type supplier would always mimic a high type if the buyer could not impose an audit at all. (Formally, by Lemma 1, $Q_L^N = d_0$ and $Q_H^N = d_1$ if $\eta_0^N < \theta_L \leq \eta_1^N < \theta_H$, and the larger order for a high type induces the low type to misreport her true type.) We relegate the discussion of the other cases to Online Appendix D; in those cases, we have either similar insights or trivial solutions. Proposition 4 characterizes the possible PBNE for all possible values of c_A and γ_A. Note that the babbling equilibrium always exists as a PBNE in cheap talk games (see, e.g., §7 of Sobel, 2014). In the following discussion, we will mainly focus on whether a truth revealing equilibrium exists.

Proposition 4. *Assuming that $\eta_0^N < \theta_L \leq \eta_1^N < \theta_H$ (and so $Q_L^N = d_0$ and $Q_H^N = Q_H^A = d_1$), the PBNE of our cheap talk game is characterized as follows:*

- *Case (i): $\gamma_A \leq 1 - \frac{\theta_L(p-w)}{(1-\theta_L)(\delta p-p+w)}$ (and so $Q_L^A = d_0$)*
  - *(a) for $c_A \geq v(\theta_H, d_1, d_1)$, the babbling equilibrium is the only PBNE;*
  - *(b) for $\min\,[v(\theta_L, d_0, d_0), v(\theta_H, d_1, d_1)] \leq c_A < v(\theta_H, d_1, d_1)$, the influential truth revealing equilibrium as described in Proposition 2, along with the babbling equilibrium, exists if $\frac{d_1-d_0}{(1-\theta_L)d_1} \leq \gamma_A \leq \frac{d_1-d_0}{(1-\theta_H)d_1}$, and the babbling equilibrium is the only PBNE otherwise;*
  - *(c) for $c_A < \min\,[v(\theta_L, d_0, d_0), v(\theta_H, d_1, d_1)]$, the babbling equilibrium is the only PBNE.*
- *Case (ii): $\gamma_A > 1 - \frac{\theta_L(p-w)}{(1-\theta_L)(\delta p-p+w)}$ (and so $Q_L^A = d_1$)*
  - *(a) for $c_A \geq v(\theta_H, d_1, d_1)$, the babbling equilibrium is the only PBNE;*
  - *(b) for $\min\,[v(\theta_L, d_0, d_1), v(\theta_H, d_1, d_1)] \leq c_A < v(\theta_H, d_1, d_1)$, the influential truth revealing equilibrium as described in Proposition 2, along with the babbling equilibrium, exists if $\frac{d_1-d_0}{(1-\theta_L)d_1} \leq \gamma_A \leq \frac{d_1-d_0}{(1-\theta_H)d_1}$, and the babbling equilibrium is the only PBNE otherwise;*
  - *(c) for $c_A < \min\,[v(\theta_L, d_0, d_1), v(\theta_H, d_1, d_1)]$, a non-influential truth revealing equilibrium, along with the babbling equilibrium, exists.*

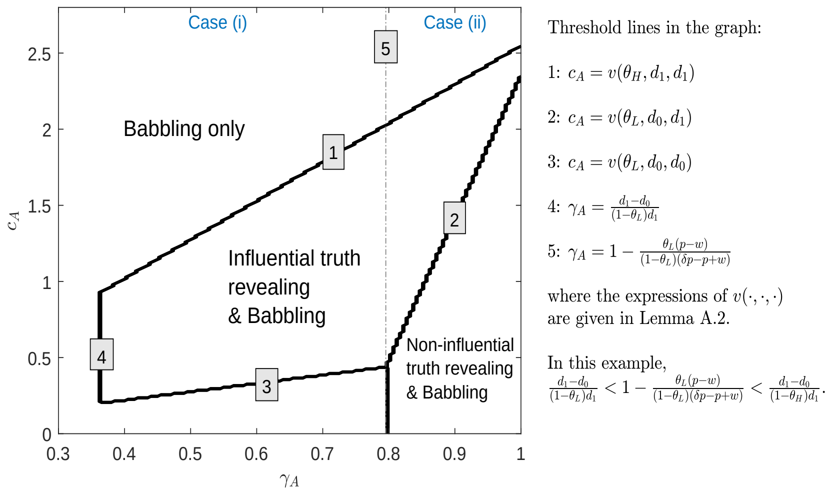


Figure 4: Impact of the audit cost and accuracy on the PBNE.

Proposition 4 is illustrated by a numerical example as shown in Figure 4.[15] Recall that $v(\theta, q^N, q^A)$ represents the value of audit when the supplier has an SR level θ and the buyer's order quantities are q^N and q^A without and with an audit, respectively. Under the assumption of Proposition 4, the value of audit associated with a high-type supplier is given by $VoA(\theta_H) = v(\theta_H, d_1, d_1)$. We first note that if the audit cost c_A exceeds $v(\theta_H, d_1, d_1)$ then the buyer will not audit the high-type supplier and hence the truth revealing equilibrium will not exist; see Cases (i)(a) and (ii)(a) of the proposition.

In Case (i) of Proposition 4, the audit accuracy γ_A is relatively low such that passing an audit does not increase the buyer's estimate of the no-violation probability sufficiently to induce it to place a larger order with a low-type supplier (i.e., $Q_L^N = Q_L^A = d_0$). Hence, $VoA(\theta_L) = v(\theta_L, d_0, d_0)$. In Case (i)(c), the audit cost is lower than both $VoA(\theta_L)$ and $VoA(\theta_H)$, and so the buyer audits both types, which eliminates the truth revealing equilibrium. If the audit cost is in the intermediate range as specified in Case (i)(b), then the buyer audits only the high-type supplier; the associated accuracy interval restriction ensures that the supplier's IC condition holds under the buyer's differentiated audit strategy, and the truth revealing equilibrium can exist. In Case (ii) of Proposition 4, the audit accuracy γ_A is high enough such that a low-type supplier passing an audit would receive a larger order than would a low type who was not audited (i.e., $Q_L^N = d_0$ and $Q_L^A = d_1$). Thus, $VoA(\theta_L) = v(\theta_L, d_0, d_1)$. In Case (ii)(c), the audit cost is lower than both $VoA(\theta_L)$ and $VoA(\theta_H)$ and the buyer would therefore conduct an audit and

[15] In general, the threshold $1 - \frac{\theta_L(p-w)}{(1-\theta_L)(\delta p - p + w)}$ in the proposition can be either greater or less than $\frac{d_1-d_0}{(1-\theta_L)d_1}$ and $\frac{d_1-d_0}{(1-\theta_H)d_1}$. In the example of Figure 4, we focus on a representative instance where $\theta_H = 0.85$, $\theta_L = 0.45$, $p = 10, w = 8, c = 6$, $\delta = 1$, $g = 17$, $d_1 = 10$, $d_0 = 8$ and therefore $\frac{d_1-d_0}{(1-\theta_L)d_1} < 1 - \frac{\theta_L(p-w)}{(1-\theta_L)(\delta p-p+w)} < \frac{d_1-d_0}{(1-\theta_H)d_1}$. We numerically verified whether an influential or non-influential truth revealing equilibrium exists for various combinations of c_A and γ_A.

conditionally order d_1 for both types; consequently, a non-influential truth revealing equilibrium exists where both types truthfully report.[16] Case (ii)(b) is analogous to Case (i)(b), in which an influential truth revealing equilibrium exists when the audit cost and accuracy are moderate.

The key message of Proposition 4 is that the influential truth revealing equilibrium occurs when the audit cost c_A or the audit accuracy γ_A is moderate. Intuitively, this is because if the audit is very efficient (inefficient) in terms of its cost or accuracy, the buyer will audit both types (neither type) instead of following the differentiated auditing strategy which induces influential truthful communication. Our findings suggest that supplier self-reporting may not bring much value if the buyer has very efficient or very poor audit capability.

4.2.3 Effect of Audit Cost on the Buyer's Profit

Having characterized the influence of the audit cost on the possible equilibria, we now explore its effect on the resulting profit of the buyer. A babbling equilibrium is always supported as a PBNE even when a truth revealing equilibrium exists. To have a unique prediction on the buyer's payoff in equilibrium, we refine the set of PBNE according to the "no incentive to separate" (NITS) condition proposed in Chen et al. (2008), a commonly used criterion for equilibrium refinement in cheap talk games (see, e.g., Chu et al., 2017). The NITS condition states that the sender with the lowest type (weakly) prefers its equilibrium payoff to the payoff it would receive if the receiver knew its type and responded optimally; see Definition 2 of Chen et al. (2008).

In our problem, the NITS condition is automatically satisfied in any truth revealing equilibrium. However, if the low-type supplier strictly prefers an influential truth revealing equilibrium to a babbling equilibrium, then the babbling equilibrium will fail the NITS refinement and can thus be eliminated from consideration. To see why the NITS refinement is reasonable, suppose that a babbling equilibrium plays out while the low-type supplier prefers the influential truth revealing equilibrium (which is also supported as a PBNE). Then, the low-type supplier may credibly self-signal with an out-of-equilibrium message, communicating that her type is actually lower than what (on average) the buyer thinks in the current babbling equilibrium. It is natural for the buyer to interpret such an out-of-equilibrium message as coming from the low-type supplier.[17] Thus, the babbling equilibrium which fails the NITS criterion is unlikely to be stable in practice. More detailed justification of the NITS refinement can be found in §4 of Chen et al. (2008).

[16]We note that the non-influential truth revealing equilibrium, if it exists, will result in the same payoffs for the buyer and the supplier as in a babbling equilibrium because the buyer's optimal decisions are independent of the supplier's report and thus identical in both equilibria.

[17]As discussed in §4.2 of Chen et al. (2008), a NITS-surviving equilibrium would survive the existence of "downward verifiable" messages such that a supplier can only prove that her type is no greater than her true type.

Intuitively, the low-type supplier would strictly prefer an influential truth revealing equilibrium (in which she is not audited) to a babbling equilibrium if the buyer finds it optimal to audit in the babbling equilibrium. Because in the influential truth revealing equilibrium the buyer audits only the high-type supplier, he would indeed conduct an audit in the babbling equilibrium if he believes that the supplier is sufficiently likely to be a high type. The following lemma formalizes this intuition and establishes that when an influential truth revealing equilibrium exists, the babbling equilibrium fails the NITS refinement if the prior probability of the supplier being a high type, i.e., $\lambda(\theta_H)$ is higher than a threshold.

Lemma 2. *When an influential truth revealing equilibrium exists, it always survives the NITS condition, whereas there exits a $\hat{\lambda} \in (0,1)$ such that the babbling equilibrium fails to satisfy the NITS condition if the prior probability $\lambda(\theta_H) \geq \hat{\lambda}$ and $d_0/d_1 > \rho(\theta_L, \gamma_A)$. In the non-NITS babbling equilibrium, the buyer chooses to audit the supplier.*

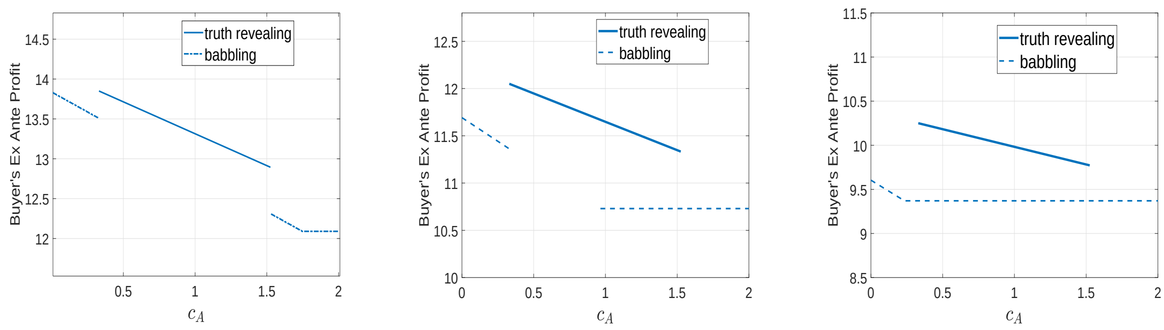
Under the conditions stated in Lemma 2, we can eliminate the babbling equilibrium from consideration whenever an influential truth revealing equilibrium exists; specifically, in Cases (i)(b) and (ii)(b) of Proposition 4, the influential truth revealing equilibrium, whenever it exists, is the unique NITS-surviving equilibrium.[18]

Now, we are ready to study the impact of the audit cost c_A on the buyer's ex ante expected profit in equilibrium. This profit is denoted by $\Pi_b^* = \max_{a\in\{0,1\}} \Pi_b(a)$, where $\Pi_b(a)$ is defined in (2). Applying Lemma 2, we can focus on the equilibrium that satisfies the NITS condition. We will continue to restrict attention to supplier SR levels $\eta_0^N < \theta_L \leq \eta_1^N < \theta_H$ as in Proposition 4; in this case, $Q_L^N = d_0$ and $Q_H^N = d_1$.

Proposition 5. *Given $\eta_0^N < \theta_L \leq \eta_1^N < \theta_H$ and $\frac{1-\theta_H}{1-\theta_L} > \frac{\delta g-(p-w)d_0}{\delta g+\delta p(d_1-d_0)-(p-w)d_1}$, under the condition stated in Lemma 2, the buyer's ex ante profit Π_b^* may strictly increase in c_A at any γ_A such that $\frac{d_1-d_0}{(1-\theta_L)d_1} \leq \gamma_A < \min\left(1 - \frac{\theta_L(p-w)}{(1-\theta_L)(\delta p-p+w)}, \frac{d_1-d_0}{(1-\theta_H)d_1}\right)$.*

Counter to what one might expect, the buyer can benefit from a more costly audit with the same accuracy level. This is not to say that it always benefits (it does not) but that it can benefit. To understand this, recall from Proposition 4 Case (i) that the truth revealing equilibrium may not be sustained as c_A decreases. This can be seen in Figure 4 for γ_A less than 0.8 (approximately); the truth revealing equilibrium switches to a babbling equilibrium if the audit cost c_A is low enough.

[18]However, in Case(ii)(c) of Proposition 4, both the non-influential truth revealing equilibrium and the babbling equilibrium survive the NITS refinement as the buyer takes identical actions in these equilibria.



(a) $\lambda(\theta_H) = 0.8$ (b) $\lambda(\theta_H) = 0.6$ (c) $\lambda(\theta_H) = 0.4$

Figure 5: Impact of auditing cost c_A on the buyer's ex ante profit; equilibria refined as per the NITS criterion

For both the truth revealing equilibrium and the NITS-surviving babbling equilibrium, Figure 5 depicts the buyer's ex ante expected profit for a fixed audit accuracy of $\gamma_A = 0.6$ as the audit cost c_A is increased from 0 to 2 and where the other parameters are the same as in our earlier example presented in Figure 4. This is done for three different priors on the supplier type: $\lambda(\theta_H)$ equal to 0.8, 0.6 and 0.4. For $\lambda(\theta_H) = 0.8$, the babbling equilibrium is eliminated in all instances and the buyer benefits from a higher audit cost when the equilibrium switches from a babbling to a truth revealing one at $c_A = 0.3$ (approximately); see the discontinuous jump up in profit. At other values of c_A, the buyer benefits from a lower audit cost. For $\lambda(\theta_H) = 0.6$, the babbling equilibrium may survive the NITS condition when c_A is greater than 1 (approximately). Yet, there exists a lower range of audit costs, each with a unique equilibrium, that contains a value at which the buyer's profit increases (discontinuously) as c_A increases. For $\lambda(\theta_H) = 0.4$, we observed that the NITS condition did not rule out the babbling equilibrium in any of the instances. However, in the presence of multiple equilibria, the buyer will not conduct an audit at all in the babbling equilibrium and so his payoff is constant in c_A. Thus, in some sense, the buyer's payoff may still weakly increase at the value of c_A where a truth revealing equilibrium first emerges.

In summary, the buyer does not necessarily benefit from a lower audit cost. His profit may actually increase as the audit becomes more expensive when such changes switch the equilibrium. This is because a lower audit cost may entice the buyer to "over-audit", which in turn eliminates the truth revealing equilibrium. In some sense, an appropriately costly audit gives the buyer a commitment power to the differentiated auditing strategy (conducting an audit only on the high type), which in turn induces truthful information exchange with the supplier. In addition, we can show that, for an analogous reason, the buyer may not necessarily be better off as the audit becomes more accurate, i.e., γ_A, increases. Broadly speaking, our comparative statics underscores again the value of having distinct audit levels as potential options, as discussed earlier §4.2. Before soliciting supplier self-reports, the buyer may benefit from developing multiple audit options with an appropriately expensive one with relatively high accuracy.

4.2.4 Social Impact of Supplier Self-reporting

We have shown that supplier self-reporting can be truthful under certain conditions. Clearly, when truthful communication occurs, the buyer is better off because the supplier's actual SR level is revealed with self-reporting. However, it is not clear whether supplier self-reporting benefits society. In this subsection, we examine the effect of self-reporting on the social damage associated with production. We focus on production-based social damage rather than consumption-based damage because SR-related damage often depends on the production quantity. For instance, a leakage of hazardous materials at a factory is likely to cause more harm to worker health and safety and to the environment if the factory is producing a larger order quantity. Thus, we use $SD(\theta) = \phi(\theta)Q$ to measure the expected social damage when a buyer places an order quantity Q with a type-θ supplier, where $\phi(\theta)$ represents the probability that a violation occurs when the buyer sources from a supplier with an SR level θ.[19] If the buyer conducts an audit, then $\phi(\theta) = (1-\theta)(1-\gamma_A)$; otherwise, $\phi(\theta) = 1-\theta$. Let SD_S and SD_{NS} denote the (ex ante) expected social damage in equilibrium with and without supplier self-reporting, respectively. Note that SD_{NS} is identical to SD_S if the game with self-reporting reaches a babbling or a non-influential equilibrium because in these equilibria the buyer's decisions are independent of the supplier's report. Hence, SD_S differs from SD_{NS} only when supplier self-reporting is truthful and influential in equilibrium. In the following proposition, we show that supplier self-reporting will increase the expected social damage if it induces an influential truth revealing equilibrium as the unique NITS-surviving equilibrium.

Proposition 6. *Suppose that conditions (i) and (ii) of Proposition 2 hold such that an influential truth revealing equilibrium exists. If the babbling equilibrium fails the NITS refinement, supplier self-reporting increases the expected social damage, i.e.,* $SD_S > SD_{NS}$.

It can be shown that in any non-NITS babbling equilibrium the buyer conducts an audit with a conditional order quantity of d_1; this is also the buyer's action without self-reporting. However, in the influential truth revealing equilibrium the buyer audits only the high-type supplier. Thus, supplier self-reporting enables the buyer to audit the supplier more selectively than what he would do without self-reporting. Consequently, it increases the expected social damage due

[19]The specification of $SD(\theta)$ is assumed for ease of exposition. Our results can be extended as long as the damage is linearly increasing in the buyer's order quantity when a violation occurs.

to the reduced use of audits. Note that Proposition 6 does not include the case where self-reporting leads to a NITS-surviving babbling equilibrium along with an influential truth revealing equilibrium. In such a case, the buyer does not audit in the babbling equilibrium, or equivalently, in the setting without self-reporting. So, self-reporting can instead increase the use of audit if the game reaches a truth revealing equilibrium. As a result, when multiple equilibria exist in the game with self-reporting, supplier self-reporting weakly reduces the expected social damage in the sense that it strictly decreases the damage if the truth revealing equilibrium plays out but has no impact otherwise. Figure 6 illustrates the above discussion with a numerical example.[20]

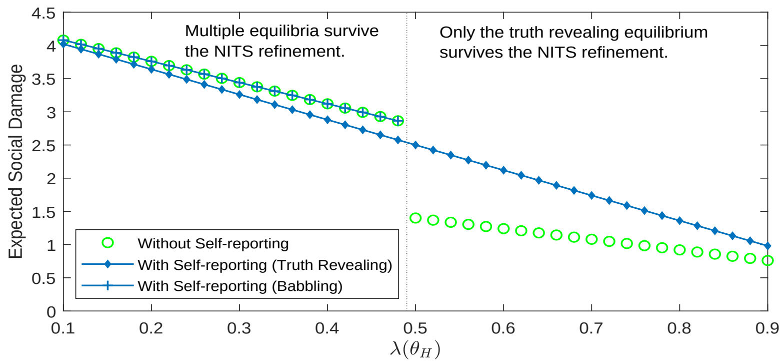


Figure 6: Effect of supplier self-reporting on the social damage; equilibria refined as per the NITS criterion

In summary, we find that although supplier self-reporting always improves the buyer's payoff, it increases the expected social damage when it induces truthful communication as the unique NITS-surviving equilibrium. This suggests that if social impact is a goal then further action should be taken following supplier self-reporting; for instance, the buyer could engage with low-performing suppliers to help them improve their SR performance in the medium and long term. Leaving improvement aside, we show later that self-reporting can reduce the expected social damage if an unscrupulous supplier can evade the buyer's audit (Proposition 8 in §5.2) or if the audit takes place after production (Proposition B.2 in Online Appendix B).

5 Extensions

5.1 Value of Pre-assessment

Corollary 2 established that suppliers below a certain SR level will not truthfully report. This suggests that a buyer should shape its approved supplier base in advance of self-reporting such

[20]The prior probability $\lambda(\theta_H)$ is varied from 0.1 to 0.9 to illustrate the case with multiple equilibria and the one with a unique equilibrium. The other parameters are set as $c_A = 0.5$, $\gamma_A = 0.6$, $\theta_H = 0.85$, $\theta_L = 0.45$, $p = 10, w = 8, c = 6$, $\delta = 1$, $g = 17$, $d_1 = 10$ and $d_0 = 8$.

that all approved suppliers have some minimum SR capability. In practice this shaping can be done by implementing pre-assessment audits but such audits impose a cost on the buyer and so it is important to examine the value of pre-assessment. To that end, we now extend our model as follows. Initially, the buyer has a pool of potential suppliers in which any supplier can be one of the three types: unqualified-, low- and high-types. We use $\theta_U < \theta_L < \theta_H$ to denote the SR levels of these three types, respectively, and their prior probabilities are denoted by $\lambda(\theta_i)$ where $i = U, L$ and H accordingly. Before the self-reporting stage, the buyer may conduct a pre-assessment audit which will detect and eliminate any unqualified supplier from the supply base but it does not distinguish between low- and high-types which reflects the fact that in practice pre-assessment audits are not as especially thorough and are focused on eliminating very low capability suppliers. As a result, the pre-assessment audit can enhance the prior distribution of the supplier type from the above three-point distribution to a two-point distribution under which the supplier can be either high- or low-type with probabilities $\lambda(\theta_H)/[\lambda(\theta_H) + \lambda(\theta_L)]$ and $\lambda(\theta_L)/[\lambda(\theta_H) + \lambda(\theta_L)]$, respectively.[21] So, if the pre-assessment is implemented, the game will be reduced to our base model with two supplier types. Let Π_b^* present the buyer's ex ante profit in the equilibrium of the two-type game. Without any pre-assessment, we will have a cheap talk game involving three supplier types. Let $\Pi_b^\dagger$ denote the buyer's ex ante profit in the equilibrium of the three-type game. The value of pre-assessment to the buyer (ignoring cost) is then given by $\Pi_b^* - \Pi_b^\dagger$.

To focus on the most interesting cases, we assume that $\theta_U < \eta_0^N$, i.e., θ_U is the SR level of a low performing supplier who would not be used at all under symmetric information (see Corollary 2) and that θ_H and θ_L are in the truth revealing intervals characterized in Proposition 3. The latter assumption ensures that the buyer can benefit from truthful communication after pre-assessment.

Proposition 7. *Assume that θ_H and θ_L are in the truth revealing intervals characterized in Proposition 3, and that $\theta_U \leq \eta_0^N$. If no pre-assessment is conducted, then the following holds for the three-type cheap talk game:*

(i) There exists a threshold Λ such that an influential communication with partial separation $\{\theta_U, \theta_L\}$ and $\{\theta_H\}$ is supported in equilibrium if $\lambda(\theta_U) < \Lambda$ whereas the babbling equilibrium where the communication is not informative at all is the only PBNE if $\lambda(\theta_U) \geq \Lambda$.

(ii) When the influential equilibrium with partial separation exists, it survives the NITS condition while the babbling equilibrium does not if $d_0/d_1 > \rho(\theta_U, \gamma_A)$ and $\lambda(\theta_H)$ is greater than

[21] More precisely, we assume that there are sufficiently many potential suppliers. Thus, after the pre-assessment stage, the buyer can always find a supplier who passed the pre-assessment.

a threshold such that the buyer chooses to audit in the babbling equilibrium.

Suppose that $\lambda(\theta_L)/\lambda(\theta_H)$ is constant as $\lambda(\theta_U)$ varies and that the NITS condition in part (ii) is satisfied, the value of pre-assessment $\Pi_b^* - \Pi_b^\dagger$ grows as $\lambda(\theta_U)$ increases, and jumps upward at $\lambda(\theta_U) = \Lambda$.

With an unqualified type, fully-truthful communication which reveals every type is no longer possible. Nevertheless, partial separation may occur in equilibrium: the high-type supplier reports a unique message but the unqualified and the low types choose the same message such that the buyer can distinguish only the high type from the other two types. Such partial separation is possible when the probability of the unqualified type $\lambda(\theta_U)$ is smaller than a threshold Λ. This is because the buyer's optimal decision in response to the message from the bundle of types $\{\theta_U, \theta_L\}$ will be the same as what he would do for type θ_L if $\lambda(\theta_U)$ is small enough, in which case the rationale for truthful communication as discussed in our base model remain valid for the type partition $\{\theta_U, \theta_L\}$ and $\{\theta_H\}$. However, if $\lambda(\theta_U)$ is relatively large, the buyer will order nothing when dealing with a bundle of types $\{\theta_U, \theta_L\}$, and therefore partial separation is no longer incentive-compatible. As a result, self-reporting will not be informative at all.

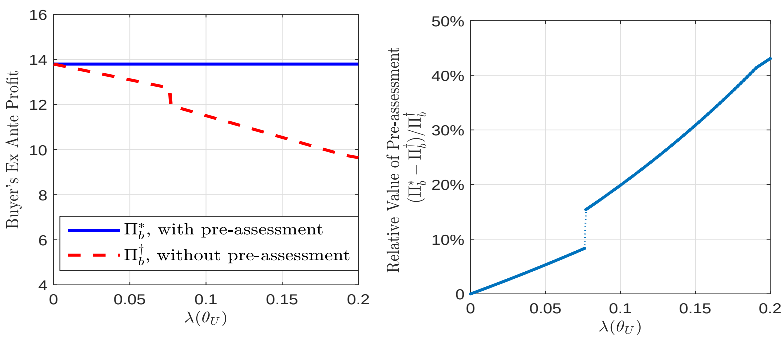


Figure 7: Illustration of the value of pre-assessment; equilibria refined as per the NITS criterion

Assuming that the likelihood ratio between the low- and high-types is fixed, Proposition 7 establishes that the value of pre-assessment, $\Pi_b^* - \Pi_b^\dagger$, in increasing in the probability of the unqualified type $\lambda(\theta_U)$ but the increase can have a discontinuous jump. Figure 7 illustrates the buyer's profits (with and without pre-assessment) and the relative value of pre-assessment as a function of $\lambda(\theta_U)$.[22] The jump in value occurs because the buyer's payoff with pre-assessment Π_b^* is constant in $\lambda(\theta_U)$ whereas his payoff without pre-assessment $\Pi_b^\dagger$ depends on whether partial separation occurs in the three-type game. For $\lambda(\theta_U) < \Lambda$, even without pre-assessment,

[22] In the example, $\theta_U = 0.05$, $\theta_L = 0.45$, $\theta_H = 0.85$, $c_A = 0.4$ and $\gamma_A = 0.6$, and the other parameters are the same as in the example of Figure 4. We set $\lambda(\theta_L)/\lambda(\theta_H) = 1/4$ vary $\lambda(\theta_U)$ from 0 to 0.2; this setting guarantees a unique NITS-surviving equilibrium (by statement (ii) of Proposition 7).

the buyer can benefit to some extent from supplier self-reporting due to the partial separation equilibrium in the three-type game. However, if $\lambda(\theta_U) \geq \Lambda$ then the presence of the unqualified type completely suppresses informative communication, and therefore there is even greater value to eliminating the unqualified type through pre-assessment.

As noted earlier, pre-assessment is worthwhile only if its value $\Pi_b^* - \Pi_b^\dagger$ exceeds its cost. Thus, Proposition 7 indicates that for a given cost, pre-assessment should be implemented if and only if the probability of the unqualified type exceeds a certain threshold. Finally, we note that the above discussion focuses on the case where truthful communication can occur in the two-type game without the unqualified type. If truthful communication is not permitted in equilibrium even in the two-type game, then the value of pre-assessment will be lower than that discussed above.

5.2 Supplier Evasion of an Audit and the Social Impact of Self-reporting

In our base model, the supplier could not influence the auditing outcome if the buyer chose to audit. In reality, however, an unscrupulous supplier might exert effort to pass a buyer's audit through bribery or deception of auditors (Manschot, 2018). In what follows, we extend our base model to allow the supplier to take some evasive action that guarantees she passes an audit irrespective of whether there is a violation or not. When deciding whether to exert this effort, at cost c_e, the supplier is aware of her SR level θ (i.e., the probability of a violation) but not the actual occurrence of a violation. See Plambeck and Taylor (2016) for a similar assumption and detailed justifications. We assume the decision to exert effort is made after the supplier knows whether she will be audited. This clearly applies to settings with announced audits but can also apply to so-called surprise audits because in practice the auditor often needs to ask the supplier in advance for access to the factory and necessary documentation (Manschot, 2018).[23] We assume that the buyer and the supplier know the possibility of evasion and its cost c_e at the beginning of the cheap talk game. The sequence of events is as follows: (i) the supplier realizes her SR level θ and sends a self-report to the buyer; (ii) based on the self-report, the

[23] As commented in Manschot (2018, p20), "In practice, however, a more appropriate term for these 'unannounced' or 'surprise' audits would be 'semi-announced' audits – since in order for the auditor to have access to the computer systems, files, locked offices and other information necessary to make his/her assessment, the right personnel needs to be at the factory premises to support the auditor in his work. Suppliers will therefore always need a heads up before the auditor arrives – albeit as late as possible, e.g. not earlier than 72 hours in advance. In case of bad will, suppliers can potentially still cover up any bad practices that they want to sweep under the carpet – even if the audit is a surprise audit." Thus, in this model extension we focus on the setting where the supplier's evasion decision takes place after the supplier knows whether she will be audited. That said, we acknowledge that there are cases in which an audit is conducted without any prior notice. In such cases, the supplier's evasion effort is made before knowing the buyer's audit decision. This alternative sequence requires a different modeling approach which involves a simultaneous-move game between the buyer and the supplier. It remains an open question whether our results continue to hold under this alternative setting.

buyer determines whether to conduct an audit and how much to order (possibly conditional on the audit outcome); (iii) the supplier, if being audited, decides whether to evade at the cost c_e. When the buyer orders Q and conducts an audit, a type-θ supplier will evade the audit if and only if $(w-c)Q - c_e > (w-c)Q\rho(\theta, \gamma_A)$, or equivalently, $c_e < (w-c)\gamma_A(1-\theta)Q$. Henceforth, we will assume $c_e \geq (w-c)\gamma_A(1-\theta_H)d_1$ so that the high-type supplier is reliable enough to have no incentive to deceive an auditor.

In the following proposition, we establish that the influential truth revealing equilibrium described in our base model continues to exist if the evasion cost c_e exceeds $(w-c)(d_1-d_0)$. Otherwise, the low type supplier will find it optimal to disguise herself as the high type by evading the audit.[24] Furthermore, supposing that the influential truth revealing equilibrium is the unique NITS-surviving equilibrium, we characterize the impact of supplier self-reporting on the expected social damage defined in §4.2.4 with different values of c_e. Note that if the buyer conducts an audit but the supplier evades, the audit will not reduce the chance of a violation and thus will have no impact on the expected social damage.

Proposition 8. *Suppose that conditions (i) and (ii) of Proposition 2 hold such that an influential truth revealing equilibrium exists in our base model and that the low-type supplier may evade an audit at a cost of c_e. If $c_e \geq (w-c)(d_1-d_0)$, then the influential truth revealing equilibrium continues to exist, and whenever the babbling equilibrium fails the NITS refinement, we have the following:*

(i) For $c_e \geq (w-c)\gamma_A(1-\theta_L)d_1$, supplier self-reporting increases the expected social damage, i.e., $SD_S > SD_{NS}$;

(ii) for $c_e < (w-c)\gamma_A(1-\theta_L)d_1$, supplier self-reporting decreases the expected social damage, i.e., $SD_S < SD_{NS}$.

If $c_e < (w-c)(d_1-d_0)$, then the influential truth revealing equilibrium will no longer exist and thus $SD_S = SD_{NS}$.

In Case (i), the evasion cost is sufficiently high that a supplier will not take evasive action regardless of the order size. The result thus coincides with Proposition 6 discussed earlier in §4.2.4: Self-reporting increases the expected social damage due to the reduced use of audits. In Case (ii), the evasion cost is low enough to make audit evasion attractive for the low-type supplier. In this case, however, self-reporting reduces the expected damage, as opposed to Proposition 6 derived from our model. This is because the supplier, if a low type, can always

[24] When a low-type supplier may possibly evade the buyer's audit, the necessary and sufficient conditions for the existence of an influential truth revealing equilibrium in the cheap talk are provided in Proposition A.1.

secure a order quantity d_1 by evading the audit in the no self-reporting case, and this results in a larger expected social damage as compared to the self-reporting case where the low type supplier will be assigned a smaller order size of d_0.

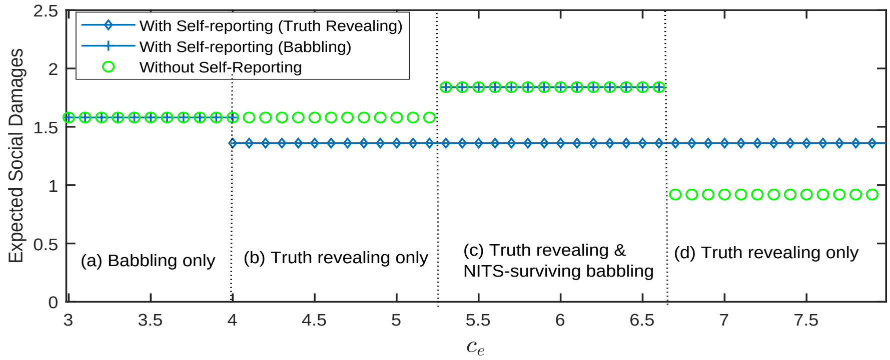


Figure 8: Effect of supplier self-reporting on the social damage; equilibria refined as per the NITS criterion

Proposition 8 does not consider the case where both the truth revealing and babbling equilibria survive the NITS refinement. We thus provide a numerical example to further illustrate the effect of supplier self-reporting. Figure 8 depicts how SD_S and SD_{NS} change as the evasion cost c_e is varied.[25] Region (a) corresponds to the case of $c_e < (w-c)(d_1-d_0)$ where, as proven in Proposition 8, no influential truth revealing equilibrium exists; regions (b) and (d) correspond to cases (ii) and (i) of Proposition 8, respectively, where the truth revealing equilibrium is the unique NITS surviving equilibrium. In region (c), the game with self-reporting permits multiple NITS-surviving equilibria,[26] and self-reporting still weakly reduces the expected social damage in the sense that it strictly reduces the damage in the truth revealing equilibrium but has no impact in the babbling equilibrium.

6 Conclusions

In this paper, we examined whether a supplier could credibly convey her private SR level to a buyer through free and unverifiable communication in a supply chain setting where the buyer's market demand was negatively affected by a publicly-revealed SR violation. We established

[25]The parameters are the same as in the example reported in Figure 6 and $\lambda(\theta_H)$ is fixed as 0.8.

[26]The babbling equilibrium survives the NITS condition because the buyer does not audit the supplier in equilibrium, and so the low-type supplier may opt for that babbling equilibrium. The value of c_e can have a subtle effect on the buyer's auditing decision in the babbling equilibrium. In region (c), a low type supplier will evade the buyer's audit only for a conditional order size d_1 but not for d_0, whereas in region (b) she would evade for both order sizes. If no self-reporting is available, conditional on the supplier passing an audit, the buyer will change his order quantity from d_1 to d_0 as c_e increases from region (b) to (c) in order to prevent the low-type supplier from evading. This increase in c_e may reduce the value of audit as the audit will no longer help the buyer place a larger order size. Thus, it is observed that the buyer chooses not to audit with an order quantity d_0 in region (c), but conducts an audit with an order quantity d_1 in region (b).

that supplier cheap talk can be truthful as a result of a complementarity between the buyer's auditing and ordering decisions. The buyer may prefer to order a larger quantity from the high-type supplier conditional on a more stringent audit level than what it would order from the low type. Truthful communication is supported in equilibrium when the high-type supplier opts for the larger conditional order whereas the low-type does not because of its higher risk of failing the audit. Furthermore, because of the possibility of truthful communication, we establish that the buyer may be better off if the audit cost is higher. Our results provide some practical guidance on managing supplier self-reports. To induce credible communication, the buyer should develop distinct audit levels as potential options, instead of precommitting to a certain audit level. Prior to the self-reporting stage, we show that it is valuable to conduct a pre-assessment that eliminates extremely low-performing suppliers. We also establish that self-reporting reduces social damage in settings where unscrupulous suppliers can cost-effectively deceive or bribe auditors but it can increase social damage if audit manipulation is sufficiently expensive.

Our work could be extended in a number of specific directions. We assumed a simple wholesale price contract with no contingent supplier penalty in the event of an SR violation. This assumption amplifies the conflict of interest between the supplier and the buyer. One could investigate our cheap talk game under more sophisticated contractual forms; for instance, the contract might stipulate the supplier shares a certain proportion of the buyer's loss in the event of an SR violation. Furthermore, our analysis was conducted under the assumption that supplier self-reporting is unverifiable. In practice, some parts of the supplier's report may be verifiable and so the case of partially verifiable self-reporting merits study. Additionally, we focused on the role of informal communication and assumed that the supplier could not leverage costly devices (such as wholesale pricing and certification) to signal her SR level. Informal communication and costly signaling may co-exist in practice, however, and it remains an open question as to whether and to what extent a costly signaling device might further enhance truthful information transmission. More broadly, our research establishes an interdependency between SR communication and supply chain strategy that could be explored from a number of different perspectives.

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Online Appendices for "Sourcing from a Self-Reporting Supplier: Strategic Communication of Social Responsibility in a Supply Chain"

This document is organized as follows. In §A, we present some technical results referenced in the main paper. In §B, we discuss the case where the buyer's audit (if any) takes place after production. In §C, we present the results for the general revenue function $R(Q|\xi)$. In §D, we include the technical results for the cases omitted in Proposition 4. The proofs of the results presented in this document are provided in the Supplementary Material of the paper, which is available from the authors upon request.

A Technical Results

Lemma A.1. *Given $R(Q|1) = p\min\{d_1, Q\}$ and $R(Q|0) = p\min\{d_0, Q\}$, Assumption 1 holds if and only if $\delta g \geq \max((p-w)d_0, (p-w)d_1 - \delta p(d_1 - d_0))$.*

Lemma A.2. *The value of audit $VoA(\theta)$ can be expressed by one of the following two cases: For $\eta_0^N > \eta_1^A$,*

$$VoA(\theta) = \begin{cases} v(\theta, d_1, d_1) \text{ if } \theta > \max(\eta_1^N, \eta_0^N) \\ v(\theta, d_0, d_1) \text{ if } \eta_0^N < \theta \leq \max(\eta_1^N, \eta_0^N) \\ v(\theta, 0, d_1) \text{ if } \eta_1^A < \theta \leq \eta_0^N \\ v(\theta, 0, d_0) \text{ if } \eta_0^A < \theta \leq \eta_1^A \end{cases} \tag{A.1}$$

and for $\eta_0^N \leq \eta_1^A$

$$VoA(\theta) = \begin{cases} v(\theta, d_1, d_1) \text{ if } \theta > \eta_1^N \\ v(\theta, d_0, d_1) \text{ if } \eta_1^A < \theta \leq \eta_1^N \\ v(\theta, d_0, d_0) \text{ if } \eta_0^N < \theta \leq \eta_1^A \\ v(\theta, 0, d_0) \text{ if } \eta_0^A < \theta \leq \eta_0^N \end{cases} \tag{A.2}$$

where

$v(\theta, d_1, d_1) = \gamma_A(1-\theta)[\delta g + \delta p(d_1 - d_0) - (p-w)d_1]$,

$v(\theta, d_0, d_1) = (p-w)(d_1-d_0) + (1-\theta)[\gamma_A(\delta g - (p-w)d_0) - (\gamma_A(p-w) + (1-\gamma_A)\delta p)(d_1 - d_0)]$,

$v(\theta, d_0, d_0) = \gamma_A(1-\theta)[\delta g - (p-w)d_0]$,

$v(\theta, 0, d_1) = (p-w)d_1 - (1-\theta)\left[\gamma_A(p-w)d_1 + (1-\gamma_A)\delta(g + p(d_1 - d_0))\right]$,

$v(\theta, 0, d_0) = (p-w)d_0 - (1-\theta)\left[\gamma_A(p-w)d_0 + (1-\gamma_A)\delta g\right]$.

Moreover, $v(\theta, 0, d_0)$ and $v(\theta, 0, d_1)$ are increasing in θ; $v(\theta, d_1, d_1)$ and $v(\theta, d_0, d_0)$ are decreasing in θ; $v(\theta, d_0, d_1)$ is increasing in θ if and only if $d_1 - d_0 > \Delta := \frac{\gamma_A[\delta g - (p-w)d_0]}{(1-\gamma_A)\delta p + \gamma_A(p-w)}$.

Lemma A.3. *The value of audit $VoA(\theta)$ satisfies the following properties: (i) If $d_1 - d_0 \leq \Delta$, then $VoA(\theta)$ is increasing in $(\eta_0^A, \eta_0^N]$ and decreasing in $(\eta_0^N, 1]$; (ii) if $d_1 - d_0 > \Delta$ and $\eta_0^N > \eta_1^A$, then $VoA(\theta)$ is increasing in $(\eta_0^A, \max(\eta_1^N, \eta_0^N)]$ and decreasing in $(\max(\eta_1^N, \eta_0^N), 1]$; (iii) if*

$d_1 - d_0 > \Delta$ and $\eta_0^N \le \eta_1^A$, then $VoA(\theta)$ is increasing in $(\eta_0^A, \eta_0^N] \cup (\eta_1^A, \eta_1^N]$ and decreasing in $(\eta_0^N, \eta_1^A] \cup (\eta_1^N, 1]$.

Proposition A.1. *Assume that a supplier may evade the buyer's audit at a cost c_e and that $(w-c)\gamma_A(1-\theta_H)d_1 \le c_e < (w-c)\gamma_A(1-\theta_L)d_1$ (i.e., the low-type supplier has an incentive to evade whereas the high type does not). An influential truth revealing equilibrium exists if and only if the following conditions hold: (a) if information was symmetric, the buyer would audit the high-type supplier and order $Q_H^A = d_1$ conditional on it passing the audit, but not audit the low type and order $Q_L^N = d_0$ from it; (b) $1 - \frac{c_e}{(w-c)d_1} \le \frac{d_0}{d_1} \le \rho(\theta_H, \gamma_A)$. Moreover, the influential revealing equilibrium, if exists, always survives the NITS refinement, whereas there exists a threshold $\hat{\lambda} \in (0,1)$ such that the babbling equilibrium fails the NITS refinement if $\lambda(\theta_H) \ge \hat{\lambda}$ and $d_0/d_1 > 1 - \frac{c_e}{(w-c)d_1}$.*

B Audit After Production

In our base model the audit is conducted prior to production, and so the supplier incurs the production cost only if it passes the audit. Our main results can be readily extended to an alternative setting in which the audit is conducted after production. In such a setting, the sequence of events is as follows. After receiving the supplier's self-reporting message, the buyer determines the order quantity and whether to conduct an audit, and then the supplier begins to produce; after production, the audit is conducted if the buyer chooses to do so; if the supplier fails the audit, the buyer will terminate the contract, i.e., purchase nothing from the supplier and turn to his outside option with zero value (Assumption 1); otherwise, the transaction will be completed and the buyer will receive his payoff according to the realized SR risk.

Although the buyer determines his order quantity before knowing the audit result, anticipating that he will cancel the order if the supplier fails, the buyer should still choose an order quantity conditional on the supplier passing the chosen audit level. Therefore, for any given belief about the supplier's SR level, the buyer's joint decisions on the auditing level and order quantity remain the same as in our base model (i.e., as characterized in §4.1). Unlike our base model, the supplier starts production prior to being audited. We assume that all the production cost is incurred before the audit outcome is realized.[27] For each unit of product, the supplier always incurs a production cost c, but may or may not receive a revenue of w, depending on the audit outcome. Thus, the supplier's expected profit is adapted to the following

$$\Pi_s(\theta, a(m), Q^N(m), Q^A(m)) = \begin{cases} (w\rho(\theta, \gamma_A) - c)Q^A(m) \text{ if } a(m) = 1 \\ (w-c)Q^N(m) \text{ if } a(m) = 0. \end{cases}$$

[27] A similar assumption is adopted in Caro et al. (2018). Our analysis can readily be adapted to a setting where the audit occurs at a certain point during production such that the supplier incurs a proportion of the total production cost by the time the audit result is known.

We assume $w\rho(\theta, \gamma_A) > c$ for all $\theta \in \{\theta_H, \theta_L\}$ such that the trade is always profitable for the supplier. The supplier's objective function and IC condition differ from that in our base model only in that $(w - c)\rho(\theta, \gamma_A)$ is replaced with $w\rho(\theta, \gamma_A) - c$. Consequently, we can extend the necessary and sufficient condition for the existence of the influential truth revealing equilibrium as follows.

Proposition B.1. *If the audit is conducted after production, an influential truth revealing equilibrium exists if and only if both following conditions hold: (i) $VoA(\theta_H) > c_A \geq VoA(\theta_L)$; (ii) under symmetric information the low-type supplier would receive $Q_L^N = d_0$ and the high type would receive $Q_H^A = d_1$ if passing the audit, and their SR levels are such that $\frac{w\rho(\theta_L,\gamma_A)-c}{w-c} \leq \frac{d_0}{d_1} \leq \frac{w\rho(\theta_H,\gamma_A)-c}{w-c}$.*

While condition (ii) is modified in this alternative setting, the key rationale for truthful communication in our base model remains valid. As before, a very low audit cost c_A may eliminate the truth revealing equilibrium in view of condition (i), and so our result on the effect of the audit cost c_A (Proposition 5) is also robust to the timing of audit.

In Proposition 6 of the main paper, we find that in the base model where the audit takes place before production, supplier self-reporting increases the expected social damage (if it results in truthful communication as a unique NITS-surviving equilibrium) as it reduces the use of audits in equilibrium and thus increases the chance of a violation occurring during production. Proposition B.2 below shows that this result is reversed when the audit happens after production.

Proposition B.2. *If the audit is conducted after production and conditions (i) and (ii) of Proposition B.1 hold such that an influential truth revealing equilibrium exists, then whenever the babbling equilibrium fails the NITS refinement, supplier self-reporting decreases the expected social damage, i.e., $SD_S < SD_{NS}$.*

The reason is that with an audit conducted after production, an audit failure will not prevent production from happening and so the production-based social damage will depends only on the production quantity and the supplier's type, whether an audit is conducted or not. In the case where supplier self-reporting leads uniquely to a truth revealing equilibrium, the buyer will order a smaller quantity once knowing the supplier is of low type than what he would do without self-reporting. We also note that the result in Proposition B.2 would change if both the truth revealing and babbling equilibria survive the NITS refinement; in such a case, the buyer would order a small quantity d_0 even without self-reporting, and self-reporting can increase the expected social damage if the truth revealing equilibrium plays out.

Our findings therefore establish that the social impact of supplier self-reporting depends on audit timing: supplier self-reporting, when leading uniquely to a truth revealing equilibrium, will reduce the expected social damage if and only if the audit is conducted after a certain point of time during production.

C General Revenue Function $R(Q|\xi)$

We now consider a general revenue function $R(Q|\xi)$ and show that our main results are robust to general quantity-dependent revenue functions. We make the following mild assumptions: (i) for any given value of ξ, the buyer's revenue $R(Q|\xi)$ is concave in the quantity Q it has to sell but the buyer receives no revenue if it has nothing to sell, i.e., $R(0|\xi) = 0$; and (ii) for any available-to-sell quantity, a publicly-revealed SR violation negatively impacts the buyer's revenue, i.e., $R(Q|1) \geq R(Q|0)$ for any $Q \geq 0$. Let Q^A and Q^N denote the profit-maximizing quantities with and without an audit, respectively. We assume that $R(Q|\xi)$ permits these two optimal quantities to be finite. When referring to specific supplier types $t \in \{H, L\}$, let Q_t^N and Q_t^A denote the maximizers of $\Pi_b^N(Q|\theta_t)$ and $\Pi_b^A(Q|\theta_t)$, respectively.

Proposition C.1. *For a general revenue function $R(Q|\xi)$, an influential truth revealing equilibrium exists if and only if the following conditions hold true: (i) $VoA(\theta_H) > c_A \geq VoA(\theta_L)$, and (ii) $\rho(\theta_L, \gamma_A) \leq \frac{Q_L^N}{Q_H^A} \leq \rho(\theta_H, \gamma_A)$.*

The necessary and sufficient conditions for existence of an influential truth revealing equilibrium for the general revenue function are given in the above proposition, which are similar in spirit to those established earlier in Proposition 2: if information were symmetric, (i) the buyer follows a differentiated auditing strategy, i.e., it would impose an audit only on the high type supplier; (ii) a high type supplier is willing to undergo the audit (because the resulting order size if it passes is sufficiently large) but a low type prefers not to be audited (because the probability of failing outweighs any benefit of a larger order size).

We note that a necessary condition for a high-type supplier to prefer to be audited is that the order size (if it passes) is strictly larger than what a low-type would receive without being audited, i.e., $Q_H^A > Q_L^N$.[28] This strict inequality will hold if the buyer's order quantity increases in the probability of no publicly-revealed violation. If $R(Q|\xi)$ is differentiable, this latter increasing property will hold if $R'(Q|1) \geq R'(Q|0)$ for all $Q \geq 0$, i.e., the buyer's marginal revenue shrinks when an SR violation is discovered by the public. We also note that a necessary condition for the low-type supplier to prefer not to be audited is that she receives a positive order without being audited. In the symmetric information case, there exists a threshold SR level below which a supplier would not receive an order even if she is not audited. This threshold is given by $\underline{\theta} = \sup\{\theta : \max_{Q \geq 0} \Pi_b^N(Q|\theta) = 0\}$. In the asymmetric information case, a low-type supplier with an SR level below this threshold would never truthfully report its type. To avoid uninteresting cases, we therefore assume that $\theta_L > \underline{\theta}$ in what follows.

In our base model, we established that the conditions for existence of an influential truth revealing equilibrium (Proposition 2) could be recast as an interval characterization of the supplier

[28] Note that to satisfy both conditions (i) and (ii), Q_H^A cannot be equal to Q_L^N, because $\rho(\theta_H, \gamma_A) = 1$ only when $\theta_H = 1$, in which case condition (i) cannot hold.

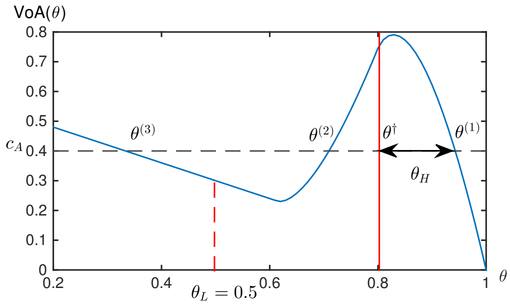

Figure C.1: Illustration of Proposition C.2: truth revealing interval of θ_H for $\theta_L = 0.5$

SR types θ_L and θ_H (Proposition 3). Using the conditions for the general case given by Proposition C.1, we now show that truth revealing intervals can be established for other reasonable revenue functions.

Random SR Customer Demand. In §4 we assumed that the market was sensitive to SR violations in the sense that demand was d_1 if there was no publicly revealed violation but demand was $d_0 < d_1$ otherwise. The selling price was p in either outcome. In effect, $d_1 - d_0$ was the demand from SR customers that would not purchase if a violation occurred and was revealed. We now allow this SR customer demand to be uncertain but continue to assume the base demand d_0 to be deterministic. In particular, let $d_1 = d_0 + \tilde{d}_{SR}$, where $\tilde{d}_{SR}$ is a continuous random variable with a nonnegative support, cumulative distribution function (cdf) $F(\cdot)$, and a probability density function (pdf) $f(\cdot)$. We assume that F has an increasing failure rate (IFR), a property satisfied by many common probability distributions (Bagnoli and Bergstrom, 2005). The revenue function is given by $R(Q|1) = pE\min(Q, d_0 + \tilde{d}_{SR})$ and $R(Q|0) = p\min(Q, d_0)$.

For the setting in which a violation (if one occurs) is discovered by the public with certainty, i.e., $\delta = 1$, we can show that the value of audit $VoA(\theta)$ is first convex and then concave over $[\underline{\theta}, 1]$, thereby characterizing the truth revealing conditions as intervals. By continuity our result holds as long as δ is sufficiently close to one, i.e., the public scrutiny is adequately stringent.

Proposition C.2. *Suppose that $R(Q|1) = pE\min(Q, d_0 + \tilde{d}_{SR})$, $R(Q|0) = p\min(Q, d_0)$, $\delta = 1$ and $F(\cdot)$ has an IFR. The following statements hold true: (i) $VoA(\theta)$ is continuously differentiable and convex-concave in $(\underline{\theta}, 1]$. Whenever $VoA'(\eta_1^N) > 0$, there may exist three thresholds $\theta^{(i)}$ where $i = 1, 2, 3$ such that under symmetric information, the buyer audits the supplier with an SR level θ if and only if $\theta \in (\underline{\theta}, \theta^{(3)}) \cup (\theta^{(2)}, \theta^{(1)})$. (ii) Under asymmetric information, for any $\theta_L \in (\max(\underline{\theta}, \theta^{(3)}), \theta^{(2)}]$, there exists two thresholds $\theta^{\dagger}(\theta_L)$ and $\theta^{\ddagger}(\theta_L)$ such that an influential truth revealing equilibrium exists if and only if $\theta_H \in (\theta^{(2)}, \theta^{(1)}) \cap [\theta^{\dagger}(\theta_L), \theta^{\ddagger}(\theta_L)]$.*

Example 1. To illustrate Proposition C.2, we consider a numerical example in which $\tilde{d}_{SR}$ follows a Gamma distribution with mean 4 and standard deviation 3. The other parameters are given by $c_A = 0.4$, $\gamma_A = 0.6$, $d_0 = 8$, $p = 10$, $w = 8$, $c = 6$, $\delta = 1$ and $g = 17$. In this example, $\underline{\theta} \approx 0.06$. As shown Figure C.1, $VoA(\theta)$ takes a similar convex-concave shape as in §4

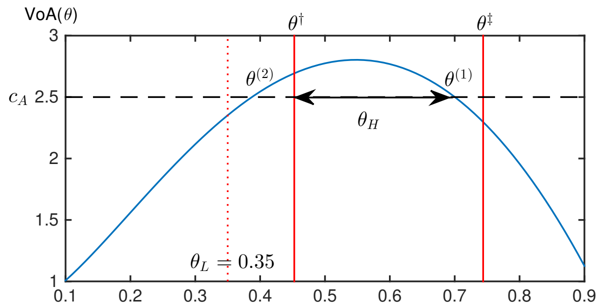

Figure C.2: Illustration of Proposition C.3: truth revealing interval of θ_H for $\theta_L = 0.35$.

except that the curve is now smooth. Since $c_A = 0.4$ crosses $VoA(\theta)$ three times at $\theta^{(3)}$, $\theta^{(2)}$ and $\theta^{(1)}$ within $(\underline{\theta}, 1]$, for any θ_L between $\theta^{(3)}$ and $\theta^{(2)}$, one can define a corresponding interval $(\theta^{(2)}, \theta^{(1)}) \cap [\theta^{\dagger}(\theta_L), \theta^{\ddagger}(\theta_L)]$ such that as long as θ_H belongs to this interval, influential truthful communication emerges in equilibrium. For instance, $\theta_L = 0.5$, $\theta^{\dagger}(\theta_L) = 0.8025$ and $\theta^{\ddagger}(\theta_L) = 1$. Consequently, an influential truth revealing equilibrium exists when $\theta_H \in [\theta^{\dagger}, \theta^{(1)})$. Note that when c_A is too low or too high, the truth revealing intervals may degenerate to empty sets.

Linear Demand with SR Violation-Dependent Market Potential. Instead of assuming a fixed selling price p in the end market that is independent of a revealed SR violation, we can instead assume that the selling price is formed according to an inverse linear demand function $p(Q) = A_{\xi} - Q$, where A_{ξ} denotes the market potential dependent on the realization of ξ. The market potential shrinks if an SR violation occurs and is revealed, i.e., $A_1 > A_0$. The revenue function is thus specified as $R(Q|\xi) = (A_{\xi} - Q)Q$. We implicitly assume that it is not optimal for the buyer to withhold some product.[29] The following proposition establishes that the structure of truth revealing intervals still holds in this setting.

Proposition C.3. *Suppose that $R(Q|\xi) = A_{\xi}Q - Q^2$ and $A_1 > A_0 > w$. The following statements hold true: (i) $VoA(\theta)$ is continuously differentiable and convex-concave. Whenever $VoA'(\theta^{\#}) > 0$ where $\theta^{\#} = \frac{1}{\gamma_A}[(1-\gamma_A)^{2/3} - (1-\gamma_A)]$, there may exist three thresholds $\theta^{(i)}$ where $i = 1, 2, 3$ such that under symmetric information, the buyer audits the supplier with an SR level θ if and only if $\theta \in (\underline{\theta}, \theta^{(3)}) \cup (\theta^{(2)}, \theta^{(1)})$. (ii) Under asymmetric information, for any $\theta_L \in (\max(\underline{\theta}, \theta^{(3)}), \theta^{(2)}]$, there exists two thresholds $\theta^{\dagger}(\theta_L)$ and $\theta^{\ddagger}(\theta_L)$ such that an influential truth revealing equilibrium exists if and only if $\theta_H \in (\theta^{(2)}, \theta^{(1)}) \cap [\theta^{\dagger}(\theta_L), \theta^{\ddagger}(\theta_L)]$.*

Example 2. Figure C.2 depicts a numerical example in which $A_1 = 18, A_0 = 10, w = 8, c = 2, \delta = 1, g = 2, c_A = 2.5$ and $\gamma_A = 0.75$. In the example, $\underline{\theta} \approx 0.09$ and the interval $(\underline{\theta}, \theta^{(3)})$ degenerates to be empty; the curve of $VoA(\theta)$ crosses $c_A = 1.8$ two times and so the buyer audits the supplier under symmetric information if and only if $\theta \in (\theta^{(2)}, \theta^{(1)})$; for any θ_L between $\underline{\theta}$

[29] Even when the buyer bears no cost for withholding inventory, the assumption is true if $A_0 + w \geq A_1$. Similar assumptions are commonly adopted in the operations management literature; for example, see Goyal and Netessine (2007), Wu and Zhang (2014).

and $\theta^{(2)}$, one can find an interval of θ_H such that influential truthful communication arises in equilibrium. For instance, given $\theta_L = 0.35$, $\theta^{(2)} < \theta^{\dagger} = 0.4526 < \theta^{(1)} < \theta^{\ddagger} = 0.7438$, and so an influential truth revealing equilibrium exists if and only if $\theta_H \in [\theta^{\dagger}, \theta^{(1)})$.

Additionally, our earlier analysis established that the buyer may not necessarily be better off when the audit incurs a lower cost (Proposition 5) because truthful communication will be sustained only when the buyer follows a differentiated auditing strategy. That rationale remains valid for general revenue functions, and we have numerically observed similar findings for the specific revenue functions discussed above.

D Effect of Audit Cost: The Cases Omitted in Proposition 4

Proposition 4 in the main paper characterizes the possible PBNE for different combinations of c_A and γ_A under the condition of $\eta_0^N < \theta_L \leq \eta_1^N < \theta^H$. In this appendix, we document the other cases. By Proposition 2, a necessary condition for the influential truth revealing equilibrium to exist is $Q_L^N = d_0$. By this observation, we can eliminate several trivial cases. First, notice that if $\eta_0^N \geq \theta_L$, the influential truth revealing equilibrium will never occur because $Q_L^N = 0$ in this case; see also Corollary 2. Second, if $\eta_0^N \geq \eta_1^N$, then Q_L^N must be either 0 or d_1 by Lemma 1, and the influential truth revealing equilibrium cannot occur as well. We can thus focus on the cases of $\eta_0^N < \eta_1^N$ and $\eta_0^N < \theta_L < \theta_H$. In what follows, we discuss all possible cases one by one.

If $\eta_0^N < \eta_1^N < \theta_L < \theta_H$, then all the buyer's profit-maximizing order quantities are equal to d_1 by Lemma 1, i.e., $Q_L^N = Q_H^N = Q_L^A = Q_H^A = d_1$. By Proposition 2, the influential truth revealing equilibrium does not exist. Since the order quantities are invariant to the supplier types, $VoA(\theta_H) \leq VoA(\theta_L)$ (see Lemma A.2). So, if $c_A \leq VoA(\theta_H)$ or $c_A > VoA(\theta_L)$, a non-influential truth revealing equilibrium exists in which the buyer audits both suppliers or neither of them respectively. If $VoA(\theta_H) < c_A \leq VoA(\theta_L)$, then the babbling equilibrium is the only PBNE. If $\eta_0^N < \theta_L \leq \eta_1^N < \theta_H$, then we have the results stated in Proposition 4. If $\eta_0^N < \theta_L < \theta_H \leq \eta_1^N$, then $Q_L^N = Q_H^N = d_0$, i.e., without any audit the buyer will order the same quantity from both types. Depending on the audit cost and accuracy, we further have three subcases, as summarized in the proposition below.

Proposition D.1. *Assuming that $\eta_0^N < \theta_L < \theta_H \leq \eta_1^N$ (and so $Q_L^N = Q_H^N = d_0$), the PBNE of our base model is characterized as follows:*

- *Case (i): $\gamma_A \leq 1 - \frac{\theta_H(p-w)}{(1-\theta_H)(\delta p - p + w)}$ (and so $Q_L^A = Q_H^A = d_0$)*
  - *(a) for $c_A \geq v(\theta_L, d_0, d_0)$, a non-influential truth revealing equilibrium, along with the babbling equilibrium, exists in which the buyer audits neither type.*
  - *(b) for $v(\theta_H, d_0, d_0) \leq c_A < v(\theta_L, d_0, d_0)$, the babbling equilibrium is the only PBNE.*

   (c) *for* $c_A < v(\theta_H, d_0, d_0)$*, a non-influential truth revealing equilibrium, along with the babbling equilibrium, exists in which the buyer audits both types.*

- *Case (ii):* $1 - \frac{\theta_H(p-w)}{(1-\theta_H)(\delta p-p+w)} < \gamma_A \le 1 - \frac{\theta_L(p-w)}{(1-\theta_L)(\delta p-p+w)}$ *(and so* $Q_L^A = d_0 < Q_H^A = d_1$*)*

   (a) *for* $c_A \ge \max[v(\theta_L, d_0, d_0), v(\theta_H, d_0, d_1)]$*, a non-influential truth revealing equilibrium, along with the babbling equilibrium, exists in which the buyer audits neither type.*

   (b) *for* $v(\theta_H, d_0, d_1) \le c_A < \max[v(\theta_L, d_0, d_0), v(\theta_H, d_0, d_1)]$*, the babbling equilibrium is the only PBNE.*

   (c) *for* $\min[v(\theta_L, d_0, d_0), v(\theta_H, d_0, d_1)] \le c_A < v(\theta_H, d_0, d_1)$*, the influential truth revealing equilibrium as described in Proposition 2, along with the babbling equilibrium, exists if* $\frac{d_1-d_0}{(1-\theta_L)d_1} \le \gamma_A \le \frac{d_1-d_0}{(1-\theta_H)d_1}$*, and the babbling equilibrium is the only PBNE otherwise.*

   (d) *for* $c_A < \min[v(\theta_L, d_0, d_0), v(\theta_H, d_0, d_1)]$*, the babbling equilibrium is the only PBNE.*

- *Case (iii):* $\gamma_A > 1 - \frac{\theta_L(p-w)}{(1-\theta_L)(\delta p-p+w)}$ *(and so* $Q_L^A = Q_H^A = d_1$*)*

   (a) *for* $c_A \ge \max[v(\theta_L, d_0, d_1), v(\theta_H, d_0, d_1)]$*, a non-influential truth revealing equilibrium, along with the babbling equilibrium, exists in which the buyer audits neither type.*

   (b) *for* $v(\theta_H, d_0, d_1) \le c_A < \max[v(\theta_L, d_0, d_1), v(\theta_H, d_0, d_1)]$*, the babbling equilibrium is the only PBNE.*

   (c) *for* $\min[v(\theta_L, d_0, d_1), v(\theta_H, d_0, d_1)] \le c_A < v(\theta_H, d_0, d_1)$*, the influential truth revealing equilibrium as described in Proposition 2, along with the babbling equilibrium, exists if* $\frac{d_1-d_0}{(1-\theta_L)d_1} \le \gamma_A \le \frac{d_1-d_0}{(1-\theta_H)d_1}$*, and the babbling equilibrium is the only PBNE otherwise.*

   (d) *for* $c_A < \min[v(\theta_L, d_0, d_0), v(\theta_H, d_0, d_1)]$*, a non-influential truth revealing equilibrium, along with the babbling equilibrium, exists in which the buyer audits both types.*

As shown in the above proposition, the equilibrium characterization yields very similar insights to those established in Proposition 4 of the main paper. An influential truth revealing equilibrium exists when the audit cost and accuracy are moderate. The main difference from Proposition 4 is that when the audit cost is prohibitive such that the buyer never uses it, there is a non-influential truth revealing equilibrium as in this case the buyer chooses the same order quantity $Q_L^N = Q_H^N = d_0$ when no audit is conducted.

Supplementary Material for "Sourcing from a Self-Reporting Supplier: Strategic Communication of Social Responsibility in a Supply Chain"

A Proofs of the Results in the Main Paper

Proof of Lemma 1. We first consider part (i) for any $i \in \{N, A\}$. Clearly, $\Pi_b^i(Q)$ is concave and piecewise linear in $Q \in (0, +\infty]$ with kink points at $Q = d_0$ and d_1. The slope of $\Pi_b^i(Q)$ is given by $p - w, \psi(\theta, \gamma_i, \delta)p - w$ and $-w$ for $Q \in (0, d_0]$, $Q \in (d_0, d_1]$ and $Q \in (d_1, +\infty)$, respectively. Substituting the expression of $\psi(\theta, \gamma_i, \delta)$ and rearranging the terms, we have

$$\Pi_b^i(d_1) > \Pi_b^i(d_0) \iff \theta > 1 - \frac{p-w}{(1-\gamma_i)\delta p + \gamma_i(p-w)}$$

We define a threshold $\eta_1^i := \left[1 - \frac{p-w}{(1-\gamma_i)\delta p+\gamma_i(p-w)}\right]^+$ to incorporate the possibility of $\frac{p-w}{(1-\gamma_i)\delta p+\gamma_i(p-w)} > 1$. With a goodwill loss g, we need to compare $\Pi_b^i(d_0)$ and $\Pi_b^i(d_1)$ with $\Pi_b^i(0) = 0$. Some algebra leads to the following relations:

$$\Pi_b^i(d_1) > \Pi_b^i(0) \iff \theta > 1 - \frac{(p-w)d_1}{(1-\gamma_i)\delta[g + p(d_1 - d_0)] + \gamma_i(p-w)d_1}$$

$$\Pi_b^i(d_0) > \Pi_b^i(0) \iff \theta > 1 - \frac{(p-w)d_0}{(1-\gamma_i)\delta g + \gamma_i(p-w)d_0}$$

We further define

$$\eta_0^i = \min\left(1 - \frac{(p-w)d_1}{(1-\gamma_i)\delta[g + p(d_1 - d_0)] + \gamma_i(p-w)d_1}, 1 - \frac{(p-w)d_0}{(1-\gamma_i)\delta g + \gamma_i(p-w)d_0}\right),$$

and so if $\theta \le \eta_0^i$, the optimal order quantity is zero; otherwise, $Q^i(\theta) = d_1$ for $\theta > \eta_1^i$ and $Q^i(\theta) = d_0$ for $\theta \le \eta_1^i$. As the thresholds η_0^i and η_1^i depend on i only through γ_i, we can use two functions $H_1(\gamma)$ and $H_0(\gamma)$ as defined in the lemma to summarize all the cases.

To prove part (ii), we show that $Q^N = d_1$ implies $Q^A = d_1$ and that $Q^N = d_0$ implies that $Q^A = d_1$ or d_0. We omit the dependence of Q^i on θ for ease of notation; note that the proof applies to any given θ. First, we show that $H_1(\gamma)$ is (weakly) decreasing in γ. If $\delta p \le p - w$, then $H_1(\gamma) = 0$ for all γ. If $\delta p > p - w$, then it is straightforward to see that $H_1(\gamma)$ is (weakly) decreasing in γ. This proves that $H_1(\gamma)$ is (weakly) decreasing in γ and so $\eta_1^A = H_1(\gamma_A) \le \eta_1^N = H_1(\gamma_N)$. Consequently, for any given θ, $\theta > \eta_1^N$ implies $\theta > \eta_1^A$, and so $Q^N = d_1$ implies $Q^A = d_1$. By assumption 1 and Lemma A.1, it follows that $H_0(\gamma)$ is decreasing in γ as well, which implies that $\eta_0^A \le \eta_0^N$. Thus, $\theta > \eta_0^N$ implies $\theta > \eta_0^A$. From part (i), it follows that if $Q^N = d_1$ then Q^A must be either d_1 and d_0. We can therefore conclude that $Q^N \le Q^A$.

Part (iii) is an immediate consequence of expression (4). $\square$

Proof of Proposition 1. By Lemma A.3, $VoA(\theta)$ is unimodal unless $d_1 - d_0 > \Delta$ and $\eta_0^N \leq \eta_1^A$. Since it is optimal audit if and only if $VoA(\theta) > c_A$, the unimodality of $VoA(\theta)$ implies part (ii). If $d_1 - d_0 > \Delta$ and $\eta_0^N \leq \eta_1^A$, then $VoA(\theta)$ is bi-modal, and so c_A may at most cross $VoA(\theta)$ four times at segments $v(\theta, 0, d_0)$, $v(\theta, d_0, d_0)$, $v(\theta, d_0, d_1)$ and $v(\theta, d_1, d_1)$, respectively. From Lemma A.2, the corresponding intersections can be derived as follows.

$$\begin{aligned}
\theta^{(4)} &= 1 - \frac{(p-w)d_0 - c_A}{\gamma_A(p-w)d_0 + (1-\gamma_A)\delta g} \\
\theta^{(3)} &= 1 - \frac{c_A}{\gamma_A[\delta g - (p-w)d_0]} \\
\theta^{(2)} &= 1 - \frac{c_A - (p-w)(d_1 - d_0)}{\gamma_A[\delta g - (p-w)d_0] - [\gamma_A(p-w) + (1-\gamma_A)\delta p](d_1 - d_0)} \\
\theta^{(1)} &= 1 - \frac{c_A}{\gamma_A[\delta g + \delta p(d_1 - d_0) - (p-w)d_1]}
\end{aligned} \tag{A.1}$$

Note that for ease of exposition, we allow the intervals $(\theta^{(4)}, \theta^{(3)})$, $(\theta^{(2)}, \theta^{(1)})$ and (θ^{-}, θ^{+}) to be empty. $\square$

Proof of Proposition 2. We prove the proposition in a more general setting where the buyer chooses between two audit accuracy levels $\gamma_A > \gamma_N \geq 0$. Clearly, it reduces to the setting where the buyer chooses to audit or not to audit when the lower accuracy level $\gamma_N = 0$. The following lemma is useful for the proof.

Lemma A.1. *The ratio $\frac{\rho(\theta,\gamma_A)}{\rho(\theta,\gamma_N)}$ is strictly increasing in θ.*

Proof. By definition, $\frac{\rho(\theta,\gamma_A)}{\rho(\theta,\gamma_N)} = \frac{1-(1-\theta)\gamma_A}{1-(1-\theta)\gamma_N}$. Taking the derivative of this ratio with respect to θ yields $\frac{\partial}{\partial\theta}\left[\frac{\rho(\theta,\gamma_A)}{\rho(\theta,\gamma_N)}\right] = \frac{\gamma_A - \gamma_N}{(1-(1-\theta)\gamma_N)^2} > 0$. $\square$

We first argue that $a_H = 1$ and $a_L = 0$, i.e., condition (i), is a necessary condition for an influential truth revealing equilibrium to exist. First, note that no influential truth revealing equilibrium can exist if $a_H = a_L$, i.e., the buyer should impose the same audit level on both types under symmetric information. Recall that Lemma 1 has established that $Q_H^N \geq Q_L^N$ and $Q_H^A \geq Q_L^A$. As we require the desired equilibrium to be influential, it follows that the L-type supplier would always mimic the H-type if $a_H = a_L = 1$ ($a_H = a_L = 1$) since the strict inequality $Q_H^A > Q_L^A$ ($Q_H^N > Q_L^N$) must hold. Next, notice that there cannot be a truth revealing equilibrium if $a_H = 0$ and $a_L = 1$, because if so, the supplier's IC condition is reduced to $\rho(\theta_H, \gamma_N)Q_H^N \geq \rho(\theta_H, \gamma_A)Q_L^A$ and $\rho(\theta_L, \gamma_A)Q_L^A \geq \rho(\theta_L, \gamma_N)Q_H^N$. The former part of the IC condition implies $Q_H^N \geq \frac{\rho(\theta_H,\gamma_A)}{\rho(\theta_H,\gamma_N)}Q_L^A$. By Lemma A.1, it follows that $Q_H^N \geq \frac{\rho(\theta_H,\gamma_A)}{\rho(\theta_H,\gamma_N)}Q_L^A > \frac{\rho(\theta_L,\gamma_A)}{\rho(\theta_L,\gamma_N)}Q_L^A$. However, the latter part of the IC condition implies $\frac{\rho(\theta_L,\gamma_A)}{\rho(\theta_L,\gamma_N)}Q_L^A \geq Q_H^N$, which leads to a contradiction.

Next, we can without loss of generality focus on the subgame with $a_H = 1$ and $a_L = 0$. In this case, the supplier's IC condition reduces to

$$Q_H^A \rho(\theta_H, \gamma_A) \geq Q_L^N \rho(\theta_H, \gamma_N) \text{ and } Q_L^N \rho(\theta_L, \gamma_N) \geq Q_H^A \rho(\theta_L, \gamma_A). \tag{A.2}$$

Under the specification of the revenue function, it is obvious that Q_L^N cannot be zero. Furthermore, Since $Q_L^N = d_1$ implies $Q_L^A = Q_H^N = Q_H^A = d_1$ (by Lemma 1), the equilibrium cannot be influential is $Q_L^N = d_1$. Hence, Q_L^N must be d_0 in the desired equilibrium. To satisfy (A.2), it is necessary to have $Q_H^N = d_1$ and $d_1\rho(\theta_H, \gamma_A) \geq d_0\rho(\theta_H, \gamma_N)$ and $d_1\rho(\theta_L, \gamma_A) \leq d_0\rho(\theta_L, \gamma_N)$. In summary of the above discussion, conditions (i) and (ii) are necessary for the existence of an influential truthful reveal equilibrium. Clearly, they ensure the supplier not to mimic the other type, and are also sufficient. $\square$

Proof of Proposition 3. By condition (ii) of Proposition 2, we must have $\eta_0^N < \theta_L \leq \eta_1^N$ such that $Q_L^N = d_0$. We can thus restrict to $\theta_L, \theta_H \in (\eta_0^N, 1]$ without loss of generality. If $d_1 - d_0 \leq \Delta$, then $VoA(\theta)$ is monotonically decreasing for all $\theta \in (\eta_0^N, 1]$ (by Lemma A.3), and so condition (i) of Proposition 2 cannot hold. By Lemma A.3, it can be seen that condition (i) of Proposition 2 cannot be satisfied if either $c_A \geq VoA(\eta_1^N)$ or $c_A \leq VoA(\max(\eta_1^A, \eta_0^N))$. So, $d_1 - d_0 > \Delta$ and $VoA(\max(\eta_1^A, \eta_0^N)) < c_A < VoA(\eta_1^N)$ are necessary for the existence of the influential truth revealing equilibrium.

Condition (i) of Proposition 2 requires θ_H and θ_L to fall into the audit and no-audit intervals, respectively, under symmetric information as characterized in Proposition 1. However, for Condition (ii) to hold, a low-type supplier must have a high enough SR level to be used (receiving an order of d_0) if she is not audited.[30] The final inequalities in Condition (ii) can be reduced to $\theta_L \in (\eta_0^N, \theta^\dagger]$ and $\theta_H \in [\theta^\dagger, 1]$ where $\theta^\dagger := \frac{d_0 - (1-\gamma_A)d_1}{\gamma_A d_1}$. The ranges of θ_H and θ_L required by both conditions jointly constitute the truth revealing intervals as stated in the proposition. $\square$

Proof of Proposition 4. In cheap talk games, the babbling equilibrium always exists as a PBNE (see, e.g., §7 of Sobel, 2014). In the following, we focus on when a truth revealing equilibrium exists. Since $\eta_0^N < \theta_L \leq \eta_1^N$, η_1^N must be positive and equal to $1 - \frac{p-w}{(1-\gamma_N)\delta p + \gamma_N(p-w)}$, which implies $\delta p > p - w$. This in turn implies $\eta_1^A = 1 - \frac{p-w}{(1-\gamma_A)\delta p + \gamma_A(p-w)}$ is also positive. So, in the analysis we can without loss of generality omit the $[\cdot]^+$ operator in the expressions of η_1^N and η_1^A. Notice that $Q_L^A = d_0$ if $\theta_L \leq \eta_1^A$ and $Q_L^A = d_1$ otherwise. The inequality $\theta_L \leq \eta_1^A$ is equivalent to $\gamma_A \leq 1 - \frac{\theta_L(p-w)}{(1-\theta_L)(\delta p - p + w)}$.

So, for case (i), $Q_L^N = Q_L^A = d_0$ and $Q_H^N = Q_H^A = d_1$. The buyer intends to audit the low-(high-)type supplier under symmetric information if and only if $c_A < v(\theta_L, d_0, d_0)$ ($c_A <$

[30] The intervals can be nonempty only when $VoA(\theta)$ is increasing over some range of $(\eta_0^N, 1]$ (i.e., when $d_1 - d_0 > \Delta$, which corresponds to panels (c) and (d) of Figure 2) and when c_A has at least two intersections with $VoA(\theta)$ in $(\eta_0^N, 1]$, i.e., $VoA(\max(\eta_1^A, \eta_0^N)) < c_A < VoA(\eta_1^N)$. Additionally, $Q_L^N = d_0$ implies $\eta_0^N < \eta_1^N$, although we generally allow η_0^N to be larger than η_1^N in §4.1.

$v(\theta_H, d_1, d_1))$. If $c_A \geq \max[v(\theta_L, d_0, d_0), v(\theta_H, d_1, d_1)]$, the buyer audits neither type and so the low type supplier would always mimic the high type since $Q_L^N < Q_H^N$. Consequently, the babbling equilibrium is the only PBNE. If $v(\theta_H, d_1, d_1) \leq c_A \leq \max[v(\theta_L, d_0, d_0), v(\theta_H, d_1, d_1)]$, the buyer intends to audit only the low type, leading to the babbling equilibrium as well by Proposition 2. Therefore, we can combine these two cases as case (i)(a) stated in the proposition. If $\min[v(\theta_L, d_0, d_0), v(\theta_H, d_1, d_1)] \leq c_A < v(\theta_H, d_1, d_1)$, then the buyer intends to audit only the high-type supplier. Thus, by Proposition 2, an influential truth revealing equilibrium exists if the inequalities $\frac{\rho(\theta_L,\gamma_A)}{\rho(\theta_L,\gamma_N)} \leq \frac{d_0}{d_1} \leq \frac{\rho(\theta_H,\gamma_A)}{\rho(\theta_H,\gamma_N)}$ hold, which is equivalent to the interval of γ_A stated in the proposition. Otherwise, the babbling equilibrium is the only PBNE. This leads to case (i)(b). If $c_A < \min[v(\theta_L, d_0, d_0), v(\theta_H, d_1, d_1)]$, the buyer audits both types, and consequently the low-type supplier will mimic the high type because $Q_L^A < Q_H^A$ in case (i). So, the babbling equilibrium is the only PBNE in case (i)(c)

For case (ii), $Q_L^N = d_0$ and $Q_L^A = Q_H^N = Q_H^A = d_1$. Different from Case (i), now the buyer intends to audit the low type under symmetric information if and only if $c_A < VoA(\theta_L) = v(\theta_L, d_0, d_1)$. The three subcases can be derived in a similar fashion. Note that when $c_A < \min[v(\theta_L, d_0, d_1), v(\theta_H, d_1, d_1)]$ such that the buyer audits both types, a non-influential truth revealing equilibrium exists because the (conditional) order quantities are the same for both types.

□

Proof of Lemma 2. Let $a_B \in \{0, 1\}$ denote the buyer's auditing decision in the babbling equilibrium where $a_B = 1$ if the buyer chooses to audit and $a_B = 0$ otherwise. Let Q_B^N (Q_B^A) denote the buyer's order quantity without (with) an audit in the babbling equilibrium. The babbling equilibrium fails to satisfy the NITS condition if $(w - c)Q_L^N > \Pi_s(\theta_L, a_B, Q_B^N, Q_B^A)$ where the left-hand side is the supplier's payoff under symmetric information/truth revealing equilibrium when an influential truth revealing equilibrium exists. Note that in the babbling equilibrium, $\left(\sum_{\theta\in\Theta} \lambda(\theta)\rho(\theta, \gamma_A)\right)$ is the probability of the supplier passing an audit from the buyer's perspective, which is calculated based on the prior probabilities. After the supplier passes an audit, the buyer can update his belief based on the audit result, and the resulting posterior belief is given by $\tilde{\beta}(\theta) = \frac{\rho(\theta,\gamma_A)\lambda(\theta)}{\sum_{\theta'\in\Theta} \rho(\theta',\gamma_A)\lambda(\theta')}$. Therefore, the value of audit in the babbling equilibrium for the buyer can be written as

$$
\begin{aligned}
VoA_B &= \left(\sum_{\theta\in\Theta} \lambda(\theta)\rho(\theta, \gamma_A)\right) \max_{Q\geq 0} \sum_{\theta\in\Theta} \tilde{\beta}(\theta)\Pi_b^A(Q|\theta) - \max_{Q\geq 0} \sum_{\theta\in\Theta} \lambda(\theta)\Pi_b^N(Q|\theta) \\
&= \max_{Q\geq 0} \sum_{\theta\in\Theta} \lambda(\theta)\rho(\theta, \gamma_A)\Pi_b^A(Q|\theta) - \max_{Q\geq 0} \sum_{\theta\in\Theta} \lambda(\theta)\Pi_b^N(Q|\theta)
\end{aligned}
$$

We are considering the case where an influential truth revealing equilibrium exists, which implies $Q_L^N = d_0$ and $VoA(\theta_H) > c_A \geq VoA(\theta_L)$. From $Q_L^N = d_0$, which implies $Q_L^A \geq d_0$, it follows that the order quantities in the babbling equilibrium, Q_B^N and Q_B^A, cannot be zero, and therefore the two maximization problems in the expression of VoA_B have continuous objective functions in $\lambda(\theta)$ and Q over their domains (since these profit functions are discontinuous only at $Q = 0$). By the Maximum Theorem, VoA_B is continuous in $\lambda(\theta)$. Because $VoA(\theta_H) > c_A \geq VoA(\theta_L)$ and $VoA_B \to VoA(\theta_H)$ [resp. $VoA(\theta_L)$] as $\lambda(\theta_H) \to 1$ [resp. 0], there exists a $\hat{\lambda} \in (0,1)$ such that $VoA_B > c_A$ and so $a_B = 1$ if $\lambda(\theta_H) > \hat{\lambda}$. Therefore, when $\lambda(\theta_H) \geq \hat{\lambda}$, the babbling equilibrium satisfies the NITS condition if and only if $Q_B^A \rho(\theta_L, \gamma_A) \geq Q_L^N = d_0$. Clearly, $Q_B^A \leq d_1$, and so the inequality $d_0/d_1 > \rho(\theta_L, \gamma_A)$ implies that the babbling equilibrium violates the NITS condition.

□

Proof of Proposition 5. Under the condition in Lemma 2, the buyer's ex ante profit is given by that he receives in the influential truth revealing equilibrium if such an equilibrium exists; otherwise, the buyer's ex ante profit is equal to that in the babbling equilibrium. Therefore, Π_b^* can be discontinuous in c_A at the boundary conditions for the existence of an influential truth revealing equilibrium.

To prove this proposition, we show that $\lim_{c_A \uparrow VoA(\theta_L)} \Pi_b^*$ is strictly greater than $\lim_{c_A \downarrow VoA(\theta_L)} \Pi_b^*$ under the conditions described in the proposition. Given $\gamma_A < 1 - \frac{\theta_L(p-w)}{(1-\theta_L)(\delta p - p + w)}$, we have $Q_L^A = d_0$. So, we are focusing on the case with $Q_L^N = d_0, Q_L^A = d_0$, $Q_H^N = d_1$ and $Q_H^A = d_1$. Thus, $VoA(\theta_L) = v(\theta_L, d_0, d_0)$ and $VoA(\theta_H) = v(\theta_H, d_1, d_1)$. Note that $\frac{1-\theta_H}{1-\theta_L} > \frac{\delta g - (p-w)d_0}{\delta g + \delta p(d_1 - d_0) - (p-w)d_1}$ is equivalent to $v(\theta_L, d_0, d_0) < v(\theta_H, d_1, d_1)$. So, together with $\frac{d_1 - d_0}{(1-\theta_L)d_1} \leq \gamma_A \leq \frac{d_1 - d_0}{(1-\theta_H)d_1}$, it guarantees that an influential truth revealing equilibrium exists when $v(\theta_H, d_1, d_1) > c_A \geq v(\theta_L, d_0, d_0)$ and the babbling equilibrium is the only PBNE when $c_A < v(\theta_L, d_0, d_0)$ as described in Case (i) of Proposition 4.

Therefore, $\lim_{c_A \uparrow VoA(\theta_L)} \Pi_b^*$ (resp. $\lim_{c_A \downarrow VoA(\theta_L)} \Pi_b^*$) can be calculated as the buyer's ex ante profit under the babbling (resp. influential truth revealing) equilibrium with $c_A = VoA(\theta_L)$.

It follows that

$$\begin{aligned} \lim_{c_A \downarrow VoA(\theta_L)} \Pi_b^* &= \lambda(\theta_H)\rho(\theta_H, \gamma_A)(\Pi_b^A(d_1|\theta_H) - c_A) + \lambda(\theta_L)\Pi_b^N(d_0|\theta_L) \\ &= \lambda(\theta_H)\rho(\theta_H, \gamma_A)(\Pi_b^A(d_1|\theta_H) - c_A) + \lambda(\theta_L)\rho(\theta_L, \gamma_A)(\Pi_b^A(d_0|\theta_L) - c_A) \end{aligned} \tag{A.3}$$

where the second equality follows as $c_A \downarrow VoA(\theta_L)$ such that the buyer is indifferent between auditing and not auditing the low type supplier. On the other hand, under the condition of Lemma 2, the buyer audits the supplier in the babbling equilibrium. Hence,

$$\lim_{c_A \uparrow VoA(\theta_L)} \Pi_b^* = \max_{Q \geq 0} \lambda(\theta_H)\rho(\theta_H, \gamma_A)(\Pi_b^A(Q|\theta_H) - c_A) + \lambda(\theta_L)\rho(\theta_L, \gamma_A)(\Pi_b^A(Q|\theta_L) - c_A) \tag{A.4}$$

Let Q_B^A denote the maximizer of the above problem, i.e., the buyer's optimal (conditional) order quantity in the babbling equilibrium with the stringent audit. Since Q_B^A must be positive, it suffices to consider either $Q_B^A = d_0$ or $Q_B^A = d_1$. If $Q_B^A = d_1$, then

$$\begin{aligned}
\lim_{c_A \uparrow VoA(\theta_L)} \Pi_b^* &= \lambda(\theta_H)\rho(\theta_H, \gamma_A)(\Pi_b^A(d_1|\theta_H) - c_A) + \lambda(\theta_L)\rho(\theta_L, \gamma_A)(\Pi_b^A(d_1|\theta_L) - c_A) \\
&< \lambda(\theta_H)\rho(\theta_H, \gamma_A)(\Pi_b^A(d_1|\theta_H) - c_A) + \lambda(\theta_L)\rho(\theta_L, \gamma_A)(\Pi_b^A(d_0|\theta_L) - c_A) \\
&= \lim_{c_A \downarrow VoA(\theta_L)} \Pi_b^*
\end{aligned}$$

where the strict inequality follows as $Q_L^A = d_0$ is the unique maximizer of $\Pi_b^A(Q|\theta_L)$ given $\gamma_A < 1 - \frac{\theta_L(p-w)}{(1-\theta_L)(\delta p - p + w)}$ (or equivalently, $\theta_L < \eta_1^A$) and the last equality is due to (A.3). Instead, if the $Q_B^A = d_0$, then

$$\begin{aligned}
\lim_{c_A \uparrow VoA(\theta_L)} \Pi_b^* &= \lambda(\theta_H)\rho(\theta_H, \gamma_A)(\Pi_b^A(d_0|\theta_H) - c_A) + \lambda(\theta_L)\rho(\theta_L, \gamma_A)(\Pi_b^A(d_0|\theta_L) - c_A) \\
&< \lambda(\theta_H)\rho(\theta_H, \gamma_A)(\Pi_b^A(d_1|\theta_H) - c_A) + \lambda(\theta_L)\rho(\theta_L, \gamma_A)(\Pi_b^A(d_0|\theta_L) - c_A) \\
&= \lim_{c_A \downarrow VoA(\theta_L)} \Pi_b^*
\end{aligned}$$

where the strict inequality follows as $Q_H^A = d_1$ is the unique maximizer of $\Pi_b^A(Q|\theta_H)$ by the assumption of $\theta_H > \eta_1^A$ and the last equality is due to (A.3). □

Proof of Proposition 6. The proposition is a special case of Proposition 8 when the supplier's evasion cost c_e goes to infinity. Thus, part (i) of Proposition 8 applies. □

Proof of Proposition 7. We first characterize the equilibrium in the three-type game. Clearly, full separation is not possible, as the unqualified type would always mimic other types to avoid not being used. So, we need to verify if any partial separation may occur in equilibrium.

Let us first prove that the partition, $\{\theta_U, \theta_L\}$ and $\{\theta_H\}$, is the only possible separation in equilibrium.

If the types are separated in equilibrium as the partition $\{\theta_U\}$ and $\{\theta_L, \theta_H\}$, then the buyer knows the unqualified supplier with certainty in equilibrium and will therefore order nothing from her. Consequently, the unqualified type will always have the incentive to mimic other types to attract a positive order. Thus, such a partition cannot happen in equilibrium.

Suppose that the partition, $\{\theta_L\}$ and $\{\theta_U, \theta_H\}$, is supported in equilibrium. As such, the low type supplier should report a message $\mu(\theta_L) = m_1$, different from the message from the other types $\mu(\theta_U) = \mu(\theta_H) = m_2$. In such an equilibrium, the buyer orders d_0 and does not audit once receiving message m_1, as it is his optimal decision given θ_L. To guarantee the IC conditions, the buyer must order d_1 and conducts an audit if receiving a message m_2. The existence of such an equilibrium implies that $(w-c)d_1\rho(\theta_U, \gamma_A) \geq (w-c)d_0$ (i.e., the IC condition for the unqualified type) and $(w-c)d_0 \geq (w-c)d_1\rho(\theta_L, \gamma_A)$ (i.e., the IC condition for the low type).

A contradiction arises because $\rho(\theta_U, \gamma_A) < \rho(\theta_L, \gamma_A)$.

Now we analyze the conditions under which a partial separation $\{\theta_U, \theta_L\}$ and $\{\theta_H\}$ is indeed supported in equilibrium. Let m_1 be the message from an unqualified or low type supplier, and m_2 represent the message from the high type. In such an equilibrium, once receiving a message m_2, the buyer knows for sure that the supplier is of high type and so he will order d_1 and conduct the audit. If receiving a message m_1, the buyer will make decisions based on the posterior belief $\beta(\theta_t|m_1) = \lambda(\theta_t)/[\lambda(\theta_U) + \lambda(\theta_L)]$ where $t = U$ and L and $\beta(\theta_H|m_1) = 0$.

We consider the buyer's ordering decision formulated in (1), given message m_1. By Bayes' rule, the buyer's updated belief conditional on the supplier passing an audit with accuracy γ_i (where $i \in \{N, A\}$) is given by

$$\begin{aligned}\hat{\beta}(\theta_t|m_1, \gamma_i) &= \frac{\beta(\theta_t|m_1)\rho(\theta_t, \gamma_i)}{\beta(\theta_U|m_1)\rho(\theta_U, \gamma_i) + \beta(\theta_L|m_1)\rho(\theta_L, \gamma_i) + \beta(\theta_H|m_1)\rho(\theta_H, \gamma_i)} \\ &= \begin{cases} \frac{\lambda(\theta_t)\rho(\theta_t,\gamma_i)}{\lambda(\theta_U)\rho(\theta_U,\gamma_i)+\lambda(\theta_L)\rho(\theta_L,\gamma_i)} & \text{for } t = U, L \\ 0 & \text{for } t = H \end{cases}\end{aligned}$$

Substituting the above expression into (1), we can simplify $\Pi_b^i(Q)$ as

$$\begin{aligned}\Pi_b^i(Q) &= \frac{\sum_{t\in\{U,L\}} \lambda(\theta_t)[\theta_t + (1-\delta)(1-\gamma_i)(1-\theta_t)]}{\sum_{t\in\{U,L\}} \lambda(\theta_t)\rho(\theta_t, \gamma_i)} R(Q|1) \\ &\quad + \left(1 - \frac{\sum_{t\in\{U,L\}} \lambda(\theta_t)[\theta_t + (1-\delta)(1-\gamma)(1-\theta_t)]}{\sum_{t\in\{U,L\}} \lambda(\theta_t)\rho(\theta_t, \gamma_i)}\right)[R(Q|0) - g\mathbf{1}(Q>0)] - wQ \\ &= \psi(E[\theta|m_1], \gamma_i, \delta)R(Q|1) + (1 - \psi(E[\theta|m_1], \gamma_i, \delta))[R(Q|0) - g\mathbf{1}(Q>0)] - wQ\end{aligned}$$

where $E[\theta|m_1] = \sum_{t\in\{U,L\}} \beta(\theta_t|m_1)\theta_t$, and the last equality follows by the definition of $\psi(\theta, \gamma, \delta)$ and the linearity of $\rho(\theta, \gamma)$ in θ. This simplification implies that the buyer's optimal order quantity Q^i depends on θ_U and θ_L only through the expected value $E[\theta|m_1]$.

Likewise, since $\rho(\theta, \gamma)$ is linear in θ, the buyer's ex ante profit (2) can be rewritten as

$$\pi_b(a) = a\left[\Pi_b^A(Q^A(m))\rho(E[\theta|m_1], \gamma_A) - c_A\right] + (1-a)\Pi_b^N(Q^N(m)).$$

Hence, the buyer's auditing decision depends on θ_U and θ_L only through $E[\theta|m_1]$ as well.

Therefore, when the buyer holds a posterior belief: $\beta(\theta_t|m_1) = \lambda(\theta_t)/[\lambda(\theta_U) + \lambda(\theta_L)]$ for $t = U, L$ and $\beta(\theta_H|m_1) = 0$, his optimal ordering and auditing decisions follow the same structure as proved in §4.1 except that the specific value of θ is now replaced by $E[\theta|m_1]$.

On the other hand, an influential equilibrium with partial separation $\{\theta_U, \theta_L\}$ and $\{\theta_H\}$ occurs only if the buyer's optimal decisions are $a(m_1) = 0$ and $Q^N(m_1) = d_0$ once receiving the message m_1 from a supplier that belongs to $\{\theta_U, \theta_L\}$. Thus, Proposition 3 can be adapted to the following: The influential truth revealing equilibrium with partial separation $\{\theta_U, \theta_L\}$ and

$\{\theta_H\}$ exists if and only if

- $E[\theta|m_1] \in (\eta_0^N, \min(\theta^\dagger, \theta^{(2)})]$ and $\theta_H \in (\max(\theta^\dagger, \theta^{(2)}), \theta^{(1)})$ for $\eta_0^N > \eta_1^A$;
- $E[\theta|m_1] \in (\theta^{(3)}, \min(\theta^\dagger, \theta^{(2)})]$ and $\theta_H \in (\max(\theta^\dagger, \theta^{(2)}), \theta^{(1)})$ for $\eta_0^N \leq \eta_1^A$.

Under the assumption that θ_L and θ_H are in the truth revealing intervals for the two-type game, we can recast the above condition as $\frac{\lambda(\theta_U)\theta_U+\lambda(\theta_L)\theta_L}{\lambda(\theta_U)+\lambda(\theta_L)} > \eta_0^N$ for $\eta_0^N > \eta_1^A$ and $\frac{\lambda(\theta_U)\theta_U+\lambda(\theta_L)\theta_L}{\lambda(\theta_U)+\lambda(\theta_L)} > \theta^{(3)}$ for $\eta_0^N \leq \eta_1^A$, which is equivalent to $\lambda(\theta_U) > \Lambda$ where $\Lambda = \frac{\lambda(\theta_L)(\theta_L-\eta_0^N)}{\eta_0^N-\theta_U}$ for $\eta_0^N > \eta_1^A$ and $\Lambda = \frac{\lambda(\theta_L)(\theta_L-\eta^{(3)})}{\eta^{(3)}-\theta_U}$ for $\eta_0^N \leq \eta_1^A$.

Moreover, following a similar argument used in the proof of Lemma 2, one can show that the unqualified supplier strictly prefers the influential equilibrium with partial separation to the (completely) babbling equilibrium if $d_0/d_1 > \rho(\theta_U, \gamma_S)$ and the buyer opts to audit in the babbling equilibrium. As proved earlier, the buyer's optimal decisions depend only on the (conditional) expected value of θ. Thus, in the babbling equilibrium, the buyer will audit the supplier if $\sum_{t\in\{U,L,H\}} \lambda(\theta_t)\theta_t$ is close to θ_H, which is true when $\lambda(\theta_H)$ is large enough.

To examine $\Pi_b^* - \Pi_b^\dagger$, first note that Π_b^* is a constant when $\lambda(\theta_L)/\lambda(\theta_H)$ remains constant. Thus, it suffices to prove that $\Pi_b^\dagger$ is decreasing in $\lambda(\theta_U)$ with a downward jump at $\lambda(\theta_U) = \Lambda$. Let $\lambda(\theta_L) = (1-\lambda(\theta_U))r_L$ and $\lambda(\theta_H) = (1-\lambda(\theta_U))r_H$ where r_L and r_H are two positive constants such that $r_L + r_H = 1$. As a result, $\lambda(\theta_L)/\lambda(\theta_H)$ remains as r_L/r_H for any $\lambda(\theta_U)$. If $\lambda(\theta_U) < \Lambda$, $\Pi_b^\dagger$ can be generally written as

$$\Pi_b^\dagger = (\lambda(\theta_U) + \lambda(\theta_L))\Pi_b(a_{UL}, Q_{UL}|\{\theta_U, \theta_L\}) + \lambda(\theta_H)\Pi_b(a_H, Q_H|\theta_H)$$

where (a_{UL}, Q_{UL}) and (a_H, Q_H) represent the buyer's optimal decisions for the type bundle θ_U, θ_L and for θ_H, respectively. It follows that

$$\frac{\partial \Pi_b^\dagger}{\partial \lambda(\theta_U)} = r_H[\Pi_b(a_{UL}, Q_{UL}|\{\theta_U, \theta_L\}) - \Pi_b(a_H, Q_H|\theta_H)] \leq 0$$

where the equality follows by invoking $r_L + r_H = 1$ and the inequality follows by the fact that the buyer's profit is increasing in θ under symmetric information and in the average value of θ when multiple types are pooled together. Likewise, for $\lambda(\theta_U) \geq \Lambda$, we have

$$\Pi_b^\dagger = \sum_{t\in\{U,L,H\}} \lambda(\theta_t)\Pi(a_B, Q_B|\theta_t)$$

where (a_B, Q_B) denote the buyer's optimal decision under the babbling equilibrium. It follows that

$$\frac{\partial \Pi_b^\dagger}{\partial \lambda(\theta_U)} = \Pi_b(a_B, Q_B|\theta_U) - r_L\Pi_b(a_B, Q_B|\theta_L) - r_H\Pi_b(a_B, Q_B|\theta_H) \leq 0$$

as $\Pi_b(a, Q|\theta)$ is increasing in θ for fixed a and Q. We can therefore conclude that $\Pi_b^* - \Pi_b^\dagger$ is increasing in $\lambda(\theta_U)$. Furthermore, as $\lambda(\theta_U)$ exceeds Λ, the equilibrium switches and we have

$$\lim_{\lambda(\theta_U)\uparrow\Lambda}\Pi_b^\dagger-\lim_{\lambda(\theta_U)\downarrow\Lambda}\Pi_b^\dagger=[\lambda(\theta_U)+\lambda(\theta_L)][\Pi_b(a_{UL},Q_{UL}|\{\theta_U,\theta_L\})-\Pi_b(a_B,Q_B|\{\theta_U,\theta_L\})]$$
$$+\lambda(\theta_H)[\Pi_b(a_H,Q_H|\theta_H)-\Pi_b(a_B,Q_B|\theta_H)].$$

Since (a_{UL},Q_{UL}) and (a_H,Q_H) are optimal when the buyer faces the bundle $\{\theta_U,\theta_L\}$ or a single type θ_B, respectively, $\lim_{\lambda(\theta_U)\uparrow\Lambda}\Pi_b^\dagger-\lim_{\lambda(\theta_U)\downarrow\Lambda}\Pi_b^\dagger$ must be nonnegative. Note that $\lim_{\lambda(\theta_U)\uparrow\Lambda}\Pi_b^\dagger=\lim_{\lambda(\theta_U)\downarrow\Lambda}\Pi_b^\dagger$ only if (a_{UL},Q_{UL}) and (a_H,Q_H) are identical, which cannot be true by the fact that the buyer takes different actions for $\{\theta_U,\theta_L\}$ and θ_B, i.e., the communication is influential. Thus, $\lim_{\lambda(\theta_U)\uparrow\Lambda}\Pi_b^\dagger-\lim_{\lambda(\theta_U)\downarrow\Lambda}\Pi_b^\dagger$ is strictly positive. □

Proof of Proposition 8. For part (i), $c_e\geq(w-c)\gamma_A(1-\theta_L)d_1$ implies that the low-type supplier will never evade an audit. Thus, the influential truth revealing equilibrium characterized in the base model is intact. Also note that by condition (ii) of Proposition 2, we have $d_0/d_1\geq\rho(\theta_L,\gamma_A)$, which implies $(w-c)(d_1-d_0)\leq(w-c)d_1\gamma_A(1-\theta_L)$. Thus, $c_e\geq(w-c)(d_1-d_0)$ is automatically satisfied in part (i).

In what follows, we show that when the influential truth revealing equilibrium is the unique equilibrium surviving the NITS refinement, the buyer's optimal decision without supplier self-reporting (i.e., in the babbling equilibrium of the game with self-reporting) must be $Q_B^A=d_1$ and $a_B=1$. It is easy to see that when the babbling equilibrium fails the NITS refinement, then the buyer must opt to audit in the babbling equilibrium. Otherwise, the low-type supplier will weakly prefer the babbling equilibrium in which no audit is conducted. So, the other possible decision for the buyer to take in a NITS-surviving babbling equilibrium is $Q_B^A=d_0$ and $a_B=1$. However, we show that this solution cannot be optimal by proving that it results in a lower payoff to the buyer than $a_B=0$ and $Q_B^A=d_0$. Given $a_B=0$ and $Q_B^A=d_0$, the buyer's ex ante profit is given by

$$\Pi_b(0,d_0)=\lambda(\theta_L)\Pi_b^N(d_0|\theta_L)+\lambda(\theta_H)\Pi_b^N(d_0|\theta_H) \tag{A.5}$$

Given $a_B=1$ and $Q_B^A=d_0$, the buyer's ex ante profit depends on whether the supplier will evade an audit when the conditional order is d_0. If $c_e\geq(w-c)\gamma_A(1-\theta_L)d_0$, the supplier will not evade for $Q_B^A=d_0$. The buyer's ex ante profit is equal to

$$\Pi_b(1,d_0)=\lambda(\theta_L)\rho(\theta_L,\gamma_A)\Pi_b^A(d_0|\theta_L)+\lambda(\theta_H)\rho(\theta_H,\gamma_A)\Pi_b^A(d_0|\theta_H)-c_A \tag{A.6}$$

If follows that

$$\Pi_b(1,d_0)-\Pi_b(0,d_0)=\lambda(\theta_L)v(\theta_L,d_0,d_0)+\lambda(\theta_H)v(\theta_H,d_0,d_0)-c_A$$

where $v(\theta,d_0,d_0)$ is as defined in Lemma A.2. By the existence of the influential revealing equilibrium in our base model, the buyer would not audit the low-type supplier under symmetric information. This implies that if $Q_L^A = d_0$ then $VoA(\theta_L) = v(\theta_L, d_0, d_0) \leq c_A$, or if $Q_L^A = d_1$ then $VoA(\theta_L) = v(\theta_L, d_0, d_1) \leq c_A$. Note that in the base model $Q_L^A = d_1$ implies $\Pi_b^A(d_0|\theta_L) \leq \Pi_b^A(d_1|\theta_L)$. By the definition of $v(\cdot,\cdot,\cdot)$, it follows that $v(\theta_L, d_0, d_0) \leq v(\theta_L, d_0, d_1)$. Hence, $v(\theta_L, d_0, d_0) \leq c_A$ as well. Because $v(\theta, d_0, d_0)$ is decreasing in θ by Lemma A.2, we have $v(\theta_H, d_0, d_0) \leq c_A$. Hence, $\Pi_b(1, d_0) - \Pi_b(0, d_0) \leq 0$, and thus $a_B = 1$ and $Q_B^A = d_0$ cannot be optimal.

Therefore, the buyer will set $a_B = 1$ and $Q_B^A = d_1$ in any non-NITS babbling equilibrium, leading to an expected damage $SD_{NS} = \lambda(\theta_L)(1-\theta_L)(1-\gamma_A)d_1 + \lambda(\theta_H)(1-\theta_H)(1-\gamma_A)d_1$, whereas the expected damage with supplier self-reporting is given by $SD_S = \lambda(\theta_L)(1-\theta_L)d_0 + \lambda(\theta_H)(1-\theta_H)(1-\gamma_A)d_1$. Thus, $SD_S - SD_{NS} = \lambda(\theta_L)(1-\theta_L)[d_0 - (1-\gamma_A)d_1]$. Note that the existence of the influential truth revealing equilibrium requires $d_0/d_1 \geq \rho(\theta_L, \gamma_A) = 1 - (1-\theta_L)\gamma_A > 1 - \gamma_A$, which implies $d_0 > (1-\gamma_A)d_1$. Thus, $SD_S - SD_{NS} > 0$, i.e., the expected damage is higher with supplier self-reporting than without it.

For part (ii), the low-type supplier will evade at least for a conditional order size d_1. For this case, the sufficient and necessary conditions for the existence of an influential truth revealing equilibrium have been derived in Proposition A.1. Given that the influential truth revealing equilibrium exists in our base model, We first prove that it will continue to exist for if $c_e \geq (w-c)(d_1 - d_0)$.

As the high-type supplier will not evade, the buyer will still audit the high type with a conditional order d_1 if information is symmetric. It is easy to see that for the low type, the buyer will still order d_0 without an audit as in the base model. Clearly, ordering nothing or unconditionally ordering d_1 is not optimal, as the optimal order quantity without audit is $Q_L^N = d_0$ as per Proposition 2. Suppose that the buyer wants to order d_1 with an audit, and then the low-type supplier will evade any audit since $c_e < (w-c)\gamma_A(1-\theta_L)d_1$. Anticipating that the supplier will evade, the buyer recognizes that a positive auditing outcome has no effect. Thus, his optimal order quantity will remain as d_0 as opposed to d_1. Condition (a) of Proposition A.1 is thus satisfied. Condition (b) of Proposition A.1 is also satisfied as long as $c_e \geq (w-c)(d_1 - d_0)$ and that condition(ii) of Proposition 2 holds. Therefore, the influential truth revealing equilibrium continues to exist.

Next, similar to part (i), we show that in any non-NITS babbling equilibrium, the buyer's optimal action must be $a_B = 1$ and $Q_B = d_1$. As before, it suffices to prove that $a_B = 1$ and $Q_B = d_0$ are not optimal. However, we need to consider two subcases, depending on whether the supplier will evade for a conditional order d_0.

If $c_e \geq (w-c)\gamma_A(1-\theta_L)d_0$, the supplier will not evade for a conditional order d_0. As a result, $\Pi_b(0, d_0)$ and $\Pi_b(1, d_0)$ remains the same as defined in (A.5) and (A.6), respectively. Thus, following the same argument as in the proof of part (i), we have $\Pi_b(1, d_0) - \Pi_b(0, d_0) \leq 0$.

If $c_e < (w-c)\gamma_A(1-\theta_L)d_0$, the supplier will evade for both (conditional) order sizes d_0 and d_1. Note that $\Pi_b(0,d_0)$ remains as defined (A.5) but $\Pi_b(1,d_0)$ should be adapted to

$$\Pi_b(1,d_0) = \lambda(\theta_L)\Pi_b^N(d_0|\theta_L) + \lambda(\theta_H)\rho(\theta_H,\gamma_A)\Pi_b^A(d_0|\theta_H) - c_A \tag{A.7}$$

as the audit is not informative at all when the supplier is low-type. It follow that $\Pi_b(1,d_0) - \Pi_b(0,d_0) = \lambda(\theta_H)v(\theta_H,d_0,d_0) - c_A \leq 0$ where the inequality holds since we have proved $v(\theta_H,d_0,d_0) \leq c_A$ in part (i).[31]

Therefore, $a_B = 1$ and $Q_B = d_0$ cannot be optimal in either subcase. So, in the non-NITS babbling equilibrium we must have $a_B = 1$ and $Q_B = d_1$. Consequently, the supplier, if being a low type, will evade the audit in such an equilibrium It follows that without supplier self-reporting, the expected damage is given by

$$SD_{NS} = \lambda(\theta_L)(1-\theta_L)d_1 + \lambda(\theta_H)(1-\gamma_A)(1-\theta_H)d_1$$

whereas with supplier self-reporting,

$$SD_S = \lambda(\theta_L)(1-\theta_L)d_0 + \lambda(\theta_H)(1-\theta_H)(1-\gamma_A)d_1.$$

Thus, $SD_S - SD_{NS} = -\lambda(\theta_L)(1-\theta_L)(d_1-d_0) < 0$

□

B Proofs of the Technical Results in Appendix A

Proof of Lemma A.1. Note that if a violation is detected in the audit, then the buyer's expected profit is given by

$$\Pi_b^F(Q) = \delta p \min(d_1,Q) + (1-\delta)(p\min(d_0,Q) - g\mathbf{1}(Q>0)) - wQ,$$

which is piecewise linear and concave in $Q \in (0,\infty)$. The buyer orders a zero quantity if and only if $\max_{Q>=0}\Pi_b^F(Q) \leq 0$, which is equivalent to $\max(\Pi_b^F(d_1),\Pi_b^F(d_0)) \leq 0$ in view of the piecewise linearity of the profit function. Rearranging the terms leads to the following relations:

$$\Pi_b^F(d_1) > 0 \iff \delta g < (p-w)d_1 - \delta p(d_1-d_0) \tag{F1}$$

$$\Pi_b^F(d_0) > 0 \iff \delta g < (p-w)d_0 \tag{F2}$$

The lemma follows as Assumption 1 holds true if and only if $\max(\Pi_b^F(d_1),\Pi_b^F(d_0)) \leq 0$. □

[31] Note that $v(\cdot,\cdot,\cdot)$ is as defined for our base model without supplier evasion.

Proof of Lemma A.2. The closed-form expressions of different cases are straightforward to derive. Clearly, $v(\theta, 0, d_0)$ and $v(\theta, 0, d_1)$ are both increasing in θ. The fact that $v(\theta, d_1, d_1)$ and $v(\theta, d_0, d_0)$ are both decreasing in θ follows by Assumption 1. Finally, the slope of $v(\theta, d_0, d_1)$ is positive if and only if $d_1 - d_0 > \Delta := \frac{(\gamma_A-\gamma_N)[\delta g-(p-w)d_0]}{(1-\gamma_A)\delta p+\gamma_A(p-w)}$. $\square$

Proof of Lemma A.3. This lemma follows from the expressions of $VoA(\theta)$ and the monotone properties of $v(\theta, q^N, q^A)$ characterized in Lemma A.2. $\square$

Proof of Proposition A.1. In general, with the possibility of deception in an audit, the supplier's expected profit (3) can be adapted to

$$\Pi_s(\theta, a(m), Q^N(m), Q^A(m)) = \begin{cases} (w-c)\max\left[Q^A(m) - \frac{c_e}{w-c}, Q^A(m)\rho(\theta, \gamma_A)\right] & \text{if } a(m) = 1 \\ (w-c)Q^N(m) & \text{if } a(m) = 0 \end{cases} \tag{B.1}$$

We first show that the influential truth revealing equilibrium can occur only if the buyer intends to audit the high-type and conditionally order d_1 whereas not audit the low-type and order d_0 (i.e., $Q_H^A = d_1, a_H = 1$, $Q_L^N = d_0, a_L = 0$).

In what follows, we verify that the influential truth revealing equilibrium is not sustained whenever the buyer follow any other strategies than $Q_H^A = d_1, a_H = 1$, $Q_L^N = d_0, a_L = 0$.

First, note that the supplier's profit function(B.1) is strictly increasing in the order quantity for fixed any $a(m)$. Hence, there will be one type trying to mimic the other for a larger order size whenever $a_H = a_L$. Second, if the buyer would order the same quantity from both types but conduct the audit on only one type, i.e., $a_t = 1$, $a_{t'} = 0$ and $Q_t^A = Q_{t'}^N$ for $t, t' \in \{H, L\}$ and $t \neq t'$, then type t will mimic type t' to avoid the audit since $a(m) = 0$ always leads to a higher profit whenever $Q^N(m) = Q^A(m)$ in (B.1). Similarly, if $a_t = 1$ and $Q_t^A = d_0$, $a_{t'} = 0$ and $Q_{t'}^A = d_1$, then by mimicking type t', the type-t supplier can always improve her profit from $(w-c)\max\left[d_0 - \frac{c_e}{w-c}, d_0\rho(\theta_t, \gamma_A)\right]$ to $(w-c)d_1$.

The remaining case to rule out is $a_H = 0, Q_H^N = d_0$ and $a_L = 1, Q_L^A = d_1$. The IC conditions require the following.

$$\text{For the high type: } (w-c)d_0 \geq (w-c)\max\left[d_1 - \frac{c_e}{w-c}, d_1\rho(\theta_H, \gamma_A)\right]$$

$$\text{For the low type: } (w-c)\max\left[d_1 - \frac{c_e}{w-c}, d_1\rho(\theta_L, \gamma_A)\right] \geq (w-c)d_0$$

By assumption, the high-type will not cheat in the audit, i.e., $c_e > (w-c)\gamma_A(1-\theta_H)d_1$, or equivalently, $d_1 - \frac{c_e}{w-c} < d_1\rho(\theta_H, \gamma_A)$. Thus, the high type's IC condition implies $d_0 \geq d_1\rho(\theta_H, \gamma_A)$. On the other hand, if $c_e < (w-c)\gamma_A(1-\theta_L)d_1$, then the low-type supplier cheat in the audit and earns $d_1 - \frac{c_e}{w-c}$ and so the low type's IC condition reduces to $d_1 - \frac{c_e}{w-c} \geq d_0$. The two IC conditions together imply $d_1 - \frac{c_e}{w-c} \geq d_1\rho(\theta_H, \gamma_A)$, which will not be satisfied under the assumption $d_1 - \frac{c_e}{w-c} < d_1\rho(\theta_H, \gamma_A)$.

Thus, the only possible strategy of the buyer that can result in an influential truth revealing equilibrium is $a_H = 1, Q_H^A = d_1$ and $a_L = 0, Q_L^N = d_0$.

When $(w-c)\gamma_A(1-\theta_H)d_1 \le c_e < (w-c)\gamma_A(1-\theta_L)d_1$, the low-type supplier will evade an audit if receiving a conditional order d_1. In this case, the IC condition for the high-type supplier is the same as before: $(w-c)d_1\rho(\theta_H, \gamma_A) \ge (w-c)d_0$, or equivalently, $d_0/d_1 \le \rho(\theta_H, \gamma_A)$. However, the low-type's IC condition needs to be adapted to $(w-c)d_0 \ge (w-c)d_1 - c_e$, or equivalently, $d_0/d_1 \ge 1 - \frac{c_e}{(w-c)d_1}$. These IC conditions lead to condition (b) stated in part (i) of the proposition.

For the NITS refinement, similar to the proof of Lemma 2, the low-type supplier will strictly prefer the influential revealing equilibrium over the babbling equilibrium only if the buyer chooses to audit in the babbling equilibrium. As $\lambda(\theta_H)$ is close to 1, the buyer will audit because the problem converges to the symmetric information case with a high-type supplier. Hence, there exists a threshold $\hat{\lambda}$ such that the buyer opt to audit if $\lambda(\theta_H) \ge \hat{\lambda}$. To guarantee the strict preference, we need $d_0/d_1 > 1 - \frac{c_e}{(w-c)d_1}$.

□

C Proofs of the Results for the Case with Audit after Production

Proof of Proposition B.1. We prove for the general setting where the buyer chooses between two audit accuracy levels $\gamma_A > \gamma_N \ge 0$. When $\gamma_N = 0$ (and thus $\rho(\theta, \gamma_N) = 1$), it reduces to the case where the buyer chooses between auditing and not auditing the supplier.

First, it is straightforward to verify that

$$\frac{\partial}{\partial\theta}\left[\frac{w\rho(\theta,\gamma_A) - c}{w\rho(\theta,\gamma_N) - c}\right] = \frac{(\gamma_A - \gamma_N)(w-c)w}{(w(1-\gamma_N(1-\theta)) - c)^2} > 0.$$

Therefore, $\frac{w\rho(\theta,\gamma_A)-c}{w\rho(\theta,\gamma_N)-c}$ is strictly increasing in θ. This is analogous to the monotone property of $\frac{\rho(\theta,\gamma_A)}{\rho(\theta,\gamma_N)}$ stated in Lemma A.1; see Proof of Proposition 2. The supplier's IC condition differs from that in our base model only in that $(w-c)\rho(\theta,\gamma)$ is replaced with $w\rho(\theta,\gamma) - c$. Hence, the proposition can be proved by the similar argument used in the proof of Propositions 2. □

Proof of Proposition B.2. As in our base model, the babbling equilibrium fails the NITS refinement while the influential truth revealing equilibrium survives only if the buyer chooses to audit in the babbling equilibrium. As the buyer's decision problem is identical to that in our base model, one can show that in the non-NITS babbling equilibrium, the buyer will order $Q_B^A = d_1$ conditional on the supplier passing the audit. (This is proved when we prove part (i) of Proposition 8.) So, the babbling equilibrium will fail the NITS refinement if the low-type supplier earns a profit strictly higher than what she receives in the babbling equilibrium, i.e., $(w-c)d_0 > (w\rho(\theta_L,\gamma_A)-c)d_1$.

Different from our base model, if the audit is conducted after production, an audit failure will not prevent production from happening. Thus, the production-based social damage is simply equal to $SD(\theta) = (1-\theta)Q$. With self-reporting, the expected social damage (after the NITS refinement) is thus given by $SD_S = \lambda(\theta_H)(1-\theta_H)d_1 + \lambda(\theta_L)(1-\theta_L)d_0$. Without self-reporting, the buyer will make decisions based on the prior belief, so his order quantity and auditing choice will be identical to what he would do in the babbling equilibrium. Hence, the expected social damage without self-reporting is equal to $SD_{NS} = \lambda(\theta_H)(1-\theta_H)d_1 + \lambda(\theta_L)(1-\theta_L)d_1$. Clearly, $SD_S - SD_{NS} < 0$ as $d_0 < d_1$. □

D Proofs of the Results for the General Revenue Function

D.1 Proofs of Propositions C.1, C.2 and C.3

In the following proofs, we consider a general setting where the buyer chooses between two audit accuracy levels $\gamma_A > \gamma_N \geq 0$. When $\gamma_N = 0$, it reduces to the case where the buyer chooses between auditing and not auditing the supplier.

Proof of Proposition C.1. The sufficiency of conditions (i) and (ii) follows immediately by noting that they together guarantee that the supplier's IC conditions are met. To show that they are necessary as well, we argue that an influential truth revealing equilibrium cannot emerge if under symmetric information, the buyer would impose a higher audit level on both types, or neither of them, or only the low-type. If $c_A < \min(VoA(\theta_H), VoA(\theta_L))$, then the buyer imposes the stringent audit level on both types. Unless $Q_L^A = Q_H^A$, one type will always mimic the other to attract a larger order. However, given $Q_L^A = Q_H^A$, the truth revealing equilibrium cannot be influential. Likewise, if $c_A \geq \max(VoA(\theta_H), VoA(\theta_L))$ such that the buyer does not use the stringent audit on either type, one type will always mimic the other for a larger order unless $Q_L^N = Q_H^N$, and so an influential equilibrium cannot emerge. Suppose that $VoA(\theta_L) > c_A \geq VoA(\theta_H)$ such that the buyer conduct the stringent audit only on the low-type supplier. The IC condition for the low type supplier requires that $\rho(\theta_L,\gamma_A)Q_L^A \geq \rho(\theta_L,\gamma_N)Q_H^N$, whereas that for the high type requires that $\rho(\theta_H,\gamma_N)Q_H^N \geq \rho(\theta_H,\gamma_A)Q_L^A$. It follows from the former part and Lemma A.1 (see the proof of Proposition 2), i.e., the monotone likelihood ratio of $\rho(\theta,\gamma)$, that $Q_H^N \leq \frac{\rho(\theta_L,\gamma_A)}{\rho(\theta_L,\gamma_N)}Q_L^A < \frac{\rho(\theta_H,\gamma_A)}{\rho(\theta_H,\gamma_N)}Q_L^A$, which conflicts with the latter part of the IC condition. Therefore, it is necessary to have $VoA(\theta_H) > c_A \geq VoA(\theta_L)$, i.e., condition (i). Under condition (i), the supplier's IC conditions are equivalent to condition (ii). This completes the proof. □

Proof of Proposition C.2. Within $(\underline{\theta}, 1]$, Q^N is not smaller than d_0. Hence, we can write

$R(Q|0) = pd_0$ without loss of generality, and so $r(Q|0) = 0$. Additionally, we have $r(Q|1) = p\bar{F}(Q - d_0)$ and $r'(Q|1) = -pf(Q - d_0)$. The optimal order quantities under symmetric information are given by

$$Q^i = \begin{cases} F^{-1}\left(1 - \frac{w}{\psi(\theta,\gamma_i,\delta)p}\right) + d_0 & \text{if } \theta > \eta_1^i \\ d_0 & \text{if } \theta \leq \eta_1^i \end{cases} \tag{D.1}$$

where $i \in \{N, A\}$ and η_1^i is as defined in Proposition 1. Recall that $\eta_1^A \leq \eta_1^N$. Thus, to analyze the shape of $VoA(\theta)$, we have three cases.

Case 1: $\underline{\theta} < \theta \leq \eta_1^A$. In this case, $Q^N = Q^A = d_0$, and so $VoA(\theta) = v(\theta, d_0, d_0)$, which is linearly decreasing in θ by Lemma A.2.

Case 2: $\eta_1^A < \theta \leq \eta_1^N$. In this case, $Q^N = d_0$ and $Q^A = F^{-1}\left(1 - \frac{w}{\psi(\theta,\gamma_A,\delta)p}\right) + d_0$. With $\delta = 1$, it follows that

$$\begin{aligned} VoA'(\theta) &= \left(\frac{\partial\rho(\theta,\gamma_A)}{\partial\theta}\Pi_b^A(Q^A) + \rho(\theta,\gamma_A)\frac{\partial\Pi_b^A(Q^A)}{\partial\theta}\right) - \left(\frac{\partial\rho(\theta,\gamma_N)}{\partial\theta}\Pi_b^N(d_0) + \rho(\theta,\gamma_N)\frac{\partial\Pi_b^A(d_0)}{\partial\theta}\right). \\ &= \gamma_A[R(Q^A|1) - wQ^A] + (1-\gamma_A)[R(Q^A|1) - R(Q^A|0) + g] - \gamma_N(p-w)d_0 - (1-\gamma_N)g \end{aligned} \tag{D.2}$$

It can be verified that $\lim_{\theta\downarrow\eta_1^A} VoA'(\theta) = v'(\eta_1^A, d_0, d_0)$. Thus, $VoA'(\theta)$ is continuous at $\theta = \eta_1^A$. Similar to §D.2, we can express $VoA''(\theta)$ as

$$\begin{aligned} VoA''(\theta) &= \gamma_A[r(Q^A|1) - w]\frac{\partial Q^A}{\partial\theta} + (1-\gamma_A)[r(Q^A|1) - r(Q^A|0)]\frac{\partial Q^A}{\partial\theta} \\ &= -\frac{(1-\gamma_A)^2}{[1-\gamma_A(1-\theta)]^3}\frac{[r(Q^A|1) - r(Q^A|0)]^2}{\psi_1 r'(Q^A|1) + (1-\psi_1)r'(Q^A|0)} \geq 0, \end{aligned} \tag{D.3}$$

where the second equality follows by using the Implicit Function Theorem and the last inequality holds by the concavity of $R(Q|\xi)$.

Case 3: $\eta_1^N < \theta \leq 1$. In this case, both Q^N and Q^A are interior solutions. Thus, the expression of $VoA''(\theta)$ derived in §D.2 is applicable. Substituting $\delta = 1$ and the specification of $R(Q|\xi)$ into expression (D.9) yields

$$\begin{aligned} VoA''(\theta) &= p\left(\frac{(1-\gamma_A)^2}{[1-\gamma_A(1-\theta)]^2}\frac{[\bar{F}(Q^A - d_0)]^2}{\theta f(Q^A - d_0)} - \frac{(1-\gamma_N)^2}{[1-\gamma_N(1-\theta)]^2}\frac{[\bar{F}(Q^N - d_0)]^2}{\theta f(Q^N - d_0)}\right) \\ &\leq \frac{p[\bar{F}(Q^N - d_0)]^2}{\theta f(Q^N - d_0)}\left(\frac{(1-\gamma_A)^2}{[1-\gamma_A(1-\theta)]^2} - \frac{(1-\gamma_N)^2}{[1-\gamma_N(1-\theta)]^2}\right) \leq 0 \end{aligned}$$

where the first inequality follows as $\frac{[\bar{F}(Q^A-d_0)]^2}{f(Q^A-d_0)} \leq \frac{\bar{F}(Q^A-d_0)\bar{F}(Q^N-d_0)}{f(Q^N-d_0)} \leq \frac{[\bar{F}(Q^N-d_0)]^2}{f(Q^N-d_0)}$ due to the IFR of $F(\cdot)$ and the decreasing property of $\bar{F}(\cdot)$, and the second inequality is due to the fact that $\frac{(1-\gamma)^2}{[1-\gamma(1-\theta)]^2}$ is decreasing in γ. Therefore, $VoA(\theta)$ is concave. Additionally, it can be verified that $\lim_{\theta\downarrow\eta_1^N} VoA'(\theta) = \lim_{\theta\uparrow\eta_1^N} VoA'(\theta)$. Thus, $VoA'(\theta)$ is continuous at $\theta = \eta_1^N$.

Combining the above three cases, we can conclude that $VoA(\theta)$ is convex-concave in $\theta \in (\underline{\theta}, 1]$. As a result, whenever $VoA'(\eta_1^N) > 0$, $VoA(\theta)$ will be first decreasing, then increasing and finally decreasing in θ.

Next, we analyze the supplier's IC condition: $Q_H^A \rho(\theta_L, \gamma_A)/\rho(\theta_L, \gamma_N) \leq Q_L^N \leq Q_H^A \rho(\theta_H, \gamma_A)/\rho(\theta_H, \gamma_N)$. For any fixed θ_L, Q_L^N is also determined. Notice that Q_H^A and $Q_H^A \rho(\theta_H, \gamma_A)/\rho(\theta_H, \gamma_N)$ are both monotonically increasing in θ_H by Lemma 1 and Lemma A.1. It follows that there exists two thresholds $\theta^\dagger(\theta_L)$ and $\theta^\ddagger(\theta_L)$ such that the supplier's IC condition is satisfied if and only if $\theta_H \in [\theta^\dagger(\theta_L), \theta^\ddagger(\theta_L)]$, where $\theta^\dagger(\theta_L)$ (resp. $\theta^\ddagger(\theta_L)$) is the value of θ_H such that $Q_L^N = Q_H^A \rho(\theta_H, \gamma_A)/\rho(\theta_H, \gamma_N)$ (resp. $Q_H^A = \frac{Q_L^N \rho(\theta_L, \gamma_N)}{\rho(\theta_L, \gamma_A)}$). The value of $\theta^\dagger(\theta_L)$ (resp. $\theta^\ddagger(\theta_L)$) is set to be θ_L (resp. 1) if $Q_L^N \leq Q_H^A \rho(\theta_H, \gamma_A)/\rho(\theta_H, \gamma_N)$ (resp. $Q_H^A \leq \frac{Q_L^N \rho(\theta_L, \gamma_N)}{\rho(\theta_L, \gamma_A)}$) for all $\theta_L < \theta_H \leq 1$. $\square$

Proof of Proposition C.3. In this case, $r(Q|\xi) = A_\xi - 2Q$ and $r'(Q|\xi) = -2$. Substituting these into the expression (D.9), we have

$$\begin{aligned} VoA''(\theta) &= \frac{\delta^2(A_1 - A_0)^2}{2}\left(\frac{(1-\gamma_A)^2}{[1-\gamma_A(1-\theta)]^3} - \frac{(1-\gamma_N)^2}{[1-\gamma_N(1-\theta)]^3}\right) \\ &= \frac{\delta^2(A_1 - A_0)^2}{2[1-\gamma_A(1-\theta)]^3}\left((1-\gamma_A)^2 - (1-\gamma_N)^2\left[\frac{1-\gamma_A(1-\theta)}{1-\gamma_N(1-\theta)}\right]^3\right) \end{aligned}$$

Note that $\frac{1-\gamma_A(1-\theta)}{1-\gamma_N(1-\theta)} = \frac{\rho(\theta,\gamma_A)}{\rho(\theta,\gamma_N)}$ is increasing by Lemma A.1. Moreover, it is easy to see that $VoA''(0) \geq 0$ and $VoA''(1) \leq 0$. Hence, as θ increases from 0 to 1, $VoA''(\theta)$ changes its sign from positive to negative exactly once at $\theta^\#$ where $\theta^\#$ is the unique solution such that $\frac{\rho(\theta,\gamma_A)}{\rho(\theta,\gamma_N)} = \left(\frac{1-\gamma_A}{1-\gamma_N}\right)^{2/3}$, that is, $\theta^\# = 1 - \frac{1-[(1-\gamma_A)/(1-\gamma_N)]^{2/3}}{\gamma_A - \gamma_N(1-\gamma_A)/(1-\gamma_N)]^{2/3}}$. Together with the continuity of $r(Q|\xi)$ and $r'(Q|\xi)$, we can conclude that $VoA(\theta)$ is continuously differentiable and convex-concave. Whenever $VoA'(\theta^\#) > 0$, $VoA(\theta)$ will be increasing in some range of θ.

Under symmetric information, the (interior) optimal order quantities are given by $Q^N = \frac{\psi(\theta,\gamma_N,\delta)A_1 + (1-\psi(\theta,\gamma_N,\delta))A_0 - w}{2}$ and $Q^A = \frac{\psi(\theta,\gamma_A,\delta)A_1 + (1-\psi(\theta,\gamma_A,\delta))A_0 - w}{2}$. The value of $\underline{\theta}$ is determined by the largest θ such that $\Pi_b^N(Q^N) = \frac{[\psi(\theta,\gamma_N\delta)A_1 + (1-\psi(\theta,\gamma_N,\delta))A_0 - w]^2}{4} \leq (1-\psi(\theta,\gamma_N,\delta))g$.

Thus, the supplier's IC condition can be written as $\frac{\rho(\theta_L,\gamma_A)}{\rho(\theta_L,\gamma_N)} \leq \frac{\psi(\theta_L,\gamma_N,\delta)A_1 + (1-\psi(\theta_L,\gamma_N,\delta))A_0 - w}{\psi(\theta_H,\gamma_A,\delta)A_1 + (1-\psi(\theta_H,\gamma_A,\delta))A_0 - w} \leq \frac{\rho(\theta_H,\gamma_A)}{\rho(\theta_H,\gamma_N)}$. For any given θ_L, the first inequality holds if and only if θ_H is smaller than a threshold denoted by $\theta^\ddagger$ as $\psi(\theta_H, \gamma_A, \delta)A_1 + (1-\psi(\theta_H, \gamma_A, \delta))A_0 - w$ is increasing in θ_H. The second inequality holds if and only if θ_H exceeds a threshold denoted by $\theta^\dagger$ since both $\psi(\theta_H, \gamma_A, \delta)A_1 + (1-\psi(\theta_H, \gamma_A, \delta))A_0 - w$ and $\rho(\theta_H, \gamma_A)$ are increasing in θ_H. $\square$

D.2 Derivatives of $VoA(\theta)$ for twice differentiable $R(Q|\xi)$

In this subsection, we derive the general expressions for $VoA'(\theta)$ and $VoA''(\theta)$, assuming that $R(Q|\xi)$ is twice differentiable with respect to Q for all ξ, and that Q^N (resp. Q^A) is an interior maximizer of $\Pi_b^N(Q)$ (resp. $\Pi_b^A(Q)$) satisfying the following first-order conditions.

$$\psi(\theta, \gamma_N, \delta)r(Q^N|1) + (1-\psi(\theta, \gamma_N, \delta))r(Q^N|0) = w, \tag{D.4}$$

$$\psi(\theta,\gamma_A,\delta)r(Q^A|1)+(1-\psi(\theta,\gamma_A,\delta))r(Q^A|0)=w. \tag{D.5}$$

For brevity, we define shorthand notation $\rho_i=\rho(\theta,\gamma_i)$ and $\phi_i=\phi(\theta,\gamma_i,\delta)$ where $i\in\{0,1\}$. Recall that $VoA(\theta)=\rho_1\Pi_b^A(Q^A)-\rho_0\Pi_b^N(Q^N)$ where $\Pi_b^A(Q^A)=\psi_1 R(Q^A|1)+(1-\psi)[R(Q^A|0)-g]-wQ^A$ and $\Pi_b^N(Q^N)=\psi_0 R(Q^N|1)+(1-\psi_0)[R(Q^N|0)-g]-wQ^N$.

By the Envelope Theorem, it follows that $\frac{\partial\Pi_b^A(Q^A)}{\partial\theta}=\frac{\partial\psi_1}{\partial\theta}[R(Q^A|1)-R(Q^A|0)+g]$ and $\frac{\partial\Pi_b^N(Q^N)}{\partial\theta}=\frac{\partial\psi_0}{\partial\theta}[R(Q^N|1)-R(Q^N|0)+g]$. We can then differentiate $VoA(\theta)$ with respect to θ as follows:

$$\begin{aligned}VoA'(\theta)&=\left(\frac{\partial\rho_1}{\partial\theta}\Pi_b^A(Q^A)+\rho_1\frac{\partial\Pi_b^A(Q^A)}{\partial\theta}\right)-\left(\frac{\partial\rho_0}{\partial\theta}\Pi_b^N(Q^N)+\rho_0\frac{\partial\Pi_b^N(Q^N)}{\partial\theta}\right)\\&=\gamma_A[R(Q^A|1)-wQ^A]+\delta(1-\gamma_A)[R(Q^A|1)-R(Q^A|0)+g]\\&\quad-\gamma_N[R(Q^N|1)-wQ^N]-\delta(1-\gamma_N)[R(Q^N|1)-R(Q^N|0)+g]\end{aligned} \tag{D.6}$$

where the equality follows by plugging in $\frac{\partial\rho_i}{\partial\theta}=\gamma_i$, $\frac{\partial\psi_i}{\partial\theta}=\frac{\delta(1-\gamma_i)}{[1-\gamma_i(1-\theta)]^2}$ for $i\in\{0,1\}$ and rearranging the terms. Applying the Implicit Function Theorem in (D.4) and (D.5), one can obtain

$$\frac{\partial Q^N}{\partial\theta}=-\frac{\delta(1-\gamma_N)}{[1-\gamma_N(1-\theta)]^2}\frac{r(Q^N|1)-r(Q^N|0)}{\psi_0 r'(Q^N|1)+(1-\psi_0)r'(Q^N|0)} \tag{D.7}$$

$$\frac{\partial Q^A}{\partial\theta}=-\frac{\delta(1-\gamma_A)}{[1-\gamma_A(1-\theta)]^2}\frac{r(Q^A|1)-r(Q^A|0)}{\psi_1 r'(Q^N|1)+(1-\psi_1)r'(Q^N|0)} \tag{D.8}$$

With the above expressions and (D.4) and (D.5), we can derive the following:

$$\begin{aligned}VoA''(\theta)&=\gamma_A[r(Q^A|1)-w]\frac{\partial Q^A}{\partial\theta}+\delta(1-\gamma_A)[r(Q^A|1)-r(Q^A|0)]\frac{\partial Q^A}{\partial\theta}\\&\quad-\gamma_N[r(Q^N|1)-w]\frac{\partial Q^N}{\partial\theta}-\delta(1-\gamma_N)[r(Q^N|1)-r(Q^N|0)]\frac{\partial Q^N}{\partial\theta}\\&=-\frac{\delta^2(1-\gamma_A)^2}{[1-\gamma_A(1-\theta)]^3}\frac{[r(Q^A|1)-r(Q^A|0)]^2}{\psi_1 r'(Q^A|1)+(1-\psi_1)r'(Q^A|0)}\\&\quad+\frac{\delta^2(1-\gamma_N)^2}{[1-\gamma_N(1-\theta)]^3}\frac{[r(Q^N|1)-r(Q^N|0)]^2}{\psi_0 r'(Q^N|1)+(1-\psi_0)r'(Q^N|0)}\end{aligned} \tag{D.9}$$

E Proof of the Results in Online Appendix D

Proof of Proposition D.1. The babbling equilibrium always exists. So, we focus on the existence of the truth revealing equilibrium in the proof. Since $\eta_1^N\geq\theta_H$, η_1^N must be positive and equal to $1-\frac{p-w}{(1-\gamma_N)\delta p+\gamma_N(p-w)}$, which implies $\delta p>p-w$. Consequently, $\eta_1^A=1-\frac{p-w}{(1-\gamma_A)\delta p+\gamma_A(p-w)}$ is also positive. We can therefore omit the $[\cdot]^+$ operators in the definitions of η_1^N and η_1^A.

The three subcases depends on the relations among θ_L, θ_H and η_1^A.

Case (i) corresponds to the case of $\theta_L\leq\theta_H\leq\eta_1^A$. By Lemma 1, $Q_L^A=Q_H^A=d_0$. In this case, we have $v(\theta_L,d_0,d_0)\geq v(\theta_H,d_0,d_0)$ by Lemma A.2. When $c_A<v(\theta_H,d_0,d_0)$, the buyer audits both types, leading to a non-influential truth revealing equilibrium. The case of

$c_A \geq v(\theta_H, d_0, d_0)$ is similar in which the buyer does not audit at all. When c_A is between $v(\theta_H, d_0, d_0)$ and $v(\theta_L, d_0, d_0)$, the buyer intends to audit the low-type only but the low-type supplier will mimic the high type, and so the babbling equilibrium is the only PBNE.

Case (ii) corresponds to the case of $\theta_L \leq \eta_1^A < \theta_H$. By Lemma 1, $Q_L^A = d_0, Q_H^A = d_1$. So, the buyer intends to audit the low-(high-)type supplier under symmetric information if and only if $c_A < v(\theta_L, d_0, d_0)$ ($c_A < v(\theta_H, d_0, d_1)$). As the relation between $v(\theta_L, d_0, d_0)$ and $v(\theta_H, d_0, d_1)$ are indeterminate, there are four subcases as stated in the proposition. In subcase (a), the buyer never audits. In subcase (b), the buyer audits only the low type under symmetric information, leading to the babbling equilibrium. In subcase (c), the buyer audits only the high type under symmetric information and so the influential truth revealing equilibrium arises if the inequalities $\rho(\theta_L, \gamma_A) \leq d_0/d_1 \leq \rho(\theta_H, \gamma_A)$ holds (by Proposition 2), which is equivalent to the interval of γ_A given in the proposition. In subcase (d), the only PBNE is the babbling equilibrium because the buyer intends to audit both types along with different (conditional) order quantities $Q_L^A = d_0$ and $Q_H^A = d_1$.

Case (iii) corresponds to the case of $\eta_1^A < \theta_L < \theta_H$. By Lemma 1, $Q_L^A = Q_H^A = d_1$. So, the buyer intends to audit the low-(high-)type supplier under symmetric information if and only if $c_A < v(\theta_L, d_0, d_1)$ ($c_A < v(\theta_H, d_0, d_1)$). One can derive four subcases in an analogous manner to Case (ii). Subcase (d) differs from that in Case (ii) because now when the buyer intends to audit both types, he would (conditionally) order the same quantity from both types, i.e., $Q_L^A = Q_H^A = d_1$.

$\square$