Can a Supplier's Yield Risk Be Truthfully Communicated via Cheap Talk?

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Abstract

Problem definition: When a firm (buyer) outsources the production of a new product/component to a supplier subject to random yield, a major challenge is that the supplier's yield is usually private information. In practice, yield information is often shared via nonbinding communication, e.g., supplier self-assessment report. We examine whether such communication can be truthful and credible. **Methodology/results:** We analyze a cheap talk game in which, given a simple contract that specifies the prices for each unit ordered and for each effective unit delivered, the supplier first communicates its yield level and then the buyer determines an order quantity. We prove that truthful communication can emerge in equilibrium. To do so, we first show that if knowing the supplier's type, the buyer will either inflate or reduce the order quantity to cope with a lower yield, depending on the product's market potential. Under asymmetric information, the supplier will truthfully communicate its type if (i) the buyer with a high market potential intends to inflate the order quantity for a lower yield but the buyer with a low market potential prefers to do the reverse, (ii) the supplier is uncertain about the product's market potential which is the buyer's private information and anticipates that a hard-to-make product is more likely to have a higher market potential. **Managerial implications:** Truthful cheap-talk communication can emerge in equilibrium when the product's market size and yield are negatively correlated. Truthful communication always benefits the buyer and consumers and may benefit the supplier if the product has sufficient market potential and the supplier's production cost is not too high. Moreover, the buyer can be better off paying more for the input quantity (although part of the output is defective) or paying a higher wholesale rate if the adjustment in payment terms enhances communication credibility.

Key words: random yield, cheap talk, supply chain management, information asymmetry

1 Introduction

Production processes are imperfect in many industries such as agriculture, biochemicals, pharmaceuticals and semiconductor manufacturing. Defects that fail to function properly or meet the desired quality standard can account for a significant proportion of production yield. For instance, in semiconductor manufacturing, the yield losses of new-generation semiconductor chips are reported to be as significant as 30% to 45% (Lovejoy, 2023; Sohail, 2023). When a firm outsources production to a supplier subject to random yield losses, the outsourcing firm often needs to factor in the supplier's yield risk when making an order decision, because the effective quantity delivered is influenced by both the order size and the random yield distribution (Yano and Lee, 1995; Tang et al., 2014). In reality, however, a supplier's production yield is usually not visible to downstream firms. Jacob (2016) reports that in a survey concerning the management of supplier defects, lack of visibility into supplier quality is identified as a top challenge. Based on our conversation with a leading semiconductor company in Europe, the lack of knowledge about yield is a major challenge when they hire a supplier to produce their newly-designed optical parts, since little historical data are available to gauge the supplier's yield.

Although in some settings sophisticated contract mechanisms can be imposed to solve the issue of asymmetric information,[1] extensive evidence has shown that simple contracts are widely preferred and adopted in practice (see, e.g., Kalkanci et al., 2011; Kayış et al., 2013; Hwang et al., 2018). Consequently, firms often rely on nonbinding communication (Farrell and Rabin, 1996; Berman et al., 2019) when sharing yield information. For instance, a common approach used in practice is supplier self-assessment: Sourcing firms ask suppliers to complete a self-assessment survey about their capability for meeting certain quality objectives (see, e.g., ON Semiconductor, 2012). However, the information provided through self-assessment is usually difficult to verify and cannot be construed as a contractual commitment. Given that communication is nonbinding and unverifiable, can a supplier's yield be truthfully shared with a sourcing firm? If so, when and how can supply chain firms benefit from such communication?

To the best of our knowledge, no existing research has answered the above questions. In this paper, we attempt to fill this gap by analyzing a cheap talk model. We consider a supply chain in which a firm (hereafter, the buyer) outsources the production of a new product, or a key component for the product, to a supplier subject to random yield. The supplier has private knowledge about its average yield level. The supplier first communicates its yield level to the buyer via a costless and unverifiable message, and then the buyer determines an order

[1]For example, according to the mechanism design theory, a sourcing firm can elicit a supplier's private information through offering a menu of screening contracts (e.g., Yang et al., 2009; Chaturvedi and Martínez-de Albéniz, 2011).

quantity. The buyer's payment is based on a simple, pre-determined contract that specifies only flat rates for each unit ordered and each unit delivered. Conventional wisdom might suggest that, if communication is nonbinding, one supplier type will always misreport to attract a larger order quantity from the buyer and so truthful and credible communication is not sustained in equilibrium. However, we prove that truthful communication can emerge in equilibrium. To do so, we first show that if knowing the supplier's type, the buyer may either inflate or reduce the order quantity to cope with a lower yield, depending on the product's market potential. Then, we analyze a setting with two-sided asymmetric information in which the supplier has private yield information whereas the buyer has better information about the market potential. In such a setting, the supplier will truthfully communicate its type if (i) the buyer with a high market potential intends to inflate the order quantity for a lower yield but the buyer with a low market potential prefers to do the reverse, and (ii) the supplier anticipates that a product hard to make (i.e., with a low yield) is more likely to have a higher market potential.

We make the following main contributions to the literature. First, to our knowledge, this is the first paper that studies cheap-talk communication of a supplier's yield risk. Our results indicate that truthful communication can emerge in equilibrium when the product's market size and yield are negatively correlated. Our findings add to the literature by identifying a novel rationale for truthful cheap talk in a supply chain. Furthermore, we show that truthful communication always benefits the buyer and consumers and may benefit the supplier when the product has sufficient market potential and the supplier's production cost is not too high. Due to the benefit of truthful communication, the buyer can be better off paying more for the input quantity (although part of the output is defective) or paying a higher wholesale rate if the adjustment in payment terms enhances the credibility of communication.

2 Literature Review

There is a large body of literature on the management of yield or supply uncertainty (Yano and Lee, 1995; Tomlin and Wang, 2011). Numerous papers have studied the sourcing strategies from a buyer's perspective (e.g., Tomlin, 2006; Dada et al., 2007; Tomlin, 2009; Wang et al., 2010; Federgruen and Yang, 2011; Kouvelis et al., 2018; Dong et al., 2022; Geng et al., 2023). Some papers develop game-theoretical models to explore the strategic interactions among decentralized firms when yield-risk information is symmetric. For example, Tang and Kouvelis (2011) study the sourcing strategies of two horizontally competing firms with unreliable suppliers. Tang et al. (2014) consider a decentralized supply chain in which the buyer may incentivize the supplier to improve yield reliability by direct subsidy or inflating the order quantity. Hwang et al.

(2018) study the supply chain performance under the wholesale price contract determined by a bargaining game between supply chain parties.

Our work is related to the stream of literature on supply chain management with asymmetric supply-risk information. Gurnani and Shi (2006) examine a bargaining game in which the buyer and the supplier have heterogeneous beliefs about supply reliability. Yang et al. (2009) and Yang et al. (2012) study a buyer's contract design problems to elicit the private supply-risk information of supplier(s). Chaturvedi and Martínez-de Albéniz (2011) also study the optimal design of a buyer's procurement contract but allow the supplier's private information to be two-dimensional in both supply reliability and production cost. Gümüş et al. (2012) analyze a signaling game in which a supplier may convey its private information to a buyer through offering price and quantity guarantees. Furthermore, some papers consider quality risks in outsourcing with a better-informed supplier. Kaya and Özer (2009) study the optimal sourcing contracts when a supplier has private quality cost information and its product quality is noncontractible. Their analysis highlights the impact of these two risk factors on supply chain profits and the resulting quality. Nikoofal and Gümüş (2018) consider the case where a supplier is privately informed of a quality-failure probability, and compare output- and action-based incentive mechanisms. Unlike the above papers, we focus on a cheap talk setting in which, given a simple supply chain contract, a supplier shares its yield information via unverifiable and nonbinding communication.

Our work is built on a cheap talk framework, pioneered by Crawford and Sobel (1982), and is thus related to the literature on the cheap talk games in operations management contexts. Allon and Bassamboo (2011a) study a retail setting where a retailer communicates inventory availability information to influence strategic customers' purchase decisions. Focused on service settings, Allon et al. (2011) consider a service provider making unverifiable announcements to convey queue-length information to customers, and Allon and Bassamboo (2011b) explore the role of postponing delay announcements. Beer and Qi (2024) examine inter-firm communication of quality output in collaborative projects.

In the literature on cheap talk in operations management settings, a few papers have investigated cheap-talk communication of demand information between supply chain members. Researchers have identified various reasons why a downstream retailer can truthfully communicate its demand forecast with an upstream manufacturer via cheap talk, including repeated interaction (Ren et al., 2010), responsive wholesale pricing by a manufacturer (Chu et al., 2017), horizontal competition (Shamir and Shin, 2015), vertical ownership (Avinadav and Shamir, 2023), and communication of both forecast mean and accuracy (Lu and Tomlin, 2024). A series of papers have explored the role of trust and trustworthiness in truthful supply chain communication (Özer et al., 2014; Özer and Zheng, 2016, 2018). In particular, the seminal paper by Özer et al. (2011) develops a trust-embedded behavioral model and shows that trust and trustworthy behaviors can induce truthful communication of demand forecasts. Özer et al. (2018) explore various formats of assistance through which a manufacturer can effectively communicate its private product-appeal information to a retailer. Li et al. (2022) compare two behavioral theories, the trust-embedded model and level-k bounded rationality, with a structural estimation approach, and find that the trust-embedded model better explains truthful cheap talk. In this paper, we consider cheap-talk communication of a supplier's yield information and identify a novel rationale (different from those found in the above papers) why such information can be truthfully communicated.

More relevant to our paper are the studies that explore the cheap-talk communication of supply-risk information. To the best of our knowledge, only a few papers fall into this category. Berman et al. (2019) study a supply chain in which a manufacturer privately knows the probability of successfully releasing a new product whereas a retailer allocates its capacity between the new product and a traditional one. The authors uncover the role of a buyer's operational flexibility (i.e., the option to postpone the capacity allocation decision) in inducing truthful communication about the success probability of a new product release. Lu and Tomlin (2022) show that a supplier's social responsibility performance can be credibly communicated with a buyer when the buyer determines an order quantity along with a social audit level. In their setting, over-reporting induces a large order size but may result in a stringent audit. As a result, the supplier may find it optimal to truthfully communicate. In this paper, we examine the communication of a supplier's yield uncertainty. Different from the above papers, the supply risk in our setting comes from a random yield factor following a general probability distribution, and the rationale for truthful communication arises from the buyer's ordering strategies to cope with random yield.

3 The Model

Let us first give an overview of the model and then present the details. We consider a supply chain consisting of a supplier (she) and a buyer (he). The buyer (e.g., an original equipment manufacturer) outsources the production of a new product, or a key component of the product, to the supplier. The supplier's production process is subject to random yield, i.e., the effective output is stochastically proportional to the input quantity. The supplier has private information about her average yield level, while the buyer has better demand information. The supplier first communicates her yield level to the buyer via cheap talk. Then, the buyer determines an order quantity. After the production yield is realized, the buyer makes a payment based on a given simple contract as will be introduced below.

Supplier's Production Yield. We use a random variable ξ_t with support $[0, 1]$ to denote the supplier's production yield, where t is referred to as the supplier's type. The supplier's type can be either high or low, i.e., $t \in T = \{H, L\}$. Let θ_t represent the mean of ξ_t and assume $\theta_H > \theta_L$, i.e., the high-type supplier has a higher average yield than the low type. Note that two supplier types are commonly assumed for tractability in the literature on asymmetric supply-side information (Yang et al., 2009; Gümüş et al., 2012; Berman et al., 2019; Lu and Tomlin, 2022). The effective amount of production output from a type-t supplier is equal to ξ_t times the amount of production input. The supplier incurs a production cost c for each unit of production input. Our main results can be readily extended if c is type-dependent. In the base model, we assume that the coefficient of variation (cv) of ξ_t is identical across types, denoted by δ. Consequently, the variance of ξ_t is given by $\sigma_t^2 = \delta^2\theta_t^2$. Note that for all $t \in T$, $\theta_t \leq 1/(1 + \delta^2)$ is required such that there exists a well-defined probability distribution for $\xi_t \in [0, 1]$.[2] In §5.2, we show that all our results can be generalized when the cv is type-dependent. The supplier privately learns her type t, and then communicates a type $t' \in T$ to the buyer via a costless and unverifiable message. The communicated type t' may or may not be her actual type t.

Buyer's Demand Function. We adopt a standard linear inverse demand function widely used in the random yield literature (e.g., Deo and Corbett, 2009; Tang and Kouvelis, 2011; Xiao, 2020). The buyer faces an inverse demand function $P(x|m) = a_m - bx$, where x is the available-to-sell quantity, $m \in M = \{H, L\}$ presents the market state, and the product's market potentials are given by $a_H \geq a_L$. So, $m = H$ (resp. $m = L$) represents a high (resp. low) market state. For a given available-to-sell quantity x and a given market state m, the buyer collects a revenue of $xP(x|m) = x(a_m - bx)$.[3] In practice, brand-owning manufacturers often conduct extensive market research when they design a new product (Jiang et al., 2016). Thus, the buyer can possess superior knowledge about the market state m. To capture this fact, we assume that the prior probability distribution of market state m, denoted by $\mu(m)$, is common knowledge but the buyer privately learns the realization of m before making an ordering decision.

Furthermore, the product's market potential and the supplier's yield level can be generally correlated. In high-tech manufacturing industries, a low yield of a product often results from

[2]Because ξ_t is within $[0, 1]$, for a given mean θ_t, its variance σ_t^2 must be bounded above by $(1 - \theta_t)\theta_t$ where the bound is tight when ξ_t is a Bernoulli random variable (Bhatia and Davis, 2000). Therefore, we assume $\delta^2 \leq (1 - \theta_t)/\theta_t$, or equivalently, $\theta_t \leq 1/(1 + \delta^2)$, for all $t \in T$ such that a well-defined yield distribution exists for each type.

[3]As is common in the literature (e.g., Tang and Kouvelis, 2011), we implicitly assume that the buyer will not hold back some quantity for sale, which is true if market potentials are sufficiently high compared to the purchase cost.

emerging production technology required. Meanwhile, the product tends to have a high market potential because, due to the early-stage technology, there is usually a limited supply of similar products in the market. Hence, we allow the market state m and the supplier type t to be interdependent, and denote their joint probability distribution by p_{tm}. The joint probability distribution is common knowledge. To rule out degenerate cases, assume $p_{tm} > 0$ for all $t \in T$ and $m \in M$. The buyer and the supplier privately observe m and t, respectively, which reflects the fact that the buyer has better market information, whereas the supplier has better knowledge regarding the technologies available to make the product. In the high-tech setting described above, the buyer's market state m and the supplier's yield type t tend to be negatively correlated.

Supply Chain Contract and the Buyer's Order Decision. We consider a simple contract that has been widely adopted in the random yield literature (e.g., Tomlin and Wang, 2005; Tomlin, 2009; Wang et al., 2010). The contract specifies two parameters (w, η): The buyer pays a flat rate ηw for each unit ordered and a flat rate $(1-\eta)w$ for each effective unit delivered where $0 \leq \eta \leq 1$ and w will be referred to as the wholesale rate. As evidenced in Tang and Kouvelis (2011), the buyer often pays for at least a portion of the supplier's input capacity in agricultural, high-tech and biotech industries. The value of η thus reflects to what extent the buyer pays for input costs. If $\eta = 0$, the buyer pays only for the amount actually delivered, regardless of the input; if $\eta = 1$, the buyer pays for the amount ordered, which can represent a situation where the buyer is responsible for the cost of the input quantity, regardless of the effective output. If Q units are ordered and the supplier is of type t, the available-to-sell quantity for the buyer will be $x = \xi_t Q$ and the total payment will be $\eta w Q + (1-\eta) w \xi_t Q$. To focus on the role of cheap talk while abstracting away from other costly mechanisms, we follow a standard assumption in the cheap talk literature (e.g., Berman et al., 2019; Lu and Tomlin, 2022) that the contract parameters w and η are exogenously given. As a result, any information communicated by the supplier is nonbinding and costless, i.e., a cheap-talk message.[4]

Given that the supplier communicates a type t' and the market state is m, the buyer forms a posterior belief about the supplier's true type t, denote by $\beta(t|t', m)$. Based on his posterior belief, the buyer determines an order quantity Q to maximize his expected profit as expressed below.

$$\Pi_B(Q|t', m) = \sum_{t \in T} \beta(t|t', m) E_{\xi_t} \left[\xi_t Q P(\xi_t Q|m) - \eta w Q - (1-\eta) w \xi_t Q \right]. \tag{1}$$

[4]Note that our cheap talk game is different from mechanism design settings. In a typical mechanism design problem, the buyer has the power to offer a menu of take-it-or-leave-it contracts and also commit to the order quantities specified in the menu. In our cheap talk setting, we do not assume any bargaining or commitment power for the buyer. In this regard, our model aligns with the context of high-tech outsourcing where buying firms do not have a dominant bargaining power because only a limited number of suppliers are available due to highly specialized production technologies.

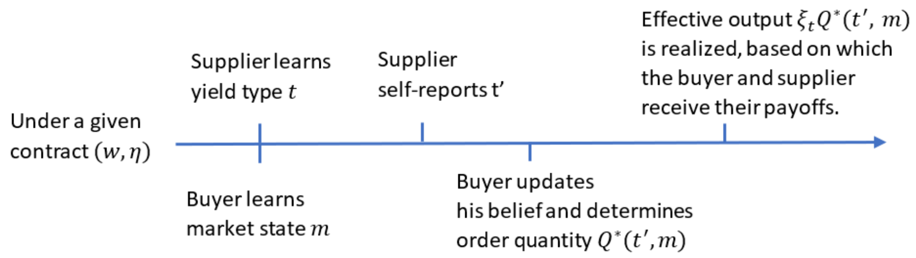

Figure 1: Sequence of events

Let $Q^*(t', m)$ denote the optimal order quantity that maximizes (1). To rule out uninteresting cases, we assume that given w and η, it is always profitable for the buyer to procure a positive quantity and for the supplier to produce, irrespective of the market state and supplier type. Mathematically, this is equivalent to assuming $a_m\theta_t > \eta w + (1-\eta)w\theta_t > c$ for all m and t.

It is worth noting that in the above we have assumed the supplier's input quantity to be equal to the buyer's order quantity Q. This assumption is not uncommon in the random yield literature (e.g., Wang et al., 2010; Tang and Kouvelis, 2011) and fits many practical situations where, for instance, the buyer provides the amount of raw materials to the supplier for production. Nevertheless, in §5.3, we show that our results can be generalized to an alternative setting in which the supplier can choose an internal production lot size different than the buyer's order quantity.

Cheap Talk Communication. Anticipating the buyer's ordering decision $Q^*(t', m)$ as described above, the supplier chooses a type $t' \in T$ to communicate so as to maximize her expected profit:

$$\Pi_S(t'|t) = \sum_{m \in M} \mu(m|t) E_{\xi_t}\left[(\eta w + (1-\eta)w\xi_t - c_t)Q^*(t', m)\right] \tag{2}$$

where $\mu(m|t) = \frac{p_{tm}}{\sum_{m' \in M} p_{tm'}}$ represents the probability of the market state being m conditional on a supplier type t.

The sequence of events, as illustrated in Figure 1, is summarized as follows: (i) The supplier privately learns her type t while the buyer privately learns the market state m; (ii) the supplier communicates a type t' to the buyer; (iii) given t' and the market state m, the buyer updates his belief and determines an order quantity $Q(t', m)$; (iv) random yield ξ_t is realized and the payment is made accordingly. We use the Perfect Bayesian Equilibrium (PBE) as the solution concept. Formally, we define the supplier's messaging strategies as a mapping $r : T \to T$. For any $t, t' \in T$, $r(t) = t'$ represents that a type-t supplier communicates her type as t'. The supplier's messaging strategy $r^*(t)$, the buyer's ordering strategy $Q^*(t', m)$ and his posterior belief $\beta(t|t', m)$ constitute a PBE if the following conditions hold:

(i) The supplier's messaging strategy maximizes her expected profit (2), i.e, $r^*(t) \in \arg\max_{t' \in T} \Pi_S(t'|t)$ for each $t \in T$ where $\Pi_S(t'|t)$ is defined in (2);

(ii) the buyer's ordering strategy maximizes his expected profit, i.e., $Q^*(t', m) \in \arg\max_{Q \geq 0} \Pi_B(Q|t', m)$ for all $t' \in T$ and $m \in M$ where $\Pi_B(Q|t', m)$ is defined in (1);

(iii) the buyer updates his belief according to Bayes' rule on the equilibrium path, i.e., $\beta(t|t', m) = \frac{\mathbf{1}(r(t)=t')\lambda(t|m)}{\sum_{\hat{t} \in T} \mathbf{1}(r(\hat{t})=t')\lambda(\hat{t}|m)}$ if $\sum_{\hat{t} \in T} \mathbf{1}(r(\hat{t}) = t')\lambda(\hat{t}|m) > 0$, where $\mathbf{1}(x)$ is the indicator function which equals one (resp. zero) if statement x is true (resp. false), and $\lambda(t|m) = p_{tm}/\sum_{\hat{t} \in T} p_{\hat{t}m}$ is the buyer's prior belief about the supplier's type t conditional on market state m. There is no restriction on $\beta(t|t', m)$ off equilibrium path, i.e., for t' such that $\sum_{\hat{t} \in T} \mathbf{1}(r(\hat{t}) = t')\lambda(\hat{t}|m) = 0$.

We will refer to a PBE as a *truth-revealing equilibrium* if the supplier of each type truthfully communicates her type in equilibrium, i.e., $r(t) = t$ for all $t \in T$, and as a *babbling equilibrium* if there exists some type misreporting, i.e., $r(t) \neq t$ for some $t \in T$. By part (iii) of the definition above, in any truth-revealing equilibrium the buyer's posterior belief will be $\beta(t|t, m) = 1$ for all $t \in T$, i.e., the supplier's type will be credibly communicated to the buyer. In a babbling equilibrium, however, the buyer's posterior belief will remain as the prior $\lambda(t|m)$. In general, a babbling equilibrium always exists in cheap talk games (Sobel, 2014). Therefore, in line with the cheap talk literature (e.g., Allon and Bassamboo, 2011a; Berman et al., 2019; Lu and Tomlin, 2022), the primary goal of our analysis is to show whether and under what condition a truth-revealing equilibrium can emerge.

4 Results

4.1 The Buyer's Order Quantity under Symmetric Information

We start by characterizing the buyer's optimal order quantity if he knows the supplier's type t. In this case, the buyer determines the order quantity based on both a_m and θ_t. For ease of notation, in this subsection we will suppress subscripts m and t and denote the buyer's optimal order quantity by $Q^r(\theta, a)$ given any market potential a and average yield θ ~~where superscript s represents symmetric information~~. By assumptions, we consider any value of θ within $(\underline{\theta}(a), \bar{\theta}]$ where $\underline{\theta}(a) = \frac{\eta w}{a-(1-\eta)w}$ and $\bar{\theta} = \frac{1}{1+\delta^2}$. The lower bound $\underline{\theta}(a)$ ensures a positive order quantity from the buyer while the upper bound $\bar{\theta}$ guarantees the existence of a valid probability distribution with mean θ and cv δ.

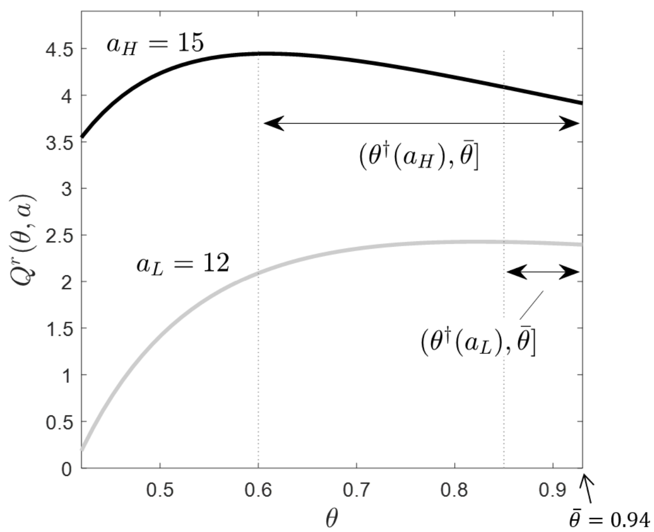

Figure 2: Effect of the average yield θ and market potential a on the buyer's optimal order quantity under symmetric information [Parameter values: $b = 1$, $w = 7$, $\eta = 0.5$, and $\delta = 0.25$; as a result, the valid values of θ are between $\underline{\theta}(a_L) = 0.41$ and $\bar{\theta} = 0.94$.]

The following proposition provides a closed-form expression of $Q^r(\theta, a)$ and characterizes some useful properties. Throughout the paper, we use "increasing" and "decreasing" in a weak sense. All the proofs of our results are presented in Online Appendix B.

Proposition 1 (Buyer's Optimal Ordering Strategy under Symmetric Information). *Suppose that information is symmetric. For any given market potential a and average yield θ such that $\underline{\theta}(a) < \theta \le \bar{\theta}$, the buyer's optimal order quantity is given by*

$$Q^r(\theta, a) = \frac{[a - (1-\eta)w]\theta - \eta w}{2b(1+\delta^2)\theta^2}. \tag{3}$$

Moreover, the following properties of $Q^r(\theta, a)$ hold: (i) For any given market potential a, $Q^r(\theta, a)$ is increasing in θ for $\theta \in (\underline{\theta}(a), \theta^\dagger(a)]$ while decreasing in θ for $\theta \in (\theta^\dagger(a), \bar{\theta}]$ where $\theta^\dagger(a) = \frac{2\eta w}{a-(1-\eta)w}$. (ii) $Q^r(\theta, a)$ is log-submodular in (θ, a).

As established in property (i), for any given market potential a, the buyer's optimal order quantity is not monotone in the average yield θ. When θ decreases from its maximum level $\bar{\theta}$, to compensate for the potential yield loss, the buyer may find it optimal to inflate his order size relative to $Q^r(\bar{\theta}, a)$, the optimal order quantity given $\theta = \bar{\theta}$. If θ is in a range $(\theta^\dagger(a), \bar{\theta}]$, the buyer will inflate more as θ becomes lower. In other words, Q^r is decreasing in θ. However, if θ is below the threshold $\theta^\dagger(a)$, the cost of yield losses will outweigh the benefit of inflating the order size, and the buyer will instead reduce the order quantity as θ decreases. In other words, Q^r is increasing in θ. We note that while a sourcing firm's order-inflation strategy has been documented in the random yield literature (e.g., Tang et al., 2014), our subsequent analysis will uncover its implication on the strategic communication of a supplier's yield risk.

Moreover, the product's market potential a influences the buyer's ordering strategies. Notice that the threshold $\theta^{\dagger}(a)$ is a decreasing function of a. Therefore, the higher the market potential, the wider the order-inflation range $(\theta^{\dagger}(a), \bar{\theta}]$; see Figure 2 for an illustration that the order-inflation range shrinks as a becomes smaller. This is intuitive: With a higher market potential, the cost of yield losses will more likely be justified by the higher profit margin and so the order-inflation strategy will be preferred. Formally, property (ii) establishes that the buyer's order quantity is log-submodular in the market potential and average yield. The log-submodularity of $Q^r(\theta, a)$ implies that the ratio $Q^r(\theta_L, a)/Q^r(\theta_H, a)$ increases with a. That is, the relative "inflation" in the buyer's order quantity as the yield level reduces from θ_H to θ_L is increasing in the market potential. As we will show in §4.2, the buyer's potentially opposite ordering strategies along with its dependence on the market potential serve as an important driver for truthful communication of yield risk when information is asymmetric.

4.2 Truthful Communication under Asymmetric Information

We proceed to explore whether or not a supplier's yield level can be truthfully communicated in equilibrium when the supplier's type t is unknown to the buyer. Anticipating that the buyer will choose a profit-maximizing order quantity based on his posterior belief, a supplier of type t chooses a $t' \in T$ to communicate so as to maximize her expected profit $\Pi_S(t'|t)$ defined in (2). For a truth-revealing equilibrium to exist, the following incentive compatibility (IC) condition must hold:

$$\Pi_S(t|t) \geq \Pi_S(t'|t) \text{ for all } t, t' \in T \text{ and } t \neq t'. \tag{4}$$

That is, for each supplier type, it is optimal to truthfully communicate. It is straightforward to see that the supplier's profit-maximizing problem is equivalent to choosing a type t' to communicate in order to attract the largest expected order quantity $\sum_{m \in M} Q^*(t', m)\mu(m|t)$. In any truth-revealing equilibrium, we must have $Q^*(t, m) = Q^r(\theta_t, a_m)$ for all $t \in T$ and $m \in M$ where $Q^r(\cdot)$ is the buyer's optimal ordering strategy under symmetric information as characterized in Proposition 1. Consequently, one can simplify the supplier's IC condition to the following:

$$Q^r(\theta_t, a_H)\mu(H|t) + Q^r(\theta_t, a_L)\mu(L|t) \geq Q^r(\theta_{t'}, a_H)\mu(H|t) + Q^r(\theta_{t'}, a_L)\mu(L|t) \text{ for all } t, t' \in T \text{ and } t \neq t'. \tag{5}$$

We first notice that a truth-revealing equilibrium can trivially exist if the buyer's optimal order quantity happens to be identical for both yield levels under both market states (i.e., $Q^r(\theta_H, a_m) = Q^r(\theta_L, a_m)$ for both $m \in M$); in such a case, truthful communication is weakly supported in equilibrium simply because the supplier will receive the same order quantity regardless of which type to communicate. We say a truth-revealing equilibrium to be *noninfluential* if the buyer's order size is independent of the supplier's message in both market states; we say a truth-revealing equilibrium to be *influential* otherwise.

Lemma 1. *When $a_H = a_L$ (resp. $a_H > a_L$) , any truth-revealing equilibrium, if exists, must be noninfluential (resp. influential).*

Lemma 1 establishes that if the two market potentials coincide, a truth-revealing equilibrium can emerge only when it is noninfluential. To focus on interesting cases, we will henceforth assume $a_H > a_L$ and so, by Lemma 1, any truth-revealing equilibrium, if exists, will be influential. In the remainder of the paper, whenever we say a truth-revealing equilibrium, we refer to an influential one unless otherwise specified.

To better explain why an (influential) truth-revealing equilibrium can exist, let us first present two necessary conditions for truthful communication.

Lemma 2 (Operational Driver)**.** *A truth-revealing equilibrium exists only if $Q^r(\theta_H, a_H) \leq Q^r(\theta_L, a_H)$ and $Q^r(\theta_H, a_L) \geq Q^r(\theta_L, a_L)$, i.e., with a high market potential, the buyer intends to order more from a low-type supplier than from a high-type supplier, whereas with a low market potential, the reverse is true.*

The above lemma reveals that for truthful communication to occur in equilibrium, it must be the case that the buyer with a high market potential prefers an order inflation strategy to cope with a lower yield, but the buyer with a low market potential intends to reduce the order quantity for a lower yield. Suppose instead that irrespective of the market potential, the buyer always inflates the order quantity to cope with a lower yield level. Then, both supplier types will communicate the yield level as θ_L instead of θ_H, because doing so will always attract a larger order quantity. As a consequence, truthful communication will not be sustained in equilibrium. Similarly, if the buyer always reduces his order quantity for a lower yield, both types will claim to be a high type such that truthful communication is impossible in equilibrium. Therefore, for truthful communication to happen, the buyer needs to prefer opposite ordering strategies under different market potentials. Furthermore, by the log-submodularity of $Q^r(\theta, a)$, i.e., property (ii) in Proposition 1, the ratio $Q^r(\theta_L, a)/Q^r(\theta_H, a)$ increases with the market potential a. This implies that whenever the buyer prefers opposite ordering strategies with different market potentials, it must be the case that he opts to inflate the order size for a lower yield given a high market potential but reverse his strategy given a low market potential, as stated in Lemma 2. We label the necessary condition in Lemma 2 as an *operational* driver for truthful communication.

Recall that the supplier aims to attract an (expected) order as large as possible when communicating her type. The buyer's distinct ordering strategies under different market potentials lead to a nontrivial trade-off for the supplier, since the market potential is uncertain to the supplier. Provided the operational driver discussed above in effect, the low-type supplier will communicate truthfully if she believes there is a relatively large chance for the market potential to be high such that the buyer will inflate order quantity for a lower yield level; on the other hand, the high-type supplier will communicate honestly if she believes the market potential is more likely to be low such that the buyer will order more from a high-type supplier than a low type. Such a desired informational structure is formally characterized in Lemma 3 below.

Lemma 3 (Informational Driver)**.** *A truth-revealing equilibrium exists only if the distribution of market state m has a decreasing monotone likelihood ratio (MLR) with respect to the supplier's type t: $\frac{\mu(H|H)}{\mu(L|H)} \leq \frac{\mu(H|L)}{\mu(L|L)}$.*

The MLR property in Lemma 3 implies that given a product hard to make (i.e., with a low yield level) with current technology, the supplier would believe the product's market potential more likely to be high, whereas given an easy-to-make product, the supplier would believe otherwise. Notably, this negative correlation echoes the practice mentioned earlier. In high-tech industries, low yield is often due to the early stage of production technology. When observing a low yield level, the supplier can infer that the technology required to produce the product is in its early stage, which usually implies that the market for such a product is not saturated and so the product's market potential tends to be high.

Generally speaking, the two-sided private information (i.e., information about the supplier's yield and the buyer's market potential) can lead to a preference reversal whereby the supplier prefers communicating different types under different market potentials, making truthful communication possible in equilibrium. This is reminiscent of Berman et al. (2019) in which the authors examine a manufacturer privately informed of the probability of releasing a new product and a retailer with a fixed capacity to allocate between the new product and a traditional one. The manufacturer communicates a product-release probability in hopes that the retailer chooses to postpone the capacity allocation after uncertainty is resolved. Berman et al. (2019) show that because the retailer's postponement choice depends on both the product-release probability and the total capacity, the manufacturer can have a preference reversal in communication, leading to truthful cheap talk, if the retailer's capacity and the manufacturer's product-release probability constitute two-sided private information with a decreasing MLR property. Different from Berman et al. (2019), the preference reversal in our model is a consequence of the buyer's optimal order strategies under random yield, i.e, either inflating or reducing the order size depending

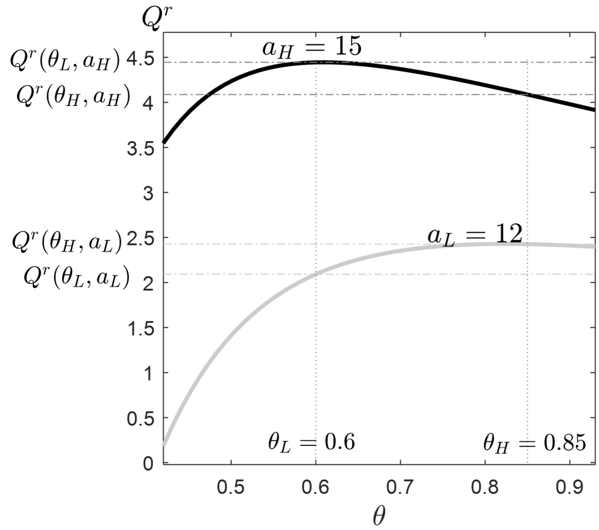

Figure 3: Illustration of the operational driver [Parameter values: $a_H = 15$, $a_L = 12$, $b = 1$, $w = 7$, $\eta = 0.5$, $\delta = 0.25$]

on his market potential. Thus, we contribute to the literature by identifying a new operational driver and supply chain context with random yield such that a preference reversal can emerge and induce truthful communication.

To summarize, a truth-revealing equilibrium may emerge only if (i) the buyer with a high market potential intends to inflate the order quantity in response to a lower yield but the buyer with a low market potential prefers a reverse strategy, and (ii) the market potential is uncertain to the supplier and negatively correlated with the yield level. The sufficient and necessary condition for a truth-revealing equilibrium to exist can be derived as follows.

Proposition 2. *A truth-revealing equilibrium exists if and only if*

$$\sum_{m \in M} \mu(m|H) a_m \leq \left[1 + \eta \left(\frac{1}{\theta_H} + \frac{1}{\theta_L} - 1 \right) \right] w \leq \sum_{m \in M} \mu(m|L) a_m. \tag{6}$$

Condition (6) can hold only if the low-type supplier expects the market potential to be higher than the high type, i.e., $\sum_{m \in M} \mu(m|L) a_m \geq \sum_{m \in M} \mu(m|H) a_m$, as implied by the decreasing MLR property. A truth-revealing equilibrium will emerge when the two yield levels θ_L and θ_H are such that the buyer's ordering strategies are properly aligned with the supplier's incentive for truthful communication. In what follows, we consider a concrete numerical example.

Example 1. We consider the same model parameters as used earlier in Figure 2: $a_H = 15$, $a_L = 12$, $b = 1$, $w = 7$, $\eta = 0.5$ and $\delta = 0.25$. The supplier can be one of the two types with average yield levels $\theta_L = 0.6$ and $\theta_H = 0.85$, respectively. Under symmetric information, with a high market potential, the buyer's optimal order quantity from each supplier type is given by $Q^r(\theta_H, a_H) = 4.09$ and $Q^r(\theta_L, a_H) = 4.44$, whereas with a low market potential, the order quantities are $Q^r(\theta_H, a_L) = 2.43$ and $Q^r(\theta_L, a_L) = 2.09$. As such, given $a_H = 15$, the buyer will order more from a low-type supplier than from a high type (i.e., inflate the order size to compensate for a lower yield); in contrast, given $a_L = 12$, he will order more from the high type than from the low type (i.e., reduce the order size for fear of a lower yield). See Figure 3 for an illustration. These opposite ordering strategies echo the operational driver as stated in Lemma 2.

To illustrate the role of the informational driver, let us assume that the joint probability distribution p_{tm} for supplier type t and market state m is given by $p_{HH} = 0.25 + \rho$, $p_{HL} = 0.25 - \rho$, $p_{LH} = 0.25 - \rho$ and $p_{LL} = 0.25 + \rho$ where $\rho \in [-0.25, 0.25]$ captures the correlation between the product's market potential and the supplier's yield level. Then, a high-type supplier believes that the market state follows the following probability distribution: $\mu(H|H) = \frac{1+4\rho}{2}$ and $\mu(L|H) = \frac{1-4\rho}{2}$, whereas a low-type supplier believes $\mu(H|L) = \frac{1-4\rho}{2}$ and $\mu(L|L) = \frac{1+4\rho}{2}$. Substituting these probabilities into the IC condition (5) or the sufficient and necessary condition (6) stated in Proposition 2, we can show that a truth-revealing equilibrium exists if and only if $\rho \leq -0.0082$. Therefore, in this example, a very slight negative correlation between the yield level and the product's market potential is sufficient for the supplier's yield information to be fully revealed via cheap talk.

Two corollaries follow by the truth-revealing condition (6), indicating when a truth-revealing equilibrium is more likely to exist.

Corollary 1. *Let $a_H = \bar{a} + \Delta$ and $a_L = \bar{a} - \Delta$ where $\Delta > 0$. If the truth-revealing equilibrium exists for some Δ, then with anything else fixed, it must exist as well for any $\Delta' > \Delta$. That is, a truth-revealing equilibrium is more likely to exist as the difference between a_H and a_L increases.*

Corollary 1 indicates the existence of a truth-revealing equilibrium is more probable as the product's market potential has greater dispersion to the supplier. Intuitively, a greater variability in market potential amplifies the role of the buyer's distinct ordering strategies given different market potentials, thereby facilitating truthful communication.

Corollary 2. *There exists no truth-revealing equilibrium if $\eta = 0$, i.e., the buyer pays only for the amount delivered. Provided $\sum_{m \in M} \mu(m|H) a_m \leq w \sum_{t \in T} (1/\theta_t)$ and the MLR property stated in Lemma 3, there are two thresholds $0 < \eta_{\min} \leq \eta_{\max} \leq 1$ such that a truth-revealing equilibrium exists if and only if $\eta \in [\eta_{\min}, \eta_{\max}]$.*

Recall that the buyer pays ηw for each unit ordered and $(1 - \eta) w$ for each unit delivered, and η reflects to what extent the buyer is responsible for the cost of production input. Corollary 2 implies that truthful communication is possible only if the buyer pays for at lease some input

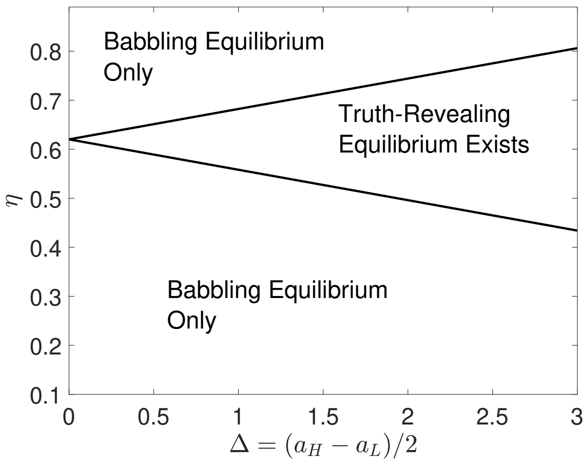

Figure 4: Illustration of Corollaries 1 and 2: Effects of $\Delta = (a_H - a_L)/2$ and η on the existence of truth-revealing equilibrium [$\theta_H = 0.85$, $\theta_L = 0.6$, $b = 1$, $w = 7$, $a_H = 15 + \Delta$, $a_L = 15 - \Delta$, $p_{HH} = p_{LL} = 0.05$, $p_{HL} = p_{LH} = 0.45$]

costs, i.e., $\eta > 0$. It is easy to see from Proposition 1 that if $\eta = 0$, the buyer will always inflate the order size to cope with a lower yield, since he bears no cost for the input quantity. Only when the buyer covers some cost of the input, he will trade off the gain from inflating the order size against the potential yield loss, which gives rise to the potentially opposite ordering strategies (i.e., the operational driver) that is necessary for truthful communication. Our finding indicates that supply chain firms can achieve credible information sharing by properly specifying how much the buyer is responsible for input costs, instead of resorting to more sophisticated mechanisms. See Figure 4 for a numerical illustration of the results discussed above. Notably, it is common in practice that buying firms pay for certain costs based on the input, although a portion of the output may be defective. For instance, in the pharmaceutical industry, clients may pay on a "dollars per batch" or "time and materials" basis even when some batches fail (Tomlin, 2009, p.194); in high-tech and biotech industries, eventually buyers pay for their supplier's startup volume, instead of simply the output (Tang and Kouvelis, 2011, p.441). Our result corroborates this practice by uncovering an informational value of sharing the cost of yield loss across supply chain members. In §4.4, we will further explore the impact of η on the buyer's expected profit.

4.3 Implications on Supply Chain Profits and Consumer Surplus

We examine the impact of truthful communication on supply chain profits and consumer surplus. To this end, we compare the buyer's and the supplier's ex ante profits as well as the (expected) consumer surplus under the truth-revealing equilibrium with their counterparts absent truthful communication. Note that the case without communication is equivalent to a babbling equilibrium in the cheap talk game where the communication is not informative at all.

Denote the buyer's ex ante profits with and without truthful communication, respectively, by Π_B^r and Π_B^b, where superscripts r and b represent the truth-revealing and babbling equilibria, respectively. Let $Q^b(a_m)$ represent the buyer's optimal order quantity for any market state $m \in M$ without communication (or equivalently, under the babbling equilibrium). The buyer's ex ante profits can be written as $\Pi_B^r = \sum_{m\in M}\sum_{t\in T} p_{tm}\pi_B(Q^r(\theta_t, a_m), t, m)$ and $\Pi_B^b = \sum_{m\in M}\sum_{t\in T} p_{tm}\pi_B(Q^b(a_m), t, m)$ where $\pi_B(Q, t, m)$ represents the buyer's profit as a function of order quantity Q and conditional on t and m. Similarly, denote by Π_S^r and Π_S^b the supplier's ex ante profits with and without communication, respectively, and we have $\Pi_S^r = \sum_{t\in T}\sum_{m\in M} p_{tm}(\eta w + (1-\eta)w\theta_t - c)Q^r(\theta_t, a_m)$, and $\Pi_S^b = \sum_{t\in T}\sum_{m\in M} p_{tm}(\eta w + (1-\eta)w\theta_t - c)Q^b(a_m)$. The consumer surplus conditional on any Q and realized yield ξ is given by $cs(Q,\xi) = \frac{bQ^2\xi^2}{2}$. The consumer surpluses with and without communication are thus given by $CS^r = \frac{b(1+\delta^2)}{2}\sum_{t\in T}\sum_{m\in M} p_{tm}\theta_t^2 Q^r(\theta_t, a_m)^2$ and $CS^b = \frac{b(1+\delta^2)}{2}\sum_{t\in T}\sum_{m\in M} p_{tm}\theta_t^2 Q^b(a_m)^2$, respectively.

A key difference between the cases with and without communication is that, absent communication, the order quantity $Q^b(a_m)$ depends only on the buyer's market state, independent of the supplier's type. In contrast, the order quantity with truthful communication, $Q^r(\theta_t, a_m)$, depends on both market state and supplier type. In other words, truthful communication enables the buyer to tailor his order size to the supplier's yield risk. In Lemma A.1 of Appendix A, we establish that, whenever truthful communication occurs in equilibrium, $Q^r(\theta_L, a_H) > Q^b(a_H) > Q^r(\theta_H, a_H)$ and $Q^r(\theta_L, a_L) < Q^b(a_L) < Q^r(\theta_H, a_L)$. This is because the buyer, without any updated yield information, is not able to inflate or reduce the order quantity according to the actual yield level.

Define shorthand notation $\gamma_m = \frac{(a_m-(1-\eta)w)\theta_L\theta_H - \eta w(\theta_H+\theta_L)}{p_{Lm}\theta_L^2 + p_{Hm}\theta_H^2}$ for all $m \in M$. The following proposition shows that truthful communication, whenever occurring in equilibrium, improves the buyer's ex ante profit as well as the consumer surplus, whereas its impact on the supplier's payoff is somewhat nuanced.

Proposition 3. *If truthful communication occurs in equilibrium, compared to the case without truthful communication, the following holds true: (i) The buyer is better off, i.e.,* $\Pi_B^b < \Pi_B^r$*; (ii) the supplier is better off, i.e.,* $\Pi_S^b < \Pi_S^r$*, if and only if* $\left(\sum_{m\in M} p_{Hm}p_{Lm}\gamma_m\right)(\hat{w} - c) > 0$ *where* $\hat{w} = \eta w + (1-\eta)w\theta_L\theta_H/(\theta_L + \theta_H)$*; (iii) the consumer surplus is improved, i.e.,* $CS^b < CS^r$*.*

Clearly, truthful communication benefits the buyer since it enables a more informed purchase decision. The consumer surplus (conditional on realized yield) is an increasing function of the actual output volume, i.e., ξQ. Intuitively, with truthful communication, the buyer can choose Q that more effectively matches the actual yield factor, thereby enhancing the expected output and consumer surplus. Whether the supplier benefits from truthful communication depends on the condition stated in part (ii) of Proposition 3. The term $\sum_{m\in M} p_{Hm}p_{Lm}\gamma_m$ increases with market potentials a_H and a_L;[5] the threshold $\hat{w}$ reflects the supplier's (expected) marginal revenue.[6] If the supplier's expected profit margin is high enough (i.e., $\hat{w} > c$), truthful communication improves the supplier's payoff for high market potential a_m's; however, if the supplier's profit margin is low, truthful communication benefits the supplier for low a_m's. Recall that compared to the case of no communication, truthful information exchange enables the buyer with a low (resp. high) market potential to order more (resp. less) from the low-type supplier but less (resp. more) from the high type. Therefore, the effect of truthful communication on the supplier is determined jointly by the market potential and the supplier's profit margin.

Our discussion has so far focused on comparing cases with and without truthful communication. Which case is a more reasonable equilibrium outcome in our cheap talk game? Recall that a babbling equilibrium, where no credible communication occurs, is always supported as a PBE in any cheap talk game (Sobel, 2014). Therefore, to obtain a clear prediction of the equilibrium outcome, we need to refine the set of PBE when a truth-revealing equilibrium exists. To this end, we adopt the "no incentive to separate" (NITS) criterion (Chen et al., 2008), a commonly used criterion in the literature for equilibrium selection in cheap talk games (see, e.g., Chu et al., 2017; Lu and Tomlin, 2022). An equilibrium satisfies the NITS criterion if the lowest-type sender weakly prefers its equilibrium payoff to the payoff that it would receive by revealing its true type. An equilibrium that violates the NITS criterion is unlikely in practice, as a low type has an incentive to self-signal through an out-of-equilibrium message. See Chen et al. (2008), Chu et al. (2017) and Lu and Tomlin (2022) for more justifications of the NITS criterion.

Lemma 4. *A truth-revealing equilibrium, whenever it exists, always satisfies the NITS criterion; however, a babbling equilibrium will violate the NITS if and only if*

$$\sum_{m\in M} p_{Lm}p_{Hm}\gamma_m > 0. \tag{7}$$

As mentioned earlier, $\sum_{m\in M} p_{Lm}p_{Hm}\gamma_m$ is increasing in a_H and a_L, and so Lemma 4 implies that if market potentials are large enough, the babbling equilibrium will fail the NITS refinement such that the truth-revealing equilibrium is a more reasonable outcome played out in the cheap talk game. Based on the NITS criterion and focusing on the case of $\sum_{m\in M} p_{Lm}p_{Hm}\gamma_m > 0$,

[5] As formally proved in Lemma A.1 of Appendix A, whenever a truth-revealing equilibrium exists, $\gamma_H > 0$ and $\gamma_L < 0$.

[6] The type-t supplier's expected marginal revenue $\eta w + (1-\eta)w\theta_t$ can be written as $\hat{w} + (1-\eta)w\theta_t^2/(\theta_L+\theta_H)$.

we are now able to formally characterize how cheap-talk communication influences supply chain profits and consumer surplus, as compared to the case where supply chain members do not engage in communication.

Proposition 4. *If a truth-revealing equilibrium exists while the babbling equilibrium fails the NITS refinement, i.e.,* (6) *and* (7) *hold, then cheap-talk communication always benefits the buyer and consumer surplus. Moreover, there exists two thresholds* $\hat{w} < \tilde{w}$ *such that cheap-talk communication benefits the supplier and, hence, the supply chain for* $c \leq \hat{w}$*, hurts the supplier but improves the total supply chain profit for* $\hat{w} < c \leq \tilde{w}$*, and hurts both the supplier and the supply chain for* $c > \tilde{w}$*, where* $\hat{w}$ *is as defined in Proposition 3 and* $\tilde{w}$ *is lower bounded by* $\frac{[a_H+(1-\eta)w]\theta_L\theta_H+\eta w(\theta_L+\theta_H)}{2(\theta_L+\theta_H)}$.

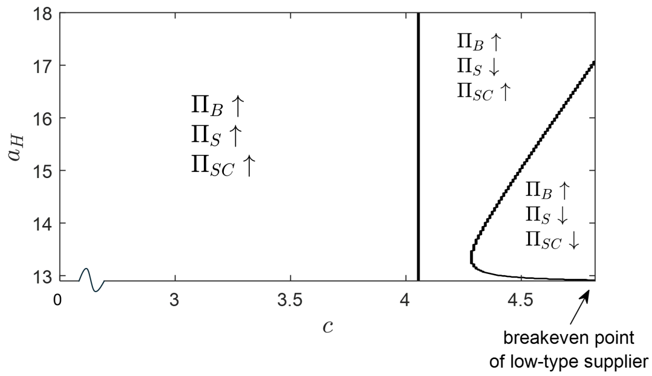

Figure 5: Illustration of Proposition 4: Impact of cheap-talk communication on supply chain profits; $\uparrow$ ($\downarrow$) indicates that the corresponding profit is improved (reduced) by cheap talk; parameters are chosen such that (6) and (7) are satisfied: $a_L = 12$, a_H varied from 12.9 to 18, $b = 1$, $\theta_L = 0.5$, $\theta_H = 0.9$, $\delta = 0.2$, $p_{LH} = p_{LH} = 0.45; p_{HH} = p_{LL} = 0.05$, $w = 7$ and $\eta = 0.38$.

The impacts of cheap-talk communication on the buyer and consumer surplus are immediately implied by parts (i) and (iii) of Proposition 3. When the truth-revealing equilibrium is the only outcome surviving the NITS refinement (which occurs if market potentials are large enough), we are able to simplify the condition for a supplier to be better off previously established in part (ii) of Proposition 3: As long as the supplier's profit margin is not too low, cheap-talk communication will improve the supplier's payoff and hence the total supply chain profit. Figure 5 presents a numerical example to illustrate Proposition 4. In this example, production cost c is varied between 0 and $c_{max} = (1-\eta)w\theta_L + \eta w = 4.83$ where c_{max} is the cost such that the low-type supplier's expected profit margin equals zero. As shown in Figure 5, cheap-talk communication makes both the buyer and supplier better off unless $c > 4.06$, or equivalently, the low-type supplier's profit margin is smaller than 0.77. The total supply chain profit can be hurt by cheap-talk communication only if c is even higher. Our results indicate that provided sufficient market potentials (such that the babbling fails the NITS refinement), a supplier can benefit from cheap-talk communication under rather mild conditions.

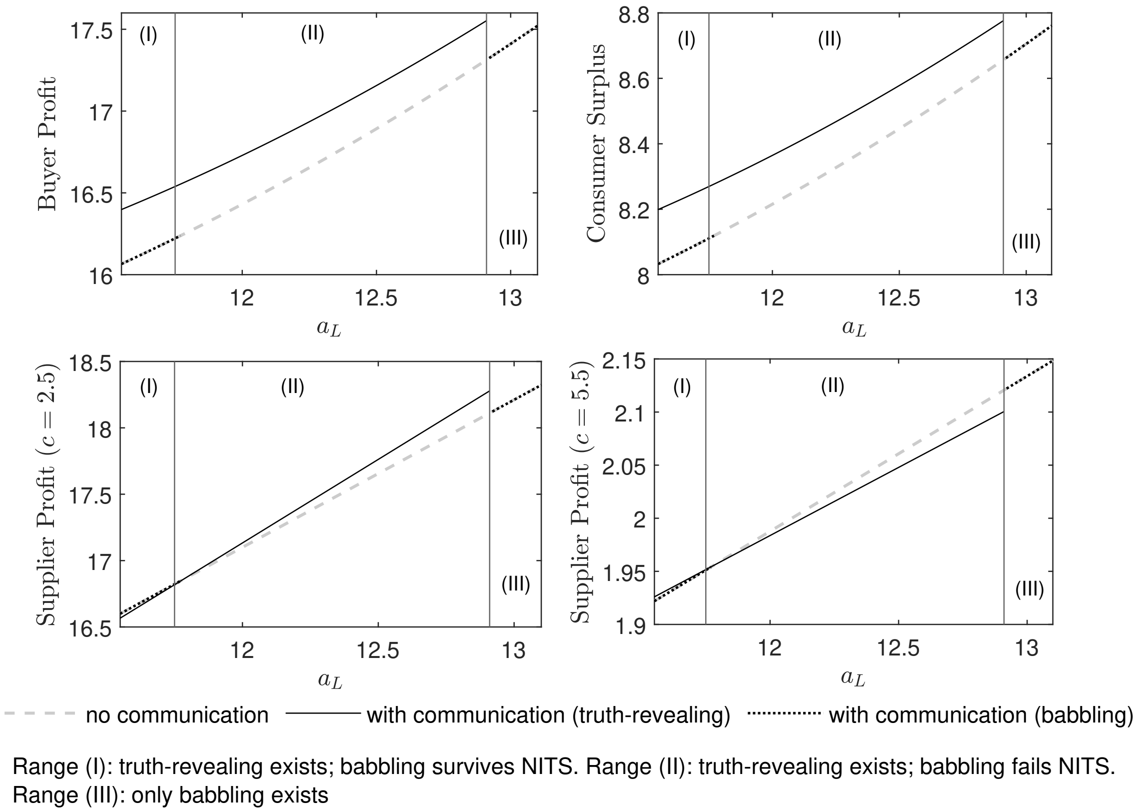

Figure 6: Impact of cheap-talk communication on the buyer, the supplier, and the consumer surplus; in case of multiple equilibria, the NITS refinement applies; parameters are set as $w = 7$, $\eta = 0.6$, $c = 2$, $\theta_L = 0.5$, $\theta_H = 0.9$, $\delta = 0.05$, $p_{HH} = p_{LL} = 0.225$, $p_{HL} = p_{LH} = 0.275$, $b = 1$, $a_H = 20$, and a_L varied as shown in the figure.

Figure 6 provides a numerical illustration of the impact of cheap-talk communication on each supply chain member and the consumer surplus while the NITS refinement is applied in case of multiple equilibria. Communication has an impact only when a truth-revealing equilibrium exists as in Regions (I) and (II). When the truth-revealing equilibrium exists and the babbling equilibrium fails the NITS refinement as in Region (II), cheap-talk communication always benefits supply chain members and consumers except for the supplier with a high production cost $c = 5.5$ (in the bottom-right panel of Figure 6). For lower values of a_L as in Region (I), the babbling equilibrium cannot be ruled out by the NITS criterion and so both the truth-revealing and babbling equilibria could be an outcome.[7]

In summary, we show that cheap-talk communication benefits the buyer and consumers; moreover, it can also improve the supplier's payoff under mild conditions. We emphasize that

[7]In such cases, we can still conclude that cheap talk benefits the buyer and consumers in a weak sense since it never makes the buyer and consumers worse off.

although the magnitude of the profit improvement varies with parameter values, such improvement is achieved by pure cheap talk, thus requiring no cost or investment at all. Our results indicate that supply chain members may have money being left on the table if they are unaware of the potential of information sharing via nonbinding communication.

4.4 Impact of Contract Parameters

We now analyze how contract parameters (w, η) influence the buyer's payoff. Being unaware of the potential of truthful communication, one might anticipate that, with w being fixed, the buyer is always worse off with a higher η as he pays more for the production input and not all of it may yield an effective output; similarly, with η being fixed, one may jump to a conclusion that a higher w always hurts the buyer. Nevertheless, as will be shown below, none of these assertions is necessarily true when cheap-talk communication takes place, because a change in η or w may influence the truthfulness of communication.

Proposition 5. *Suppose that the decreasing MLR property in Lemma 3 holds with strict inequality, i.e.,* $\frac{\mu(H|H)}{\mu(L|H)} < \frac{\mu(H|L)}{\mu(L|L)}$. *(i) With all else fixed, if* $\sum_{m \in M} \mu(m|H) a_m < w \sum_{t \in T} (1/\theta_t)$, *there exists* $\eta^\dagger \in (0, 1)$ *such that the buyer's ex ante profit strictly increases with* η *at* $\eta = \eta^\dagger$. *(ii) With all else fixed, if* $\sum_{m \in M} \mu(m|H) a_m < \frac{1-\eta+\eta \sum_{t \in} (1/\theta_t)}{1-\eta+\eta/\theta_L} a_L$, *there exists* $w^\dagger$ *such that the buyer's ex ante profit strictly increases with* w *at* $w = w^\dagger$.

Proposition 5 establishes that the buyer can be strictly better off as η or w increases. This is because, with an increase in η or w, the equilibrium may switch from babbling to truth-revealing. (The NITS refinement is applied to rule out the babbling equilibrium when both equilibria exist.) As shown earlier, truthful communication benefits the buyer and so the buyer's expected profit is discontinuous in η and w, jumping upward at the point when the equilibrium switches. The strict MLR property implies that at these discontinuous points the babbling equilibrium fails the NITS refinement.[8]

As illustrated in Figure 7(a), while the buyer's payoff is decreasing in η within each equilibrium case, he can be strictly better off as η increases, leading to a switch from the babbling to the truth-revealing equilibrium; see the discontinuous point at approximately $\eta = 0.39$ on the left panel of 7(a). Meanwhile, truthful communication also benefits the supplier and consumer surplus (see the middle and right panels of Figure 7(a)). Figure 7(b) presents similar results when w varies with η fixed at 0.45. Our findings indicate that, intriguingly, by paying more for the input quantity or paying at a higher wholesale rate can result in a higher payoff for

[8] The closed-form expressions of discontinuous points $\eta^\dagger$ and $w^\dagger$ can be found in the proof of Proposition 5, and the conditions in parts (i) and (ii) of Proposition 5 ensure that $\eta^\dagger \in (0, 1)$ and that $w^\dagger$ is within the parameter range where the buyer places a positive order quantity with the supplier.

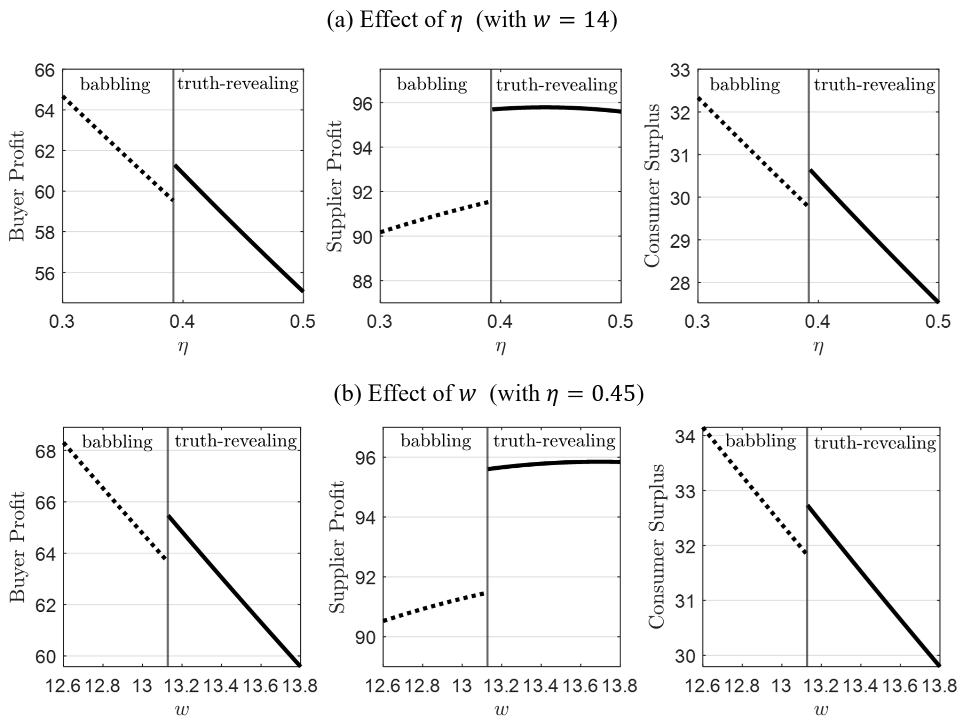

Figure 7: Effects of the contract parameters η and w on supply chain profits and consumer surplus; in case of multiple equilibria, the NITS refinement applies $[a_H = 40, a_L = 22, b = 1, c = 2, \delta = 0.2, \theta_L = 0.5, \theta_H = 0.9, p_{HH} = p_{LL} = 0.1, p_{HL} = p_{LH} = 0.4]$

the buyer. Simultaneously, this improves the supplier's payoff and consumer surplus. Certain adjustments to contract parameters, although appearing to the buyer's disadvantage, can result in a win-win-win outcome. Our result indicates that a buyer should consider paying more to its supplier not only because doing so compensates for the supplier's yield loss but also because it can enhance the credibility of communication.

5 Extensions

Our main results are robust in several model extensions. In what follows, we provide a summary of each extension that we have examined, whereas more details can be found in the Online Appendices.

5.1 Implications on Contract Negotiation

Following a standard assumption in the cheap talk literature, we have assumed contract parameters to be exogenous. In Online Appendix C, we examine a setting in which the contract parameters (w, η) are determined based on Nash bargaining before the cheap talk game. Specifically, we consider a sequence of events in which the buyer and supplier first negotiate on contract parameters w and η with their prior knowledge of the market potential and yield, and then they obtain updated market and yield information (i.e., observe m and t), respectively, and engage in cheap-talk communication as in our base model.

To analyze the implications of communication, we numerically examine two scenarios: In Scenario 1, the buyer and supplier are unaware of the possibility of truthful communication when negotiating the contract terms (w, η). As such, the Nash bargaining game is based on the firms' payoffs without communication. In Scenario 2, anticipating the possibility of truthful communication, the supply chain firms negotiate within a subset of (w, η) that induces truthful communication. Thus, the bargaining game in Scenario 2 is based on the payoffs with truthful communication whereas contract terms (w, η) must satisfy the truth-revealing condition (6). Additionally, we assume that the firms will agree on a truth-revealing contract only if they are (weakly) better off compared to Scenario 1. The detailed model formulation and numerical analysis can be found in Online Appendix C.

Our results indicate that in many cases, supply chain members can be both better off bargaining within the set of truth-revealing contracts. Therefore, the value of truthful communication, as revealed in our base-model analysis, can carry over to a setting where contract parameters are chosen as a bargaining outcome. On the other hand, there are some cases in which truthful communication cannot simultaneously benefit the two supply chain firms, depending on the firms' relative bargaining powers and the production cost. This is because forcing the contract to be truth-revealing, despite the informational value, may impact supply chain efficiency as it limits the choice of contract parameters. Our observations suggest that the efficiency loss can sometimes outweigh the informational value. In such cases, supply chain firms may need more complex contract forms if they want to achieve both truthful information sharing and high supply chain efficiency.

5.2 Type-Dependent Coefficients of Variation

In the base model, we assumed that the random yield factors ξ_t's have the same cv δ, independent of the type t. Our main results continue to hold if we relax this assumption to allow type-dependent cv, denoted by δ_t. Similar to the base model, we assume $\theta_H > \theta_L$. For each type t, we define the second moment of the yield factor ξ_t as $\chi_t = \theta_t^2(1 + \delta_t^2)$. Under this setting, we are able to generalize the existence result of a truth-revealing equilibrium (Proposition 2), as long as $\chi_H > \chi_L$, i.e., the high-type yield has a larger second moment than the low type. Note that $\chi_H > \chi_L$ includes the special cases where the two types have the same cv as in the base model, and where the two types have the same variance. While the truth-revealing condition becomes more involved as it depends on the first two moments of the yield factor, the rationale for truthful communication to emerge in equilibrium remains the same as we explained before. Furthermore, our main results on the impact of truthful communication continue to hold. Truthful communication always benefits the buyer and consumers; it can also improve the supplier's payoff under certain conditions. Furthermore, the buyer can be strictly better off as the contract parameter η or w becomes larger. We refer the reader to Online Appendix D for the details of the analysis.

5.3 Supplier-Optimized Internal Production Lot Size

In the base model, we have assumed the supplier's input quantity to be equal to the buyer's order quantity. When input materials are fully controlled by the supplier, the supplier may have the discretion to choose an *internal* production lot size that differs from the buyer's order quantity. In Online Appendix E, we consider a setting in which the supplier can choose an internal production lot size y, leading to an effective output quantity $\xi_t y$, after receiving the buyer's order Q. The buyer will not accept any excess quantity more than he ordered; therefore, the actual delivered quantity is given by $\min\{\xi_t y, Q\}$.

We first establish that given any order quantity Q from the buyer, a type-t supplier's optimal production lot size is equal to Q times a type-dependent multiplier ϕ_t^*, i.e., $y^*(Q,t) = \phi_t^* Q$. Leveraging this property, we can define a modified random yield factor $\zeta_t = \min\{\xi_t \phi_t^*, 1\}$ for each type t such that the quantity effectively delivered equals $\zeta_t Q$. Then, using the first two moments of ζ_t, we are able to generalize the main result that truthful communication can emerge in equilibrium under a similar condition as in Proposition 2. Moreover, we also present a series of results confirming the robustness of our findings regarding the implications of truthful communication on supply chain profits and consumer surplus. The detailed analysis can be found in Online Appendix E.

6 Conclusion

In this paper, we examine if a supplier's yield risk can be truthfully communicated via cheap talk when a buyer outsources the production of a new product, or a component for the product, to the supplier. We establish that although nonbinding and unverifiable, yield information shared via cheap talk can be truthful. In particular, truthful communication can emerge in equilibrium when the product's market size and yield are negatively correlated. Truthful communication always benefits the buyer and consumers and may benefit the supplier if the product has sufficient market potential and the supplier's production cost is not too high. Due to the value of communication, buying firms can be strictly better off paying more for the input quantity (although part of the output is defective) or paying a higher wholesale rate if the adjusted payment terms facilitate truthful communication.

There are several directions for future research. As the first study to explore cheap-talk communication of yield uncertainty, we have focused on a simple supply chain structure with only one supplier and one buyer. In reality, a buyer may have a supply base consisting of multiple suppliers, or a supplier may sell to multiple buyers competing in the same end market. It is valuable to explore how the upstream or downstream competition in a supply chain influences the communication of yield risk. Moreover, to mitigate yield risk, the buyer may build in-house backup capacity or resort to a quick-response supplier in case of shortage. It is worth exploring how these contingent sourcing options impact the communication of yield risk.

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Online Appendices for "Can a Supplier's Yield Risk Be Truthfully Communicated via Cheap Talk?"

A Supplemental Results for the Base Model

The following lemma characterizes some properties of the buyer's optimal quantities under the truth-revealing and babbling equilibria, which are useful for proving Propositions 3 and Lemma 4.

Lemma A.1. *Given any m, the buyer's optimal order quantity under the babbling equilibrium is given by $Q^b(a_m) = \frac{(a_m-(1-\eta)w)\left(\sum_{t\in T}\lambda(t|m)\theta_t\right)-\eta w}{2b(1+\delta^2)\left(\sum_{t\in T}\lambda(t|m)\theta_t^2\right)}$. Moreover, the optimal order quantities under the truth-revealing and babbling equilibria satisfy the following relation:*

$$Q^r(\theta_L, a_m) - Q^b(a_m) = \frac{p_{Hm}}{\theta_L^2}\Gamma_m \text{ and } Q^b(a_m) - Q^r(\theta_H, a_m) = \frac{p_{Lm}}{\theta_H^2}\Gamma_m \tag{A.1}$$

where

$$\Gamma_m = \frac{(\theta_H-\theta_L)[(a_m-(1-\eta)w)\theta_L\theta_H - \eta w(\theta_H+\theta_L)]}{2b(1+\delta^2)(p_{Lm}\theta_L^2 + p_{Hm}\theta_H^2)} \text{ for } m \in M.$$

Moreover, whenever a truth-revealing equilibrium exists, $\Gamma_L < 0$ and $\Gamma_H > 0$, and therefore $Q^r(\theta_L, a_L) < Q^b(a_L) < Q^r(\theta_H, a_L)$ and $Q^r(\theta_L, a_H) > Q^b(a_H) > Q^r(\theta_H, a_H)$.

Proof of Lemma A.1. In a babbling equilibrium, the buyer determines the order quantity based on the prior belief $\lambda(t|m) = p_{tm}/(\sum_{\hat{t}\in T} p_{\hat{t}m})$. From the first-order optimality condition, it is straightforward to show that under the babbling equilibrium, the optimal order quantity for a buyer with market potential a_m is given by

$$\begin{aligned} Q^b(a_m) &= \frac{(a_m-(1-\eta)w)\left(\sum_{t\in T}\lambda(t|m)\theta_t\right)-\eta w}{2b(1+\delta^2)\left(\sum_{t\in T}\lambda(t|m)\theta_t^2\right)} \\ &= \frac{(a_m-(1-\eta)w)\left(p_{Lm}\theta_L + p_{Hm}\theta_H\right)-\eta w(p_{Lm}+p_{Hm})}{2b(1+\delta^2)\left(p_{Lm}\theta_L^2 + p_{Hm}\theta_H^2\right)} \end{aligned}$$

where the second equality follows since $\lambda(t|m) = \frac{p_{tm}}{p_{Lm}+p_{Hm}}$ for any $t \in T$ and $m \in M$. Then, with the above expression, letting $\bar{\eta} = 1-\eta$, we can simplify $Q^r(\theta_L, a_m) - Q^b(a_m)$ as follows:

$$\begin{aligned} &Q^r(\theta_L, a_m) - Q^b(a_m) = \frac{(a_m-\bar{\eta}w)\theta_L - \eta w}{2b(1+\delta^2)\theta_L^2} - \frac{(a_m-\bar{\eta}w)\left(p_{Lm}\theta_L + p_{Hm}\theta_H\right)-\eta w(p_{Lm}+p_{Hm})}{2b(1+\delta^2)\left(p_{Lm}\theta_L^2 + p_{Hm}\theta_H^2\right)} \\ &= \frac{(a_m-\bar{\eta}w)\left(\cancel{p_{Lm}\theta_L^3} + p_{Hm}\theta_L\theta_H^2 - \cancel{p_{Lm}\theta_L^3} - p_{Hm}\theta_L^2\theta_H\right) - \eta w\left(\cancel{p_{Lm}\theta_L^2} + p_{Hm}\theta_H^2 - \cancel{p_{Lm}\theta_L^2} - p_{Hm}\theta_L^2\right)}{2b(1+\delta^2)\theta_L^2\left(p_{Lm}\theta_L^2 + p_{Hm}\theta_H^2\right)} \\ &= \frac{p_{Hm}}{\theta_L^2}\frac{(\theta_H-\theta_L)[(a_m-\bar{\eta}w)\theta_L\theta_H - \eta w(\theta_L+\theta_H))]}{2b(1+\delta^2)(p_{Lm}\theta_L^2 + p_{Hm}\theta_H^2)} = \frac{p_{Hm}}{\theta_L^2}\Gamma_m \end{aligned}$$

where Γ_m is as defined in the lemma. Similarly,

$$\begin{aligned} &Q^b(a_m) - Q^r(\theta_H, a_m) = \frac{(a_m-\bar{\eta}w)\left(p_{Lm}\theta_L + p_{Hm}\theta_H\right)-\eta w(p_{Lm}+p_{Hm})}{2b(1+\delta^2)\left(p_{Lm}\theta_L^2 + p_{Hm}\theta_H^2\right)} - \frac{(a_m-\bar{\eta}w)\theta_H - \eta w}{2b(1+\delta^2)\theta_H^2} \\ &= \frac{(a_m-\bar{\eta}w)\left(p_{Lm}\theta_L\theta_H^2 + \cancel{p_{Hm}\theta_H^3} - p_{Lm}\theta_L^2\theta_H - \cancel{p_{Hm}\theta_H^3}\right) - \eta w\left(p_{Lm}\theta_H^2 + \cancel{p_{Hm}\theta_H^2} - p_{Lm}\theta_L^2 - \cancel{p_{Hm}\theta_H^2}\right)}{2b(1+\delta^2)\theta_H^2\left(p_{Lm}\theta_L^2 + p_{Hm}\theta_H^2\right)} \\ &= \frac{p_{Lm}}{\theta_H^2}\frac{(\theta_H-\theta_L)[(a_m-\bar{\eta}w)\theta_L\theta_H - \eta(\theta_L+\theta_H))]}{2b(1+\delta^2)(p_{Lm}\theta_L^2 + p_{Hm}\theta_H^2)} = \frac{p_{Lm}}{\theta_H^2}\Gamma_m. \end{aligned}$$

Therefore, we complete the proof of (A.1).

To see whether $\Gamma_m > 0$ or < 0, note that the sign of Γ_m is identical to that of $a_m - \left[1 + \eta\left(\frac{1}{\theta_H} + \frac{1}{\theta_L} - 1\right)\right] w$. By Proposition 2, given the existence of a truth-revealing equilibrium, it must hold that

$$\mu(L|L)\left(a_L - \left[1 + \eta\left(\frac{1}{\theta_H} + \frac{1}{\theta_L} - 1\right)\right] w\right) + \mu(H|L)\left(a_H - \left[1 + \eta\left(\frac{1}{\theta_H} + \frac{1}{\theta_L} - 1\right)\right] w\right) \geq 0$$

and

$$\mu(L|H)\left(a_L - \left[1 + \eta\left(\frac{1}{\theta_H} + \frac{1}{\theta_L} - 1\right)\right] w\right) + \mu(H|H)\left(a_H - \left[1 + \eta\left(\frac{1}{\theta_H} + \frac{1}{\theta_L} - 1\right)\right] w\right) \leq 0.$$

Since $a_H > a_L$, for the above two inequalities to hold, it must be the case that $a_L - \left[1 + \eta\left(\frac{1}{\theta_H} + \frac{1}{\theta_L} - 1\right)\right] w < 0$ and $a_H - \left[1 + \eta\left(\frac{1}{\theta_H} + \frac{1}{\theta_L} - 1\right)\right] w > 0$, thus $\Gamma_L < 0 < \Gamma_H$. Hence, the relations of order quantities follow in view of (A.1). □

B Proofs of the Results in the Main Paper

Proof of Proposition 1. Under symmetric information, given any market potential a and average yield θ, the buyer's expected profit can be written

$$\Pi_B(Q) = a\theta Q - b\theta^2(1+\delta^2)Q^2 - \eta w Q - (1-\eta)w\theta Q.$$

It is straightforward to verify that $\Pi_B(Q)$ is concave in Q, and so the optimal order quantity can be found by solving the following first-order condition:

$$\frac{\partial \Pi_B}{\partial Q} = a\theta - 2b\theta^2(1+\delta^2)Q - [\eta w + (1-\eta)w\theta] = 0,$$

which leads to the closed-form expression of $Q^r(\theta, a)$ in (3). Define $z = \frac{1}{\theta}$ such that $Q^r(\theta, a) = \hat{Q}^r(z, a) = \frac{1}{2b(1+\delta^2)}\left[(a - (1-\eta)w)z - \eta w z^2\right]$. Differentiating $\hat{Q}^r(z, a)$ with respect to z, we obtain

$$\frac{\partial \hat{Q}^r(z, a)}{\partial z} = \frac{(a - (1-\eta)w) - 2\eta w z}{2b(1+\delta^2)}.$$

Hence, $\hat{Q}^r(z, a)$ is increasing in z if $z < \frac{a-(1-\eta)w}{2\eta w}$ and decreasing in z otherwise. By definition of z, equivalently, $Q^r(\theta, a)$ is decreasing in θ if $\theta > \frac{2\eta w}{a-(1-\eta)w}$ and increasing in θ otherwise. This proves property (i).

To prove property (ii), the log-submodularity of $Q^r(\theta, a)$, it suffices to check the cross-partial derivative of $\log Q^r(\theta, a)$:

$$\frac{\partial^2 \log Q^r(\theta, a)}{\partial\theta\partial a} = -\frac{\eta w}{[(a - w + \eta w)\theta - w\eta]^2} \leq 0. \tag{B.1}$$

□

Proof of Lemma 1. If $a_H = a_L = \bar{a}$, then the IC condition (5) boils down to $Q^r(\theta_H, \bar{a}) \geq Q^r(\theta_L, \bar{a})$ and $Q^r(\theta_L, \bar{a}) \geq Q^r(\theta_H, \bar{a})$, that is, $Q^r(\theta_H, \bar{a}) = Q^r(\theta_L, \bar{a})$. Any truth-revealing equilibrium must satisfy (5) and so it must be noninfluential. For the case of $a_H > a_L$, we argue that $Q^r(\theta_H, a_m) = Q^r(\theta_L, a_m)$ cannot hold for both $m \in M$. We consider two subcases with $\eta = 0$ and with $\eta > 0$, respectively. If $\eta = 0$, then $Q^r(\theta, a) = \frac{(a-w)}{2b(1+\delta^2)\theta}$, which is strictly decreasing in θ for any a; however, this contradicts with $Q^r(\theta_H, a_L) = Q^r(\theta_L, a_L)$. If $\eta > 0$, then it follows that $Q^r(\theta_H, a)/Q^r(\theta_L, a)$ is strictly decreasing in a by (B.1). Thus, $Q^r(\theta_H, a_L) = Q^r(\theta_L, a_L)$ implies that $Q^r(\theta_H, a_H) < Q^r(\theta_L, a_H)$, and $Q^r(\theta_H, a_H) = Q^r(\theta_L, a_H)$ implies that $Q^r(\theta_H, a_L) > Q^r(\theta_L, a_L)$.

Therefore, $Q^r(\theta_H, a_m) = Q^r(\theta_L, a_m)$ cannot hold for both $m \in M$. $\square$

Proof of Lemma 2. We prove the lemma by contradiction. Suppose that $Q^r(\theta_H, a_m) \geq Q^r(\theta_L, a_m)$ for both $m \in M$ with at least one of the two inequalities holding strictly. Then, in any truth-revealing equilibrium, if exists, $Q^*(H, m) \geq Q^*(L, m)$ for both $m \in M$ with at least one inequality being strict. However, it implies that reporting $t' = H$ strictly dominates reporting $t' = L$ for both types of suppliers, and so the IC condition (5) must be violated, which contradicts with the existence of truth-revealing equilibrium. Similarly, if $Q^r(\theta_H, a_m) \leq Q^r(\theta_L, a_m)$ for both $m \in M$ with at least one inequality holding strictly, both types will communicate $t' = L$ such that the truth-revealing equilibrium cannot survive.

Next, we show that the remaining case, $Q^r(\theta_H, a_H) \geq Q^r(\theta_L, a_H)$ and $Q^r(\theta_H, a_L) \leq Q^r(\theta_L, a_L)$ with at least one of the inequalities being strict, can never occur. The two inequalities, if satisfied with at least one strict, imply $\frac{Q^r(\theta_H, a_H)}{Q^r(\theta_L, a_H)} > \frac{Q^r(\theta_H, a_L)}{Q^r(\theta_L, a_L)}$. However, by part (ii) of Proposition 1, $\log Q^r(\theta, a)$ are submodular, and therefore $\frac{Q^r(\theta_H, a_H)}{Q^r(\theta_L, a_H)} \leq \frac{Q^r(\theta_H, a_L)}{Q^r(\theta_L, a_L)}$, leading to a contradiction. $\square$

Proof of Lemma 3. The IC condition (5) holds if and only if

$$[Q^r(\theta_H, a_H) - Q^r(\theta_L, a_H)]\mu(H|H) \geq [Q^r(\theta_L, a_L) - Q^r(\theta_H, a_L)]\mu(L|H) \tag{B.2}$$

and

$$[Q^r(\theta_L, a_H) - Q^r(\theta_H, a_H)]\mu(H|L) \geq [Q^r(\theta_H, a_L) - Q^r(\theta_L, a_L)]\mu(L|L). \tag{B.3}$$

These two inequalities imply that

$$[Q^r(\theta_H, a_L) - Q^r(\theta_L, a_L)]\frac{\mu(L|L)}{\mu(H|L)} \leq Q^r(\theta_L, a_H) - Q^r(\theta_H, a_H) \leq [Q^r(\theta_H, a_L) - Q^r(\theta_L, a_L)]\frac{\mu(L|H)}{\mu(H|H)}.$$

Whenever $Q^r(\theta_H, a_L) - Q^r(\theta_L, a_L) > 0$, the above inequality implies $\frac{\mu(L|L)}{\mu(H|L)} \leq \frac{\mu(L|H)}{\mu(H|H)}$, i.e., the MLR property stated in the proposition. By Lemma 2, we can without loss of generality rule out the case of $Q^r(\theta_H, a_L) - Q^r(\theta_L, a_L) < 0$. It remains to examine the case of $Q^r(\theta_H, a_L) = Q^r(\theta_L, a_L)$. If so, then the above inequality requires $Q^r(\theta_L, a_H) = Q^r(\theta_H, a_H)$ as well. However, this implies a noninfluential truth-revealing equilibrium; by Lemma 1, it cannot be the case whenever $a_H > a_L$. $\square$

Proof of Proposition 2. Rearranging the terms, we can rewrite the IC condition (5) as

$$[Q^r(\theta_H, a_H) - Q^r(\theta_L, a_H)]\mu(H|H) \geq [Q^r(\theta_L, a_L) - Q^r(\theta_H, a_L)]\mu(L|H) \tag{B.4}$$

and

$$[Q^r(\theta_L, a_H) - Q^r(\theta_H, a_H)]\mu(H|L) \geq [Q^r(\theta_H, a_L) - Q^r(\theta_L, a_L)]\mu(L|L). \tag{B.5}$$

Let $\bar{\eta} = 1 - \eta$. Substituting the expression of $Q^r(\theta, a)$ into (B.4) and multiplying both sides with $2b(1 + \delta^2)$, we have

$$\begin{aligned}&\left[(a_H - \bar{\eta}w)\left(\frac{1}{\theta_H} - \frac{1}{\theta_L}\right) - \eta w\left(\frac{1}{\theta_H^2} - \frac{1}{\theta_L^2}\right)\right]\mu(H|H)\\ \geq &\left[(a_L - \bar{\eta}w)\left(\frac{1}{\theta_L} - \frac{1}{\theta_H}\right) - \eta w\left(\frac{1}{\theta_L^2} - \frac{1}{\theta_H^2}\right)\right]\mu(L|H).\end{aligned}$$

Multiplying both sides with $\frac{\theta_L \theta_H}{\theta_H - \theta_L}$ gives

$$\left[-(a_H - \bar{\eta}w) + \eta w\left(\frac{1}{\theta_L} + \frac{1}{\theta_H}\right)\right]\mu(H|H) \geq \left[(a_L - \bar{\eta}w) - \eta w\left(\frac{1}{\theta_L} + \frac{1}{\theta_H}\right)\right]\mu(L|H).$$

Rearranging the terms and using the fact $\mu(H|H) + \mu(L|H) = 1$, we obtain $a_H\mu(H|H) + a_L\mu(L|H) \leq$

$w\left[1+\eta\left(\frac{1}{\theta_L}+\frac{1}{\theta_H}-1\right)\right]$. Following similar steps, we can simply (B.5) as

$$w\left[1+\eta\left(\frac{1}{\theta_L}+\frac{1}{\theta_H}-1\right)\right] \leq a_H\mu(H|L)+a_L\mu(L|L).$$

Putting together the two inequalities completes the proof.

□

Proof of Corollary 1. In (6), the left-hand side $\sum_{m\in M}\mu(m|H)a_m = \bar{a}+(\mu(H|H)-\mu(L|H))\Delta$ and the right-hand side $\sum_{m\in M}\mu(m|L)a_m = \bar{a}+(\mu(H|L)-\mu(L|L))\Delta$. Since condition (6) is satisfied, it must be the case that $\mu(H|H)-\mu(L|H)\leq 0$ and $\mu(H|L)-\mu(L|L)\geq 0$. This implies that the left-hand side of (6) is decreasing in Δ, whereas the right-hand side is increasing in Δ. That is, with anything else fixed, an increase in Δ relaxes condition (6), and this completes the proof. □

Proof of Corollary 2. If $\eta = 0$, then the sufficient and necessary condition (6) for the truth-revealing equilibrium to exist will be reduced to $\sum_{m\in M}\mu(m|H)a_m \leq w \leq \sum_{m\in M}\mu(m|L)a_m$. The latter inequality cannot hold as it violates the assumption that $a_m > \eta w/\theta_t + (1-\eta)w =$ for all $m \in M$ and $t \in T$ (which guarantees the buyer to order a positive quantity from the supplier). Furthermore, since the term $\left[1+\eta\left(\frac{1}{\theta_H}+\frac{1}{\theta_L}-1\right)\right]w$ in (6) is increasing in η, the second part of the corollary follows by the fact that condition (6) holds for $\eta = 1$ provided that $\sum_{m\in M}\mu(m|H)a_m \leq w\sum_{t\in T}(1/\theta_t)$ and that $\sum_{m\in M}\mu(m|H)a_m \leq \sum_{m\in S}\mu(m|L)a_m$ (implied by the MLR property in Lemma 3), but does not for $\eta = 0$. □

Proof of Proposition 3. For part (i), truthful communication improves the buyer's payoff for any market state m, because it enables the buyer to optimize Q conditional on t. That is, without communication, $\Pi^b_{B,m} = \max_Q \sum_{t\in T}\lambda(t|m)\pi_B(Q,t,m)$, whereas with truthful communication, $\Pi^r_{B,m} = \sum_{t\in T}\lambda(t|m)\max_Q \pi_B(Q,t,m)$. Thus, $\Pi^r_{B,m} > \Pi^b_{B,m}$. The inequality holds strictly because π_B is strictly concave in Q and so there is a unique optimal quantity for each t and m. Since the relation holds for any m, it also holds for the buyer's ex ante profit.

For part (ii), letting $v_t = \eta w + (1-\eta)w\theta_t - c$ as type-t supplier's profit margin, we can write the supplier's ex ante profit with truthful communication as

$$\Pi^r_S = p_{LH}Q^r(\theta_L,a_H)v_L + p_{LL}Q^r(\theta_L,a_L)v_L + p_{HH}Q^r(\theta_H,a_H)v_H + p_{HL}Q^r(\theta_H,a_L)v_H,$$

and that without communication as

$$\Pi^b_S = p_{LH}Q^b(a_H)v_L + p_{LL}Q^b(a_L)v_L + p_{HH}Q^b(a_H)v_H + p_{HL}Q^b(a_L)v_H.$$

It follows that $\Pi^r_S > \Pi^b_S$ if and only if

$$v_L\sum_{m\in M}p_{Lm}\left[Q^r(\theta_L,a_m)-Q^b(a_m)\right] > v_H\sum_{m\in M}p_{Hm}\left[Q^b(a_m)-Q^r(\theta_H,a_m)\right]. \quad \text{(B.6)}$$

Invoking (A.1) established in Lemma A.1 in (B.6), we have $\Pi^r_S > \Pi^b_S$ if and only if

$$\left(\sum_{m\in M}p_{Hm}p_{Lm}\Gamma_m\right)\left(\frac{v_L}{\theta_L^2}-\frac{v_H}{\theta_H^2}\right) > 0. \quad \text{(B.7)}$$

Note that $\Gamma_m = \frac{(\theta_H-\theta_L)}{2b(1+\delta^2)}\gamma_m$ by definition, and

$$\frac{v_L}{\theta_L^2}-\frac{v_H}{\theta_H^2} = \frac{(\theta_H^2-\theta_L^2)\left(\eta w+(1-\eta)w\frac{\theta_L\theta_H}{\theta_L+\theta_H}-c\right)}{\theta_L^2\theta_H^2}.$$

Removing the common positive factors in (B.7), and defining $\hat{w} = \eta w + (1-\eta)w\frac{\theta_L\theta_H}{\theta_L+\theta_H}$, we obtain the condition for $\Pi_S^r > \Pi_S^b$ as stated in the part (ii).

For part (iii), writing out the expressions of CS^b and CS^r, we have

$$CS^b = \frac{b(1+\delta^2)}{2}\left(p_{HH}\theta_H^2 Q^b(a_H)^2 + p_{HL}\theta_H^2 Q^b(a_L)^2 + p_{LH}\theta_L^2 Q^b(a_H)^2 + p_{LL}\theta_L^2 Q^b(a_L)^2\right)$$

and

$$CS^r = \frac{b(1+\delta^2)}{2}\left(p_{HH}\theta_H^2 Q^r(\theta_H,a_H)^2 + p_{HL}\theta_H^2 Q^r(\theta_H,a_L)^2 + p_{LH}\theta_L^2 Q^r(\theta_L,a_H)^2 + p_{LL}\theta_L^2 Q^r(\theta_L,a_L)^2\right).$$

By rearranging the terms, we see inequality $CS^b \leq CS^r$ hold if and only if

$$\begin{aligned} &p_{HH}\theta_H^2[Q^b(a_H)^2 - Q^r(\theta_H,a_H)^2] + p_{HL}\theta_H^2[Q^b(a_L)^2 - Q^r(\theta_H,a_L)^2] \\ \leq &p_{LH}\theta_L^2[Q^r(\theta_L,a_H)^2 - Q^b(a_H)^2] + p_{LL}\theta_L^2[Q^r(\theta_L,a_L)^2 - Q^b(a_L)^2] \end{aligned}$$

With the relations established in Lemma A.1, the above can be further simplified to

$$\begin{aligned} &p_{HH}p_{LH}\Gamma_H[Q^b(a_H) + Q^r(\theta_H,a_H)] + p_{HL}p_{LL}\Gamma_L[Q^b(a_L) + Q^r(\theta_H,a_L)] \\ \leq &p_{LH}p_{HH}\Gamma_H[Q^r(\theta_L,a_H) + Q^b(a_H)] + p_{LL}p_{HL}\Gamma_L[Q^r(\theta_L,a_L) + Q^b(a_L)], \end{aligned}$$

or equivalently,

$$p_{HH}p_{LH}\Gamma_H[Q^r(\theta_H,a_H) - Q^r(\theta_L,a_H)] + p_{HL}p_{LL}\Gamma_L[Q^r(\theta_H,a_L) - Q^r(\theta_L,a_L)] \leq 0. \tag{B.8}$$

When a truth-revealing equilibrium exist, $Q^r(\theta_H,a_H) - Q^r(\theta_L,a_H) \leq 0$ and $Q^r(\theta_H,a_L) - Q^r(\theta_L,a_L) \geq 0$ by Lemma 2. At least one of these inequalities must hold strictly because we have assumed $a_H > a_L$ such that the truth-revealing equilibrium (if any) is influential per Lemma 1. Furthermore, by Lemma A.1, $\Gamma_H > 0$ and $\Gamma_L < 0$ given the existence of a truth-revealing equilibrium. Therefore, (B.8) holds strictly and so $CS^b < CS^r$, provided that a truth-revealing equilibrium exists. □

Proof of Lemma 4. Clearly, in any truth-revealing equilibrium, the low-type supplier's payoff is equal to the payoff if her type is revealed. Thus, the NITS criterion is satisfied.

The low-type supplier's expected payoff under the babbling equilibrium is equal to $\Pi_{SL}^b = (\eta w + (1-\eta)w\theta_L - c_L)\sum_{m\in M}\mu(m|L)Q^b(a_m)$, whereas her expected payoff under the truth-revealing equilibrium is given by $\Pi_{SL}^r = (\eta w + (1-\eta)w\theta_L - c_L)\sum_{m\in M}\mu(m|L)Q^r(\theta_L,a_m)$. By definition, a babbling equilibrium will violate the NITS criterion if $\Pi_{SL}^b < \Pi_{SL}^r$, or equivalently,

$$\mu(H|L)Q^b(a_H) + \mu(L|L)Q^b(a_L) < \mu(H|L)Q^r(\theta_L,a_H) + \mu(L|L)Q^r(\theta_L,a_L). \tag{B.9}$$

Rearranging the terms (B.9) yields $p_{LH}\left[Q^b(a_H) - Q^r(\theta_L,a_H)\right] < p_{LL}\left[Q^r(\theta_L,a_L) - Q^b(a_L)\right]$, which can be simplified to $p_{LH}p_{HH}\gamma_H + p_{LL}p_{HL}\gamma_L > 0$ by invoking (A.1) per Lemma A.1 and the relation $\Gamma_m = \frac{\theta_H-\theta_L}{2b(1+\delta^2)}\gamma_m$.

□

Proof of Proposition 4. By Proposition 3(ii) and Lemma 4, the condition under which the supplier's ex ante profit is improved by truthful communication will coincide with that under which the babbling equilibrium fails the NITS refinement if and only if $\eta w + (1-\eta)w\frac{\theta_L\theta_H}{\theta_L+\theta_H} > c$.

It remains to analyze the supply chain's ex ante profit. Given any order quantity Q, supplier type t, and market state m, the supply chain profit is given by $\pi_{SC}(Q,t,m) = (a_m\theta_t - c)Q - b(1+\delta^2)\theta_t^2Q^2$. The ex ante supply chain profits with and without communication can be written respectively as

$$\begin{aligned} \Pi_{SC}^r = \sum_{m\in M}&\{p_{Lm}[(a_m\theta_L - c)Q^r(\theta_L,a_m) - b(1+\delta^2)\theta_L^2Q^r(\theta_L,a_m)^2] \\ &+p_{Hm}[(a_m\theta_H - c)Q^r(\theta_H,a_m) - b(1+\delta^2)\theta_H^2Q^r(\theta_H,a_m)^2]\} \end{aligned}$$

and

$$\Pi^b_{SC} = \sum_{m\in M} \Big\{ p_{Lm}[(a_m\theta_L - c)Q^b(a_m) - b(1+\delta^2)\theta_H^2 Q^b(a_m)^2]$$
$$+p_{Hm}[(a_m\theta_H - c)Q^b(a_m) - b(1+\delta^2)\theta_H^2 Q^b(a_m)^2] \Big\}.$$

Using relations (A.1), we can simplify $\Pi^r_{SC} - \Pi^b_{SC}$ to

$$\Pi^r_{SC} - \Pi^b_{SC} = \sum_{m\in M} p_{Lm}p_{Hm}\Gamma_m A_m$$

where Γ_m is as defined in Lemma A.1 and

$$A_m = \frac{\theta_H - \theta_L}{2\theta_L^2\theta_H^2}\left([a_m + (1-\eta)w]\theta_L\theta_H + \eta w(\theta_L+\theta_H) - 2c(\theta_L+\theta_H)\right).$$

Since $\Gamma_m = \frac{\theta_H-\theta_L}{2b(1+\delta^2)}\gamma_m$, differentiating $\Pi^r_{SC} - \Pi^b_{SC}$ with respect to c yields

$$\frac{\partial(\Pi^r_{SC} - \Pi^b_{SC})}{\partial c} = -\frac{(\theta_H-\theta_L)^2(\theta_H+\theta_L)}{2b(1+\delta^2)\theta_L^2\theta_H^2}\left(\sum_{m\in M}\sum_{m\in M} p_{Lm}p_{Hm}\gamma_m\right).$$

By Lemma 4, when the babbling equilibrium fails the NITS criterion, $\sum_{m\in M}\sum_{m\in M} p_{Lm}p_{Hm}\gamma_m > 0$ and so $\Pi^r_{SC} - \Pi^b_{SC}$ is decreasing in c. Thus, there is $\tilde{w}$ such that $\Pi^r_{SC} < \Pi^b_{SC}$ if and only if $c > \tilde{w}$.

Furthermore, notice that when $c = \frac{[a_H+(1-\eta)w]\theta_L\theta_H}{2(\theta_L+\theta_H)} + \frac{\eta w}{2}$, $A_L < A_H = 0$. Together with the fact $\Gamma_H > 0$ and $\Gamma_L < 0$ provided the existence of truth-revealing equilibrium, it follows that $\Pi^r_{SC} - \Pi^b_{SC} > 0$ at $c = \frac{[a_H+(1-\eta)w]\theta_L\theta_H}{2(\theta_L+\theta_H)} + \frac{\eta w}{2}$. Thus, $\tilde{w} > \frac{[a_H+(1-\eta)w]\theta_L\theta_H + \eta w(\theta_L+\theta_H)}{2(\theta_L+\theta_H)}$. Because $\Gamma_H > 0$ implies $a_H\theta_L\theta_H > (1-\eta)w\theta_L\theta_H + \eta w(\theta_L+\theta_H)$, it follows that

$$\tilde{w} > \frac{[a_H+(1-\eta)w]\theta_L\theta_H + \eta w(\theta_L+\theta_H)}{2(\theta_L+\theta_H)} > \frac{(1-\eta)w\theta_L\theta_H + \eta w(\theta_L+\theta_H)}{\theta_L+\theta_H} = \hat{w}.$$

□

Proof of Proposition 5. For part (i), let $\eta^\dagger = \frac{\sum_{m\in M}\mu(m|H)a_m - w}{(1/\theta_H + 1/\theta_L - 1)w}$. The condition $\sum_{m\in M}\mu(m|H)a_m < w\sum_{t\in T}(1/\theta_t)$ guarantees $\eta^\dagger < 1$. Moreover, $\eta^\dagger > 0$ since by assumption $a_m > \eta w/\theta_t + (1-\eta)w \geq w$ for all m. Thus, $\eta^\dagger \in (0,1)$

Per Proposition 2, $\eta^\dagger$ is the smallest η such that (6) holds. A truth-revealing equilibrium emerges as η increases to point $\eta = \eta^\dagger$. We can further show that at $\eta = \eta^\dagger$, the babbling equilibrium actually fails the NITS refinement, and so the truth-revealing equilibrium must be played out. To see this, note that $\eta^\dagger$ is such that $\sum_{m\in M}\mu(m|H)a_m = [1+\eta^\dagger(\frac{1}{\theta_H}+\frac{1}{\theta_L}-1)]w$. Substituting this relation into the expressions of γ_m, we have

$$\gamma_H = \frac{\mu(L|H)(a_H - a_L)\theta_L\theta_H}{p_{LH}\theta_L^2 + p_{HH}\theta_H^2} \text{ and } \gamma_L = \frac{\mu(H|H)(a_L - a_H)\theta_L\theta_H}{p_{LL}\theta_L^2 + p_{HL}\theta_H^2}.$$

Consequently, by Lemma 4, the babbling equilibrium fails the NITS criterion if and only if

$$\frac{p_{LH}p_{HH}\mu(L|H)(a_H - a_L)\theta_L\theta_H}{p_{LH}\theta_L^2 + p_{HH}\theta_H^2} + \frac{p_{HL}p_{LL}\mu(H|H)(a_L - a_H)\theta_L\theta_H}{p_{LL}\theta_L^2 + p_{HL}\theta_H^2} > 0$$
$$\iff \frac{p_{LH}p_{HH}p_{HL}}{p_{LH}\theta_L^2 + p_{HH}\theta_H^2} - \frac{p_{HL}p_{LL}p_{HH}}{p_{LL}\theta_L^2 + p_{HL}\theta_H^2} > 0 \text{ (by removing common factors)}$$
$$\iff \theta_H^2 p_{HH}p_{HL}(p_{LH}p_{HL} - p_{LL}p_{HH}) > 0 \text{ (rearranging and simplifying the terms)}.$$

The above inequality holds as $p_{LH}p_{HL} - p_{LL}p_{HH} > 0$ is implied by $\frac{\mu(H|H)}{\mu(L|H)} < \frac{\mu(H|L)}{\mu(L|L)}$.

Therefore, based on the NITS refinement, as η increases to point $\eta^\dagger$, the equilibrium must change from the babbling to the truth-revealing equilibrium, which strictly improves the buyer's ex ante profit by Proposition 3.

The proof of part (ii) is similar to that of part (i). By Proposition 2, as w increases to point $w^\dagger = \frac{\sum_{m\in M}\mu(m|H)a_m}{1-\eta+\eta(1/\theta_H+1/\theta_L)}$, a truth-revealing equilibrium emerges. The inequality condition stated in part (ii) guarantees that $a_m\theta_t > \eta w^\dagger + (1-\eta)w^\dagger\theta_t$ for all m and t, i.e., the prerequisite assumption such that the buyer's order quantities are positive. Furthermore, as have been established above, at the boundary condition $\sum_{m\in M}\mu(m|H)a_m = [1+\eta(\frac{1}{\theta_H}+\frac{1}{\theta_L}-1)]w^\dagger$, the babbling equilibrium fails the NITS criterion. Therefore, as w increases to $w^\dagger$, the equilibrium must switch from the babbling to the truth-revealing equilibrium, and by Proposition 3, the buyer is strictly better. □

C Contract Negotiation Before the Cheap Talk Game

Following a standard assumption in the cheap talk literature, we have assumed contract parameters to be exogenous. In this subsection, we allow the contract parameters (w,η) to be determined based on Nash bargaining prior to the cheap talk game. In high-tech manufacturing industries as considered in this paper, outsourcing firms usually do not have many alternative suppliers because of highly specialized production technology. Hence, it is reasonable to assume that neither supply chain member has the power to monopolize contract terms, and so the Nash bargaining approach is suitable for our context.

Specifically, we consider a sequence of events in which the buyer and supplier first negotiate on contract parameters w and η with their prior knowledge of the market potential and yield, and then they obtain updated market and yield information (i.e., observe m and t), respectively, and engage in cheap-talk communication as in our base model.[9] Let $\alpha \in (0,1)$ denote the supplier's relative bargaining power, and assume the outside option of each supply chain member to be of zero value. Note that to be consistent with our base model, we assume that the firms will restrict to the simple contract form (w,η) during negotiation; bargaining within a given form of contracts has been widely considered in the supply chain literature (e.g., Feng and Lu, 2013; Hwang et al., 2018).[10]

In what follows, we will numerically examine the extent to which supply chain firms can benefit from truthful cheap-talk communication if contract parameters result from ex ante negotiation. To this end, let us consider two scenarios: In the first scenario, supply chain firms are unaware of the possibility of truthful communication when negotiating the contract terms (w,η). In the second scenario, knowing that truthful communication is possible, firms negotiate within the subset of (w,η) that induces truthful communication, i.e., satisfies the truth-revealing condition (6).

Scenario 1. Negotiation without Consideration of Truthful Communication. Being unaware of the possible truthful communication, supply chain members negotiate the contract while anticipating that their expected payoffs will be as in the case without communication, i.e., Π_B^b and Π_S^b defined in §4.3. As in Feng and Lu (2013) and Hwang et al. (2018), the contract parameters can be determined by solving

$$\max_{(w,\eta)\in\mathcal{C}} (\Pi_S^b)^\alpha(\Pi_B^b)^{1-\alpha} \tag{C.1}$$

where $\mathcal{C} = \{(w,\eta)|w \geq 0, 0 \leq \eta \leq 1\}$ represents the set of contract parameters. Let Π_S^{b*} and Π_B^{b*} represent the supplier's and the buyer's payoffs, respectively, under the solution to (C.1).

Scenario 2. Negotiation with Consideration of Truthful Communication. Supply chain members may focus on the set of truth-revealing contracts, i.e., $\mathcal{C}^* = \mathcal{C}\bigcap\{(w,\eta)|(6) \text{ is satisfied}\}$, while anticipating the payoffs under the truth-revealing equilibrium, i.e., Π_B^r and Π_S^r as defined in

[9] The assumed sequence of events aligns with many realistic situations. For instance, after a contract is determined, a supplier may need some trial production runs to obtain a better estimation of yield, and the buyer can receive better demand forecast as the selling season approaches. In some other cases, however, contract renegotiation could be allowed after m and t are realized. That will result in a bargaining problem under two-sided asymmetric information, which requires an entirely different modeling approach and is beyond the scope of the present paper.

[10] As discussed in Hwang et al. (2018), Zhou (1997) has shown that the Nash bargaining model extends to the problems with nonconvex payoff sets, thus applicable to the bargaining problems under a given contract form.

§4.3. Moreover, they may agree on a truth-revealing contract only if both of them (weakly) prefer this new contract over the original one in Scenario 1. To incorporate this, we can regard their payoffs Π_S^{b*} and Π_B^{b*} under Scenario 1 as their reservation values in the bargaining game of Scenario 2. Thus, to explore the value of truthful communication, the supply chain members can solve the following problem:

$$\max_{(w,\eta)\in\mathcal{C}^*} (\Pi_S^r - \Pi_S^{b*})^\alpha (\Pi_B^r - \Pi_B^{b*})^{1-\alpha} \text{ subject to } \Pi_S^r \geq \Pi_S^{b*},\ \Pi_B^r \geq \Pi_B^{b*}, \tag{C.2}$$

We can solve problems (C.1) and (C.2) sequentially: Whenever a solution is found to (C.2), then bargaining within the set of truth-revealing contracts will lead to Pareto-improvement on supply chain profits compared to the no-communication scenario. However, (C.2) is not always feasible because the set C^* in (C.2) is more restrictive than $\mathcal{C}$ in (C.1).

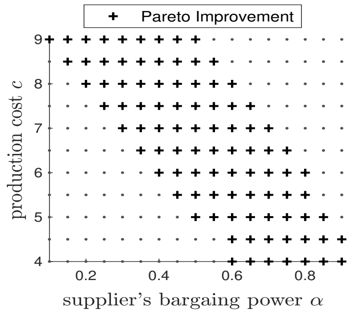

Figure C.1: Impact of truthful communication on contract negotiation

We consider a set of examples where $a_L = 25$, $a_H = 40$, $b = 1$, $\theta_L = 0.5$, $\theta_H = 0.9$, $p_{LL} = p_{HH} = 0.15$, $p_{LH} = p_{HL} = 0.35$ and $\delta = 0.3$. Following the procedure described above, we numerically solve problems (C.1) and (C.2), as the supplier's relative bargaining power α varied between 0.1 to 0.9 with increment of 0.05, and the production cost c varies between 4 and 9 with increment of 0.5. Figure C.1 shows that in many instances supply chain members can be both better off bargaining within the set of truth-revealing contracts, as indicated by the "+" markers.

Table C.1: Representative instances with $c = 6$

	Without Communication				With Communication				
α	w^{b*}	η^{b*}	Π_S^{b*}	Π_B^{b*}	w^{r*}	η^{r*}	Π_S^{r*}	Π_B^{r*}	Pareto-Improved?
0.1	6.86	1.00	12.29	116.90	n/a	n/a	n/a	n/a	0
0.2	7.71	1.00	23.25	104.95	n/a	n/a	n/a	n/a	0
0.3	8.56	1.00	32.87	93.67	n/a	n/a	n/a	n/a	0
0.4	9.42	1.00	41.15	83.08	9.48	1.00	43.24	83.44	1
0.5	10.27	1.00	48.09	73.18	10.82	0.82	48.70	75.48	1
0.6	11.11	1.00	53.70	63.99	12.39	0.65	53.87	66.78	1
0.7	11.95	1.00	57.98	55.52	14.10	0.52	57.98	58.05	1
0.8	12.77	1.00	60.97	47.79	16.26	0.39	60.97	47.96	1
0.9	13.58	1.00	62.69	40.83	n/a	n/a	n/a	n/a	0

Table C.1 reports some representative instances when the production cost $c = 6$. It is observed that supply chain profits are Pareto-improved if the supplier's bargaining power α is between 0.4 and 0.8. Consider the instance with $\alpha = 0.4$. The contract parameters without consideration of truthful communication are $w = 9.42$ and $\eta = 1$, which violates the truth-revealing condition (6) and results in profits of $\Pi_S^{b*} = 41.15$ and $\Pi_B^{b*} = 83.08$. Focusing on truth-revealing contracts, the supply chain members will reach a new contract with $w = 9.48$ and $\eta = 1$, under which the buyer pays a higher wholesale rate. (We can further verify that under this new contract, the babbling equilibrium fails the NITS refinement.) The supply chain profits are Pareto-improved to $\Pi_S^{r*} = 43.24$ and $\Pi_B^{r*} = 83.44$.

For α between 0.5 and 0.8, to benefit from truthful communication, the firms agree to increase w but lower η such that the truth-revealing condition is met. Admittedly, there are some instances in which the supply chain profits cannot be Pareto-improved, i.e., no feasible solution is found to problem (C.2), as indicated by the dots in Figure C.1. This is because forcing the contract to be truth-revealing, despite the informational value, may impact supply chain efficiency as it limits the choice of contract parameters. Our observations suggest that, depending on the supplier's bargaining power and production cost, the efficiency loss can possibly outweigh the informational value. In such cases, supply chain firms may need more complex contract forms if they want to achieve both truthful information sharing and high supply chain efficiency.

Additionally, our observations suggest that without consideration of communication, $\eta = 1$ should be the bargaining outcome, i.e., the buyer should pay for all the units ordered, regardless of how many units are defective. This makes sense because the bargaining outcome should tend to enhance the supply chain efficiency. Since the buyer is the decision maker of the order quantity, letting the buyer bear all the yield risk (i.e., $\eta = 1$) helps align the buyer's incentive with the supply chain's interest. However, when exploring the value of communication, supply chain firms must restrict to the subset of (w, η) satisfying the truth-revealing condition. As a result, we observe $\eta < 1$ in some of the bargaining outcomes under Scenario 2. These observations point to a valuable research question: Whether η is always equal to one, or as high as possible, in any Nash bargaining game between a buyer and a supplier with random yield. To answer such a question, a more in-depth analysis is needed, and let us leave it for future research.

D The Case of Type-Dependent Coefficients of Variation

In the base model, we assumed that the random yield factors ξ_t's have the same coefficient of variation (cv) δ, independent of the type t. We now relax this assumption to allow type-dependent cv, denoted by δ_t. As in the base model, we assume $\theta_H > \theta_L$. For each type t, we define the second moment of the yield factor ξ_t as $\chi_t = \theta_t^2(1 + \delta_t^2)$. We use $Q^r(t, a)$ to denote the buyer's optimal order quantity under symmetric information, as the optimal order quantity depends on both θ_t and χ_t. We present the results of this model extension in §D.1 and the technical proofs in §D.2.

D.1 Results

The following proposition confirms that as long as $\chi_H > \chi_L$, i.e., the high-type yield has a larger second moment than the low type, our main results regarding the existence of truth-revealing equilibrium continue to hold. Note that $\chi_H > \chi_L$ includes the special cases where the two types have the same cv as in the base model, and where the two types have the same variance.

Proposition D.1. *Suppose that the first and second moments of ξ_t satisfy $\theta_L < \theta_H$ and $\chi_L < \chi_H$. A truth-revealing equilibrium exists if and only if $\frac{\theta_H}{\theta_L} < \frac{\chi_H}{\chi_L}$ and*

$$\sum_{m \in M} \mu(m|H) a_m \leq \left[1 + \eta \left(\frac{\chi_H - \chi_L}{\theta_L \chi_H - \theta_H \chi_L} - 1 \right) \right] w \leq \sum_{m \in M} \mu(m|L) a_m. \tag{D.1}$$

Whenever a truth-revealing equilibrium exists, it must be the case that (1) $Q^r(L, a_H) > Q^r(H, a_H)$ and $Q^r(L, a_L) < Q^r(H, a_L)$ and (2) the distribution of market state m has a decreasing MLR with respect to supplier type t.

Proposition D.1 generalizes the truth-revealing condition derived in our base model. The generalized condition includes an inequality (D.1) similar to (6) for the base model, along with an additional requirement on the first two moments, i.e., $\frac{\theta_H}{\theta_L} < \frac{\chi_H}{\chi_L}$. Moreover, the two properties (1) and (2) in Proposition D.1 confirm that the operational and informational drivers as established in the base model remain valid under this generalized case.

Furthermore, we can generalize Lemma A.1 as follows.

Lemma D.1. *Suppose that the first and second moments of ξ_t satisfy $\theta_L < \theta_H$ and $\chi_L < \chi_H$. The buyer's order quantities under the babbling and truth-revealing equilibria satisfy the following properties:*

$$Q^r(L, a_m) - Q^b(a_m) = \frac{p_{Hm}}{\chi_L}\tilde{\Gamma}_m \text{ and } Q^b(a_m) - Q^r(H, a_m) = \frac{p_{Lm}}{\chi_H}\tilde{\Gamma}_m$$

where $\tilde{\Gamma}_m = \frac{(a_m-(1-\eta)w)(\theta_L\chi_H-\theta_H\chi_L)-\eta w(\chi_H-\chi_L)}{2b(p_{Lm}\chi_L+p_{Hm}\chi_H)}$. Moreover, whenever a truth-revealing equilibrium exists, $Q^r(L, a_L) < Q^b(a_L) < Q^r(H, a_L)$ and $Q^r(L, a_H) > Q^b(a_H) > Q^r(H, a_H)$.

Let $\tilde{\Gamma}_m = \frac{(a_m-(1-\eta)w)(\theta_L\chi_H-\theta_H\chi_L)-\eta w(\chi_H-\chi_L)}{2b(p_{Lm}\chi_L+p_{Hm}\chi_H)}$ for all $m \in M$. With this new shorthand notation, we are able to generalize all the results discussed in §4.3 and §4.4 as follows.

Proposition D.2. *Suppose that the first and second moments of ξ_t satisfy $\theta_L < \theta_H$ and $\chi_L < \chi_H$. Compared to the case of no communication, truth communication strictly improves the buyer's ex ante profit and the consumer surplus; it also strictly improves the supplier's ex ante profit if and only if $(\sum_{m\in M} p_{Lm}p_{Hm}\tilde{\Gamma}_m)(\hat{w} - c) > 0$ where $\hat{w} = \eta w + (1-\eta)w(\theta_L\chi_H - \theta_H\chi_L)/(\chi_H - \chi_L)$.*

Proposition D.3. *Suppose that the first and second moments of ξ_t satisfy $\theta_L < \theta_H$ and $\chi_L < \chi_H$. The truth-revealing equilibrium, if it exists, always survives the NITS refinement, but the babbling equilibrium fails the NITS refinement if and only if $\sum_{m\in M} p_{Lm}p_{Hm}\tilde{\Gamma}_m > 0$. Whenever the truth-revealing equilibrium exists while the babbling equilibrium fails the NITS refinement, cheap-talk communication will always make the buyer and the consumer surplus better off, and will also make the supplier better off as long as the supplier's profit margin is not too low, i.e., $c < \hat{w}$, where $\hat{w}$ is as defined in Proposition D.2.*

Proposition D.4. *Suppose that the first and second moments of ξ_t satisfy $\theta_L < \theta_H$ and $\chi_L < \chi_H$, and the decreasing MLR property holds strictly. (i) Provided $\sum_{m\in M} \mu(m|H)a_m < \frac{(\chi_H-\chi_L)w}{(\theta_L\chi_H-\theta_H\chi_L)}$, the buyer's ex ante profit strictly increases with η at some $\eta^\dagger \in (0,1)$; (ii) provided $\sum_{m\in M} \mu(m|H)a_m < \frac{(1-\eta+\eta(\chi_H-\chi_L)/(\theta_L\chi_H-\theta_H\chi_L))a_L}{1-\eta+\eta/\theta_L}$, the buyer's ex ante profit strictly increases with w at some $w = w^\dagger$.*

D.2 Proofs of the Results in §D.1

Proof of Proposition D.1. As in the base model, the buyer's optimal order quantity under symmetric information is given by $Q^r(t, a_m) = \frac{[a_m-(1-\eta)w]\theta_t-\eta w}{2b(1+\delta_t^2)\theta_t^2}$ or $\frac{[a_m-(1-\eta)w]\theta_t-\eta w}{2b\chi_t}$ using the second moment $\chi_t = \theta_t^2(1+\delta_t^2)$. But now both θ and δ are type-dependent. With type-dependent θ_t and δ_t, the IC condition for the H-type can be simplified to

$$\left(\sum_{m\in M} \mu(m|H)a_m - (1-\eta)w\right)(\theta_H\chi_L - \theta_L\chi_H) - \eta w(\chi_L - \chi_H) \geq 0$$

and that for the L-type can be written as

$$\left(\sum_{m\in M} \mu(m|L)a_m - (1-\eta)w\right)(\theta_L\chi_H - \theta_H\chi_L) - \eta w(\chi_H - \chi_L) \geq 0.$$

Altogether, the IC condition can be expressed as

$$\begin{aligned}&\left(\sum_{m\in M} \mu(m|H)a_m - (1-\eta)w\right)(\theta_L\chi_H - \theta_H\chi_L)\\ &\leq \eta w(\chi_H - \chi_L) \leq \left(\sum_{m\in M} \mu(m|L)a_m - (1-\eta)w\right)(\theta_L\chi_H - \theta_H\chi_L)\end{aligned} \tag{D.2}$$

If $\theta_L\chi_H > \theta_H\chi_L$, then the two IC conditions can be expressed as the inequality stated in the proposition by rearranging the terms.

Now we show that $\theta_L\chi_H > \theta_H\chi_L$ is also necessary. Clearly, if $\theta_L\chi_H = \theta_H\chi_L$, (D.2) cannot hold. if $\theta_L\chi_H < \theta_H\chi_L$, then the second inequality of (D.2) entails $\sum_{m\in M}\mu(m|L)a_m - (1-\eta)w < 0$. However, this contradicts with the assumption that the buyer's optimal order quantity under the given contract parameters which requires $a_m > (1-\eta)w$ for all $m \in M$.

As a result, $\theta_L\chi_H > \theta_H\chi_L$ and (D.2) are sufficient and necessary for a truth-revealing equilibrium to exist.

Moreover, for any $m \in M$, $Q^r(t, a_m) - Q^r(t, a_m) = \frac{[a_m-(1-\eta)w](\theta_H\chi_L-\theta_L\chi_H)+\eta w(\chi_H-\chi_L)}{2b\chi_H\chi_L}$. Since $\chi_H > \chi_L$, given that a truth-revealing equilibrium exists and so $\theta_L\chi_H > \theta_H\chi_L$, it is necessary that $Q^r(L, a_H) > Q^r(H, a_H)$ and $Q^r(L, a_L) < Q^r(H, a_L)$. Additionally, (D.2) can hold only if $\sum_{m\in M}\mu(m|H)a_m \le \sum_{m\in M}\mu(m|L)a_m$, thereby implying the decreasing MLR property.

□

Proof of Lemma D.1. Given that the first and second moments of ξ_t are equal to θ_t and χ_t, respectively, conditional on m, the buyer with prior belief $\lambda(t|m)$ will maximize the expected profit below:

$$\Pi_B(Q|m) = \sum_{t\in T}\lambda(t|m)[a_m\theta_t - \eta w - (1-\eta)w\theta_t]Q - b\chi_t Q^2, \tag{D.3}$$

resulting in the following optimal order quantity under the babbling equilibrium:

$$Q^b(a_m) = \frac{(a_m - (1-\eta)w)\left(\sum_{t\in T}\lambda(t|m)\theta_t\right) - \eta w}{2b\sum_{t\in T}\lambda(t|m)\chi_t}.$$

Following similar steps as in the proof of Lemma A.1, we can establish the following relations:

$$Q^r(L, a_m) - Q^b(a_m) = \frac{p_{Hm}}{\chi_L}\tilde{\Gamma}_m \text{ and } Q^b(a_m) - Q^r(H, a_m) = \frac{p_{Lm}}{\chi_H}\tilde{\Gamma}_m$$

where $\tilde{\Gamma}_m$ as defined in the lemma. According to condition (D.1) Proposition D.1, a truth-revealing equilibrium can exist only if $a_L < (1-\eta)w + \eta w\frac{\chi_H-\chi_L}{\theta_L\chi_H-\theta_H\chi_L} < a_H$ and $\theta_L\chi_H - \theta_H\chi_L > 0$. Thus, by rearranging the terms, it follows that provided the existence of a truth-revealing equilibrium, $\tilde{\Gamma}_L < 0 < \tilde{\Gamma}_H$, which implies the relationships among order quantities. □

Proof of Proposition D.2. Clearly, the buyer is better off with truthful communication due to the better informed ordering decision.

For consumer surplus, the expressions of CS^b and CS^r are slightly different than the base-model counterpart since they depend on both the first and second moments of ξ_t. However, they can be written in a similar form as follows:

$$CS^b = \frac{b}{2}\left(p_{HH}\chi_H Q^b(a_H)^2 + p_{HL}\chi_H Q^b(a_L)^2 + p_{LH}\chi_L Q^b(a_H)^2 + p_{LL}\chi_L Q^b(a_L)^2\right)$$

and

$$CS^r = \frac{b}{2}\left(p_{HH}\chi_H Q^r(H, a_H)^2 + p_{HL}\chi_H Q^r(H, a_L)^2 + p_{LH}\chi_L Q^r(L, a_H)^2 + p_{LL}\chi_L Q^r(L, a_L)^2\right).$$

So, $CS^b \le CS^r$ holds if and only if

$$\begin{aligned}&p_{HH}\chi_H[Q^b(a_H)^2 - Q^r(H, a_H)^2] + p_{HL}\chi_H[Q^b(a_L)^2 - Q^r(H, a_L)^2]\\ \le &p_{LH}\chi_L[Q^r(L, a_H)^2 - Q^b(a_H)^2] + p_{LL}\chi_L[Q^r(L, a_L)^2 - Q^b(a_L)^2].\end{aligned}$$

Invoking Lemma D.1, the above can be further simplified to

$$\begin{aligned}&p_{HH}p_{LH}\tilde{\Gamma}_H[Q^b(a_H) + Q^r(H, a_H)] + p_{HL}p_{LL}\tilde{\Gamma}_L[Q^b(a_L) + Q^r(H, a_L)]\\ \le &p_{LH}p_{HH}\tilde{\Gamma}_H[Q^r(L, a_H) + Q^b(a_H)] + p_{LL}p_{HL}\tilde{\Gamma}_L[Q^r(L, a_L) + Q^b(a_L)],\end{aligned}$$

or equivalently,

$$p_{HH}p_{LH}\tilde{\Gamma}_H[Q^r(H,a_H)-Q^r(L,a_H)]+p_{HL}p_{LL}\tilde{\Gamma}_L[Q^r(H,a_L)-Q^r(L,a_L)]\leq 0. \tag{D.4}$$

By Lemma D.1, whenever a truth-revealing equilibrium exists, $\tilde{\Gamma}_H > 0$, $Q^r(H,a_H) < Q^r(L,a_H)$, $\tilde{\Gamma}_L < 0$ and $Q^r(H,a_L) > Q^r(L,a_L)$, and so the above inequality holds strictly, implying $CS^b < CS^r$.

For the supplier, as in the proof of Proposition 3, we can reach the same result: $\Pi_S^r > \Pi_S^b$ if and only if

$$v_L\sum_{m\in M}p_{Lm}\left[Q^r(L,a_m)-Q^b(a_m)\right] > v_H\sum_{m\in M}p_{Hm}\left[Q^b(a_m)-Q^r(H,a_m)\right]. \tag{D.5}$$

Now invoking the relations established in D.1, it follows that $\Pi_S^r > \Pi_S^b$ if and only if

$$\left(\sum_{m\in M}p_{Hm}p_{Lm}\tilde{\Gamma}_m\right)\left(\frac{v_L}{\chi_L}-\frac{v_H}{\chi_H}\right)>0 \tag{D.6}$$

where we note that $\frac{v_L}{\chi_L}-\frac{v_H}{\chi_H}=\frac{(\chi_H-\chi_L)\left(\eta w+(1-\eta)w\frac{\theta_L\chi_H-\theta_H\chi_L}{\chi_H-\chi_L}-c\right)}{\chi_L\chi_H}$. Eliminating the positive factor $(\chi_H-\chi_L)/(\chi_L\chi_H)$ and letting $\hat{w}=\eta w+(1-\eta)w\frac{\theta_L\chi_H-\theta_H\chi_L}{\chi_H-\chi_L}$, we obtain the condition under which $\Pi_S^r > \Pi_S^b$.

□

Proof of Proposition D.3. Analogous to the proof of Lemma 4, a babbling equilibrium violates the NITS if and only if $p_{LH}\left[Q^b(a_H)-Q^r(L,a_H)\right] < p_{LL}\left[Q^r(L,a_L)-Q^b(a_L)\right]$. Now, substituting $Q^r(L,a_H)-Q^b(a_H)=\frac{p_{HH}}{\chi_L}\tilde{\Gamma}_H$ and $Q^r(L,a_L)-Q^b(a_L)=\frac{p_{HL}}{\chi_L}\tilde{\Gamma}_L$ as proved in Lemma D.1, we obtain an equivalent condition $\sum_{m\in M}p_{Lm}p_{Hm}\tilde{\Gamma}_m > 0$.

The rest of the proposition follows by the above condition and Proposition D.3. □

Proof of Proposition D.4. Consider the case of $\frac{\theta_H}{\theta_L} < \frac{\chi_H}{\chi_L}$ such that a truth-revealing equilibrium can possibly exist per Proposition D.1. In this case, $\frac{\chi_H-\chi_L}{\theta_L\chi_H-\theta_H\chi_L} > \frac{\chi_H-\chi_L}{\theta_H\chi_H-\theta_H\chi_L}=\frac{1}{\theta_H}\geq 1$. Hence, $\left[1+\eta\left(\frac{\chi_H-\chi_L}{\theta_L\chi_H-\theta_H\chi_L}-1\right)\right]w$ is increasing in η and w.

Then, the smallest η such that (6) holds is given by $\eta^\dagger=\frac{\sum_{m\in M}\mu(m|H)a_m-w}{\left(\frac{\chi_H-\chi_L}{\theta_L\chi_H-\theta_H\chi_L}-1\right)w}$. We have $\eta^\dagger\in(0,1)$ if and only if $\frac{(\theta_L\chi_H-\theta_H\chi_L)\sum_{m\in M}\mu(m|H)a_m}{\chi_H-\chi_L} < w < \sum_{m\in M}\mu(m|H)a_m$. By assumption that the buyer's order quantity is positive for all m and t, $w<\sum_{m\in M}\mu(m|H)a_m$ is automatically satisfied since the assumption entails $a_m > (1-\eta)w+\eta w/\theta_t > w$. Hence, provided $\sum_{m\in M}\mu(m|H)a_m < \frac{(\chi_H-\chi_L)w}{(\theta_L\chi_H-\theta_H\chi_L)}$, $\eta^\dagger\in(0,1)$.

It remains to verify that at $\eta=\eta^\dagger$, the babbling equilibrium fails the NITS refinement. Substituting $\eta^\dagger=\frac{\sum_{m\in M}\mu(m|H)a_m-w}{\left(\frac{\chi_H-\chi_L}{\theta_L\chi_H-\theta_H\chi_L}-1\right)w}$ into $\tilde{\Gamma}_m$, we have

$$\tilde{\Gamma}_H=\frac{(\theta_L\chi_H-\theta_H\chi_L)\mu(L|H)(a_H-a_L)}{2b(p_{LH}\chi_L+p_{HH}\chi_H)}\text{ and }\tilde{\Gamma}_L=\frac{(\theta_L\chi_H-\theta_H\chi_L)\mu(H|H)(a_L-a_H)}{2b(p_{LL}\chi_L+p_{HL}\chi_H)}.$$

So, by part (ii), the babbling equilibrium fails the NITS refinement if and only if

$$\frac{p_{LH}p_{HH}(\theta_L\chi_H-\theta_H\chi_L)\mu(L|H)(a_H-a_L)}{2b(p_{LH}\chi_L+p_{HH}\chi_H)}-\frac{p_{LL}p_{HL}(\theta_L\chi_H-\theta_H\chi_L)\mu(H|H)(a_H-a_L)}{2b(p_{LL}\chi_L+p_{HL}\chi_H)}>0$$

which can be further simplified to $\frac{p_{LH}}{p_{LH}\chi_L+p_{HH}\chi_H}-\frac{p_{LL}}{p_{LL}\chi_L+p_{HL}\chi_H}>0$. This inequality holds when $\frac{p_{HH}}{p_{LH}}<\frac{p_{HL}}{p_{LL}}$, or equivalently, $\frac{\mu(H|H)}{\mu(L|H)}<\frac{\mu(H|L)}{\mu(L|L)}$, i.e., the decreasing MLR property holds strictly.

Therefore, given a strictly decreasing MLR property and $\sum_{m\in M}\mu(m|H)a_m < \frac{(\chi_H-\chi_L)w}{(\theta_L\chi_H-\theta_H\chi_L)}$, the equilibrium switches from the babbling to truth-revealing equilibrium as eta increases to $\eta^\dagger \in (0,1)$, and by Part (i), the change in equilibrium type makes the buyer better off.

Similarly, we can establish that as w increases to $w^\dagger = \frac{\sum_{m\in M}\mu(m|H)a_m}{1-\eta+\eta\frac{\chi_H-\chi_L}{\theta_L\chi_H-\theta_H\chi_L}}$, the equilibrium switches from the babbling to truth-revealing type so that the buyer is better off. Moreover, the babbling equilibrium fails at $w = w^\dagger$ if the decreasing MLR property is strict, and $w^\dagger$ satisfies the assumption of $a_m\theta_t > \eta w^\dagger + (1-\eta)w^\dagger\theta_t$ for all m and t if $\sum_{m\in M}\mu(m|H)a_m < \frac{\left(1-\eta+\eta\frac{\chi_H-\chi_L}{\theta_L\chi_H-\theta_H\chi_L}\right)a_L}{1-\eta+\eta/\theta_L}$. □

E The Case of Supplier-Optimized Internal Production Lot

Our base model has assumed the supplier's input quantity to be equal to the buyer's order quantity. In this appendix, we allow the supplier to choose an internal production lot size after receiving the buyer's order. The sequence of events remains the same as in our base model except that after receiving an order quantity Q from the buyer, the supplier determines an internal input quantity y leading to an effective output quantity $\xi_t y$. It is reasonable to assume that the buyer will not accept any excess quantity more than he ordered; therefore, the actual delivered quantity is given by $x(Q,\xi) = \min\{\xi_t y, Q\}$. Given any order quantity Q, a type-t supplier determines an optimal production quantity $y^*(Q,t)$ to maximize her expected profit:

$$\pi_S(y|t,Q) = (1-\eta)wE_{\xi_t}\left[\min\{\xi_t y, Q\}\right] + \eta wQ - cy. \tag{E.1}$$

We assume that $(1-\eta)w\theta_t > c$ for all $t \in T$; otherwise, the supplier will produce nothing. We present the results of this model extension in §E.1 and the proofs in §E.2.

E.1 Results

Lemma E.1. *Given any order quantity Q, a type-t supplier's optimal production lot size is given by $y^*(Q,t) = \phi_t^* Q$ where ϕ_t^* is the unique solution to the following equation:*

$$\int_0^{1/\phi_t^*} z dF_t(z) = \frac{c}{(1-\eta)w} \quad \textit{for all } t \in T. \tag{E.2}$$

By Lemma E.1, we can define a modified random yield factor $\zeta_t = \min\{\xi_t\phi_t^*, 1\}$ for each type t such that the quantity successfully delivered equals $x(Q,\xi_t) = \zeta_t Q$ if the buyer orders Q units from a type-t supplier. With this modified yield factor, we can then formulate the cheap talk game in a similar fashion to the base model. Receiving a supplier's message $t' \in T$ and anticipating her optimal internal production lot size, the buyer determines an optimal order quantity $Q^*(t',m)$ to maximize the following expected profit:

$$\Pi_B(Q|t',m) = \sum_{t\in T}\beta(t|t',m)E\left[\zeta_t QP(\zeta_t Q) - \eta wQ - (1-\eta)w\zeta_t Q\right]. \tag{E.3}$$

At the beginning of the interaction with the buyer, the supplier chooses a type to communicate to maximize

$$\Pi_S(t'|t) = \sum_{m\in M}\mu(m|t)\left[(1-\eta)w\hat{\theta}_t + \eta w - c\phi_t^*\right]Q^*(t',m). \tag{E.4}$$

Define $\hat{\theta}_t = E[\zeta_t]$ and $\hat{\chi}_t = E[\zeta_t^2]$ as the first two moments of the modified random yield factor ζ_t. Under a mild assumption that ξ_H is equal in distribution to $(\theta_H/\theta_L)\xi_L$, written as $\xi_H \overset{d}{=} (\theta_H/\theta_L)\xi_L$, we can establish the following lemma.

Lemma E.2. *Let $\hat{\theta}_t = E[\zeta_t]$ and $\hat{\chi}_t = E[\zeta_t^2]$ be the first two moments of $\zeta_t = \min\{\xi_t\phi_t^*, 1\}$. If $\xi_H \overset{d}{=} (\theta_H/\theta_L)\xi_L$, then $\theta_L < \theta_H$ implies $\hat{\theta}_L < \hat{\theta}_H$ and $\hat{\chi}_L < \hat{\chi}_H$.*

We can now generalize our main results using the first two moments of ζ_t.

Proposition E.1. *Suppose that the production lot size is internally determined by the supplier and that $\xi_H \overset{d}{=} (\theta_H/\theta_L)\xi_L$. The following statements hold.*

(i) Under symmetric information, for any given market potential a and any random yield factor ξ, the buyer's optimal order quantity is given by $Q^r = \hat{Q}^r(\hat{\theta}, \hat{\chi}, a) = \frac{[a-(1-\eta)w]\hat{\theta}-\eta w}{2b\chi}$ where $\hat{\theta}$ and $\hat{\chi}$ are the first and second moments of the modified random yield factor $\zeta = \min\{\xi\phi^*, 1\}$, respectively; moreover, Q^r is log-submodular in the market potential a and the mean of the original yield level $\theta = E[\xi]$.

(ii) Under asymmetric information, a truth-revealing equilibrium exists if and only if $\frac{\hat{\theta}_H}{\hat{\theta}_L} < \frac{\hat{\chi}_H}{\hat{\chi}_L}$ and

$$\sum_{m\in M} \mu(m|H)a_m \leq \left[1 + \eta\left(\frac{\hat{\chi}_H - \hat{\chi}_L}{\hat{\theta}_L\hat{\chi}_H - \hat{\theta}_H\hat{\chi}_L} - 1\right)\right] w \leq \sum_{m\in M} \mu(m|L)a_m \tag{E.5}$$

In part (i) of Proposition E.1, we suppress subscript t for the results under symmetric information. Using the relation between the original and modified yield factors characterized in Lemma E.2, we are able to show that the log-supermodularity of the buyer's optimal order size with respect to the original yield level and the market potential still hold. As a result, the operational driver discussed in the base model remains in effect. It is easy to see that the informational driver discussed earlier is still necessary for a truth-revealing equilibrium to emerge. Part (ii) of Proposition E.1 generalizes the truth-revealing condition presented in Proposition 2 for the base model.

Proposition E.2. *Suppose that the production lot size is internally determined by the supplier and that $\xi_H \overset{d}{=} (\theta_H/\theta_L)\xi_L$. When a truth-revealing equilibrium occurs, compared to the case without truthful communication, (i) the buyer is better off; (ii) the supplier is better off if and only only if*

$$\left(\sum_{m\in M} p_{Hm}p_{Lm}\hat{\Gamma}_m\right)(\hat{w} - c(\phi_L^*\hat{\chi}_H - \phi_H^*\hat{\chi}_L)) > 0$$

where $\hat{w} = (\hat{\chi}_H - \hat{\chi}_L)\eta w + (\hat{\theta}_L\hat{\chi}_H - \hat{\theta}_H\hat{\chi}_L)(1-\eta)w$; (iii) the consumer surplus is improved.

Proposition E.3. *Suppose that the production lot size is internally determined by the supplier and that $\xi_H \overset{d}{=} (\theta_H/\theta_L)\xi_L$. A truth-revealing equilibrium, if exists, will survive the NITS refinement, whereas the babbling equilibrium fails if and only if $\sum_{m\in M} p_{Lm}p_{Hm}\hat{\Gamma}_m > 0$. If the truth-revealing equilibrium exists while the babbling equilibrium fails the NITS refinement, then cheap-talk communication will always improve the buyer's ex ante profit and the consumer surplus, and it will also improve the supplier's ex ante profit if $\hat{w} > c(\phi_L^*\hat{\chi}_H - \phi_H^*\hat{\chi}_L)$ where $\hat{w} = (\hat{\chi}_H - \hat{\chi}_L)\eta w + (\hat{\theta}_L\hat{\chi}_H - \hat{\theta}_H\hat{\chi}_L)(1-\eta)w$.*

Note that when the supplier optimally chooses a lot size according to the buyer's order quantity Q, in effect, the supplier's production cost per unit of order quantity Q becomes $c\phi_t^*$. As a result, the condition for a supplier to be better off with cheap talk is more involved: $\hat{w} > c(\phi_L^*\hat{\chi}_H - \phi_H^*\hat{\chi}_L)$. If the factor $\phi_L^*\hat{\chi}_H - \phi_H^*\hat{\chi}_L \leq 0$, the condition always holds; if $\phi_L^*\hat{\chi}_H - \phi_H^*\hat{\chi}_L > 0$, the condition reduces to $c < \hat{w}/(\phi_L^*\hat{\chi}_H - \phi_H^*\hat{\chi}_L)$. Hence, the main finding regarding the supplier's payoff continues to hold, i.e., the supplier is better off with cheap talk as long as her production cost c is low enough.

Proposition E.4. *Suppose that the production lot size is internally determined by the supplier, $\xi_H \overset{d}{=} (\theta_H/\theta_L)\xi_L$, and the decreasing MLR property holds strictly. (i) Provided $\sum_{m\in M} \mu(m|H)a_m < \frac{(\chi_H - \hat{\chi}_L)w}{(\hat{\theta}_L\hat{\chi}_H - \hat{\theta}_H\hat{\chi}_L)}$, the buyer's ex ante profit strictly increases with η at some $\eta^\dagger \in (0,1)$; (ii) provided $\sum_{m\in M} \mu(m|H)a_m < \frac{\left(1-\eta+\eta(\hat{\chi}_H - \hat{\chi}_L)/(\hat{\theta}_L\hat{\chi}_H - \hat{\theta}_H\hat{\chi}_L)\right)a_L}{1-\eta+\eta/\hat{\theta}_L}$, the buyer's ex ante profit strictly increases with w at some $w = w^\dagger$.*

E.2 Proofs of the Results in §E.1

Proof of Lemma E.1. It is straightforward to see that $\pi_S(y|t,Q)$ is concave in y, because $\min\{\xi_t y, Q\}$ is concave in y and the concavity is preserved under expectation. We first argue that we can without loss of generality restrict to the case of $y \geq Q$. Whenever $y < Q$, $\pi_S(y|Q,t) = [(1-\eta)w\theta_t - c]y + \eta w Q$ and the supplier can thus always increase y to improve her expected profit by the assumption of $(1-\eta)w\theta_t \geq c$.

For any $y \geq Q$, it can be verified that $E[\min\{\xi_t y, Q\}] = Q - y\int_0^{Q/y} F_t(z)dz$ and $\frac{\partial E[\min\{\xi_t y, Q\}]}{\partial y} = \int_0^{Q/y} z dF_t(z)$. Hence, we have $\frac{\partial \pi_S(y|t,Q)}{\partial y} = (1-\eta)w\int_0^{Q/y} z dF_t(z) - c$. By concavity, the optimal production lot size y^* is such that the above first-order derivative is equal to zero, and this implies that in the optimal solution $\int_0^{Q/y^*} z dF_t(z) = \frac{c}{(1-\eta)w}$. The lemma is proved by defining a factor $\phi_t^* = y^*/Q$. □

Proof of Lemma E.2. First, we show that $\phi_L^*(\theta_L/\theta_H) < \phi_H^*$ where ϕ_L^* and ϕ_H^* are as defined in Lemma E.1. For any given Q, by the optimality of ϕ_L^*, it follows that

$$y^*(Q,L) = \phi_L^* Q = \arg max_y\ (1-\eta)wE[\min(\xi_L y, Q)] - cy.$$

Given $\xi_H \stackrel{d}{=} (\theta_H/\theta_L)\xi_L$, we have

$$y^*(Q,L) = \phi_L^* Q = \arg max_y\ (1-\eta)wE[\min(\xi_H \frac{\theta_L}{\theta_H} y, Q)] - cy.$$

Define $z^*(Q,L) = \frac{\theta_L}{\theta_H} y^*(Q,L)$. It follows that $z^*(Q,L)$ maximizes

$$z^*(Q,L) = \arg\max_z (1-\eta)wE[\min(\xi_H z, Q)] - (c\theta_H/\theta_L)z \tag{E.6}$$

On the other hand, the optimality of ϕ_H^* implies that

$$y^*(Q,H) = \phi_H^* Q = \arg max_y\ (1-\eta)wE[\min(\xi_H y, Q)] - cy. \tag{E.7}$$

Note that problems (E.6) and (E.7) are both concave maximization, and the difference between the first-order derivatives (with respect to the corresponding decision variables) of their objective functions is given by $(1-\theta_H/\theta_L)c < 0$ whenever $\theta_H > \theta_L$. Therefore, the maximizer of (E.6) is smaller than that of (E.7). It follows that $z^*(Q,L) = \frac{\theta_L}{\theta_H} y^*(Q,L) < y^*(Q,H)$. Since $y^*(Q,t) = \phi_t^* Q$, we have $\phi_L^*(\theta_L/\theta_H) < \phi_H^*$. Consequently, we have

$$\min(\xi_H \frac{\theta_L}{\theta_H} \phi_L^*, 1) \leq \min(\xi_H \phi_H^*, 1) \quad \text{almost surely (a.s.)} \tag{E.8}$$

Recall that by assumption $\zeta_L = \min(\xi_L \phi_L^*, 1) \stackrel{d}{=} \min(\xi_H \frac{\theta_L}{\theta_H} \phi_L^*, 1)$ while $\zeta_H = \min(\xi_H \phi_H^*, 1)$. So, $\zeta_L \leq \zeta_H$ a.s. Taking expectation on both sides of (E.8) yields $E[\zeta_L] \leq E[\zeta_H]$. This inequality holds strictly unless $\min(\xi_H \frac{\theta_L}{\theta_H} \phi_L^*, 1) = \min(\xi_H \phi_H^*, 1)$ a.s., which can happen only if $\xi_H \frac{\theta_L}{\theta_H} \phi_L^* \geq 1$ and $\xi_H \phi_H^* \geq 1$ a.s. In this case, however, the optimality condition of ϕ_H^* (E.2) cannot hold for any positive c. Therefore, $E[\zeta_L] < E[\zeta_H]$, i.e., $\hat{\theta}_L < \hat{\theta}_H$. Similarly, for second moments, we have $E[\zeta_L^2] < E[\zeta_H^2]$, i.e., $\hat{\chi}_L < \hat{\chi}_H$. □

Proof of Proposition E.1. It is straightforward to show that the buyer's optimal order quantity Q^r is determined by the following first-order condition: $\frac{\partial \Pi_B}{\partial Q} = a\theta - 2b\hat{\chi}Q - [\eta w + (1-\eta)w\hat{\theta}] = 0$, which leads to the closed-form expression of $\hat{Q}^r(\hat{\theta}, a, \hat{\sigma})$. To prove the log-submodularity of Q^r, we first show the following lemma.

Lemma E.3. *For any $a_L \leq a_H$, $\hat{Q}^r(\hat{\theta}, \hat{\chi}, a_H)/\hat{Q}^r(\hat{\theta}, \hat{\chi}, a_L)$ is decreasing $\hat{\theta}$ while independent of $\hat{\chi}$.*

Proof. Note that $\frac{\hat{Q}^r(\hat{\theta},\hat{\chi},a_H)}{\hat{Q}^r(\hat{\theta},\hat{\chi},a_L)} = \frac{[a_H-(1-\eta)w]\theta-\eta w}{[a_L-(1-\eta)w]\theta-\eta w}$, which is independent of $\hat{\chi}$. Differentiating $\log \frac{\hat{Q}^r(\hat{\theta},\hat{\chi},a_H)}{\hat{Q}^r(\hat{\theta},\hat{\chi},a_L)}$ yields

$$\frac{\partial}{\partial\hat{\theta}} \log \left[\frac{\hat{Q}^r(\hat{\theta},\hat{\chi},a_H)}{\hat{Q}^r(\hat{\theta},\hat{\chi},a_L)}\right] = \frac{a_H-(1-\eta)w}{[a_H-(1-\eta)w]\hat{\theta}-\eta w} - \frac{a_L-(1-\eta)w}{[a_L-(1-\eta)w]\hat{\theta}-\eta w} \leq 0$$

where the inequality follows as $\frac{z}{z\hat{\theta}-\eta w}$ is a decreasing function of z. □

By Lemma E.3, for any two market potentials $a' \geq a$, $\log \hat{Q}^r(\hat{\theta},\hat{\chi},a') - \log \hat{Q}^r(\hat{\theta},\hat{\chi},a)$ is decreasing in $\hat{\theta}$, and by Lemma E.2 it must be decreasing in the mean of the original yield θ as well. Therefore, $\log Q^r = \log \hat{Q}^r(\hat{\theta},\hat{\chi},a')$ has decreasing differences in (θ, a), and so $Q^r(\theta, a)$ is log-submodular.

The truth-revealing condition under asymmetric information in part (ii) follows by Proposition D.1, because we have established $\hat{\chi}_H > \hat{\chi}_L$. □

Proof of Proposition E.2. The buyer is better off because truthful communication enables him to optimize the order quantity based on different supplier types.

For the supplier, we can first establish the following relationships among the buyer's order quantities under the babbling and truth-revealing equilibria satisfy the following properties:

$$Q^r(\hat{\theta}_L,\hat{\chi}_L,a_m) - Q^b(a_m) = \frac{p_{Hm}}{\hat{\chi}_L}\hat{\Gamma}_m \text{ and } Q^b(a_m) - Q^r(\hat{\theta}_H,\hat{\chi}_H,a_m) = \frac{p_{Lm}}{\hat{\chi}_H}\hat{\Gamma}_m \tag{E.9}$$

where $\hat{\Gamma}_m = \frac{(a_m-(1-\eta)w)(\hat{\theta}_L\hat{\chi}_H-\hat{\theta}_H\hat{\chi}_L)-\eta w(\hat{\chi}_H-\hat{\chi}_L)}{2b(p_{Lm}\hat{\chi}_L+p_{Hm}\hat{\chi}_H)}$. Moreover, whenever a truth-revealing equilibrium exists, $Q^r(\hat{\theta}_L,\hat{\chi}_L,a_L) < Q^b(a_L) < Q^r(\hat{\theta}_H,\hat{\chi}_H,a_L)$ and $Q^r(\hat{\theta}_L,\hat{\chi}_L,a_H) > Q^b(a_H) > Q^r(\hat{\theta}_H,\hat{\chi}_H a_H)$. These properties immediately follow by Lemma D.1, as the optimal quantities take the same expressions as in the case of type-dependent cv except that they are now functions of $\hat{\theta}_t$ and $\hat{\chi}_t$ based on the modified yield factor ζ_t.

When choosing her own production lot size, the supplier's expected profit margin is now given by $v_t = (1-\eta)w\hat{\theta}_t + \eta w - c\phi_t^*$ where ϕ_t^* is determined per Lemma E.1. With relations E.9 and the definition of v_t, analogous to the proof of Proposition D.2, we can show that $\Pi_S^r > \Pi_S^b$ if and only if

$$\left(\sum_{m\in M} p_{Hm}p_{Lm}\hat{\Gamma}_m\right)\left(\frac{v_L}{\hat{\chi}_L} - \frac{v_H}{\hat{\chi}_H}\right) > 0 \tag{E.10}$$

where $\frac{v_L}{\hat{\chi}_L} - \frac{v_H}{\hat{\chi}_H} = \frac{(\hat{\chi}_H-\hat{\chi}_L)\eta w+(\hat{\theta}_L\hat{\chi}_H-\hat{\theta}_H\hat{\chi}_L)(1-\eta)w-c(\phi_L^*\hat{\chi}_H-\phi_H^*\hat{\chi}_L)}{\hat{\chi}_L\hat{\chi}_H}$. Eliminating the common factor $1/(\hat{\chi}_L\hat{\chi}_H)$ leads to the condition stated in the proposition.

For the consumer surplus, the proof follows the same steps as in that of Proposition D.2.

□

Proof of Proposition E.3. The proposition follows by the condition under which the babbling fails the NITS refinement and the impact of truthful communication characterized in Proposition E.2. □

Proof of Proposition E.4. The proof is analogous to that of Proposition D.4. □

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